# **INSTITUTE OF AERONAUTICAL ENGINEERING**

(Autonomous)

Dundigal, Hyderabad-500043

## **AERONAUTICAL ENGINEERING**

#### **TUTORIAL QUESTION BANK**

Course Title	LINEAR ALGEBRA AND CALCULUS						
Course Code	AHSB02	AHSB02					
Programme	B.Tech						
Semester	I AE   CSE   IT	ECE   EEE   M	E   CE				
Course Type	Foundation						
Regulation	IARE - R18						
	r	Гheory		Pract	ical		
<b>Course Structure</b>	Lectures	Tutorials	Credits	Laboratory	Credits		
	3	1	4	-	-		
Chief Coordinator	Ms. P Rajani, Assi	stant Professor	1 1				
Course Faculty	Dr. M Anita, Profe	essor					
	Dr. S Jagadha, Pro	fessor					
	Dr. J Suresh Goud,	, Assistant Profe	ssor				
	Ms. L Indira, Assis	stant Professor					
	Mr. Ch Somasheka	ar, Assistant Prof	fessor				
	Ms. P Srilatha, Ass	sistant Professor					
	Ms. C Rachana, As	ssistant Professo	r				
	Ms. V Subba Laxn	ni, Assistant Pro	fessor				
	Ms. B Praveena, A	ssistant Professo	or				

#### **COURSE OBJECTIVES:**

The cours	The course should enable the students to:					
Ι	Determine rank of a matrix and solve linear differential equations of second order.					
II	Determine the characteristic roots and apply double integrals to evaluate area.					
III	Apply mean value theorems and apply triple integrals to evaluate volume.					
IV	Determine the functional dependence and extremum value of a function					
V	Analyze gradient, divergence, curl and evaluate line, surface, volume integrals over a vector field.					

#### **COURSE OUTCOMES (COs):**

CO 1	Determine rank by reducing the matrix to Echelon and Normal forms. Determine inverse of the matrix by Gauss Jordon Method and Solving Second and higher order differential equations with constant coefficients.
CO 2	Determine a modal matrix, and reducing a matrix to diagonal form. Evaluate inverse and powers of matrices by using Cayley-Hamilton theorem. Evaluate double integral. Utilize the concept of change order of integration and change of variables to evaluate double integrals. Determine the area.
CO 3	Apply the Mean value theorems for the single variable functions. Apply triple integrals to evaluate volume.



CO 4	Determine the maxima and minima for a function of several variable with and without constraints.
CO 5	Analyze scalar and vector fields and compute the gradient, divergence and curl. Evaluate line, surface
	and volume integral of vectors. Use Vector integral theorems to facilitate vector integration.

#### COURSE LEARNING OUTCOMES (CLOs):

#### Students, who complete the course, will have demonstrated the ability to do the following:

AHSB02.01	Demonstrate knowledge of matrix calculation as an elegant and powerful mathematical language in connection with rank of a matrix.
AHSB02.02	Determine rank by reducing the matrix to Echelon and Normal forms.
AHSB02.03	Determine inverse of the matrix by Gauss Jordon Method.
AHSB02.04	Find the complete solution of a non-homogeneous differential equation as a linear combination of the complementary function and a particular solution.
AHSB02.05	Solving Second and higher order differential equations with constant coefficients.
AHSB02.06	Interpret the Eigen values and Eigen vectors of matrix for a linear transformation and use properties of Eigen values
AHSB02.07	Understand the concept of Eigen values in real-world problems of control field where they are pole of closed loop system.
AHSB02.08	Apply the concept of Eigen values in real-world problems of mechanical systems where Eigen values are natural frequency and mode shape.
AHSB02.09	Use the system of linear equations and matrix to determine the dependency and independency.
AHSB02.10	Determine a modal matrix, and reducing a matrix to diagonal form.
AHSB02.11	Evaluate inverse and powers of matrices by using Cayley-Hamilton theorem.
AHSB02.12	Apply double integrals to evaluate area of a given function.
AHSB02.13	Utilize the concept of change order of integration and change of variables to evaluate double integrals.
AHSB02.14	Apply the Mean value theorems for the single variable functions.
AHSB02.15	Apply triple integrals to evaluate volume of a given function.
AHSB02.16	Find partial derivatives numerically and symbolically and use them to analyze and interpret the way a function varies.
AHSB02.17	Understand the techniques of multidimensional change of variables to transform the coordinates by utilizing the Jacobian. Determine Jacobian for the coordinate transformation.
AHSB02.18	Apply maxima and minima for functions of several variable's and Lagrange's method of multipliers.
AHSB02.19	Analyze scalar and vector fields and compute the gradient, divergence and curl.
AHSB02.20	Understand integration of vector function with given initial conditions.
AHSB02.21	Evaluate line, surface and volume integral of vectors.
AHSB02.22	Use Vector integral theorems to facilitate vector integration.

	MODULE - I				
	THEORY OF MATRICES AND LINEAR TRANSFORMATIONS				
S No	Part - A (Short Answer Questions	ons) Blooms Taxonomy Level	Course Outcomes	Course Learning Outcomes (CLOs)	
1	Define Orthogonal matrix.	Remember	CO 1	AHSB02.01	
2	Find the value of k such that the rank of $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2.	Remember	CO 1	AHSB02.01	
3	Prove that $\frac{1}{2} \begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is a unitary matrix.	Understand	CO 1	AHSB02.01	
4	Find the value of k such that rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2	Understand	CO 1	AHSB02.01	
5	Find the Skew-symmetric part of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$ .	Understand	CO 1	AHSB02.01	
6	Define Rank of a matrix and Skew-Hermitian matrix., Unitary matrix.	Remember	CO 1	AHSB02.01	
7	If $A = \begin{bmatrix} 3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix}$ is symmetric, then find the values of a and b.	Understand	CO 1	AHSB02.01	
8	Define orthogonal matrix .Prove that $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.	Understand	CO 1	AHSB02.01	
9	Determine the values of a, b, c when the matrix $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.	Remember	CO 1	AHSB02.01	
10	Express the matrix A as sum of symmetric and Skew-symmetric matrices. where $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$	Understand	CO 1	AHSB02.01	
11	Write the solution of the $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$	Understand	CO 1	AHSB02.04	
12	Write the solution of the $(4D^2-4D+1)y=100$	Understand	CO 1	AHSB02.04	

## TUTORIAL QUESTION BANK

13	Find the particular integral of $\frac{1}{(D^2 - 1)}x$	Understand	CO 1	AHSB02.04
14	$\frac{(D^2 - 1)}{d^3 y} = 0$	Remember	CO 1	AHSB02.04
15	Solve the differential equation $\frac{d^3 y}{dx^3} + y = 0$ Solve the differential equation $(D^2 + a^2)y = 0$	I I a long to a l	CO 1	
15	Solve the differential equation $(D^2 + a^2)y = 0$	Understand	CO 1	AHSB02.04
16	Find the particular value of $\frac{1}{(D-3)}x$ Find the particular integral of $(D^3 - D^2 + 4D - 4)y = e^x$	Understand	CO 1	AHSB02.04
17	Find the particular integral of $(D^3 - D^2 + 4D - 4)y = e^x$	Understand	CO 1	AHSB02.04
18	Solve the differential equation $\frac{1}{(D+1)(D-1)}e^{-x}$	Understand	CO 1	AHSB02.04
19	Solve the differential equation $(D^3 + D)y = 0$	Understand	CO 1	AHSB02.04
20	Solve the differential equation $(D^6 - 64)y = 0$	Remember	CO 1	AHSB02.04
1	Part - B (Long Answer Questi		<b>GO</b> 1	
1	By reducing the matrix $\begin{bmatrix} -1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$ into normal form, find its rank.	Understand	CO 1	AHSB02.02
2	Find the values of a and b such that rank of the matrix	Understand	CO 1	AHSB02.02
	$\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ is 3.			
3		Understand	CO 1	AHSB02.02
5	Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ by reducing to echelon form.	Cinderstand		And Doz.oz
4	Reduce the matrix to its normal form where	Understand	CO 1	AHSB02.03
	$A = \begin{bmatrix} -1 & -3 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$			
5	Find the Inverse of a matrix by using Gauss-Jordan method $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}.$	Understand	CO 1	AHSB02.02

6	Reduce the matrix A to its normal form where	Understand	CO 1	AHSB02.02
	$\begin{bmatrix} 0 & 1 & 2 & -2 \end{bmatrix}$			
	$A = \begin{vmatrix} 4 & 0 & 2 & 6 \end{vmatrix}$ and hence find the rank			
	$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ and hence this die tank			
7		Understand	CO 1	AHSB02.02
/	$\begin{pmatrix} 4 & 4 & -3 & 1 \end{pmatrix}$	Understand	01	АПЗВ02.02
	Find value of K such that the matrix $\begin{vmatrix} 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \end{vmatrix}$ has rank 3			
8	$\begin{pmatrix} 9 & 9 & k & 3 \end{pmatrix}$	Understand	CO 1	AHSB02.02
0		Understand	01	АПЗВ02.02
	Find the rank of the matrix $\begin{vmatrix} 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \end{vmatrix}$ by reducing to normal			
	6 8 7 5			
9	form Find the rank of the matrix, by reducing it to the echelon form	Understand	CO 1	AHSB02.02
9	Find the rank of the matrix, by reducing it to the echeron form $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \end{bmatrix}$	Understand	01	АПЗВ02.02
	$\begin{bmatrix} 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$			
10		Understand	CO 1	AHSB02.02
10	4 0 2 1	Understand	COT	AH3B02.02
	Find the rank of the $A^T$ matrix if A= $\begin{vmatrix} 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{vmatrix}$ by Echelon			
	form.			
11	Solve the differential equation $(D^2 + 1)y = cosecx$ using variation	Understand	CO 1	AHSB02.04
10	of parameter.	I In denote a d	CO 1	
12	Solve the differential equation $D^2(D^2+4)y = 96x^2 + \sin 2x - k$	Understand	01	AHSB02.04
13	Solve the differential equation $(D^2 + 6D + 9)y = sin3x$	Understand	CO 1	AHSB02.04
14	Solve the differential equation $(D^2 + 2D + 1)y = x^2$	Understand	CO 1	AHSB02.04
15	Solve the differential equation $(D^2 + 2D + 1)y = x$ Solve the differential equation	Understand	CO 1	AHSB02.04
	$(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$			
16	Solve the differential equation $(D^2 + 1)y = \sin x \sin 2x + e^x x^2$	Understand	CO 1	AHSB02.04
17	Solve the differential equation $(D^3 + 1)y = 3 + 5e^x$	Understand	CO 1	AHSB02.04
18	Solve the differential equation $(D^2 - 3D + 2)y = \cos hx$	Understand	CO 1	AHSB02.04
19	Solve the differential equation $(D^2 + 4)y = x \cos x$	Understand	CO 1	AHSB02.04
20	Solve the differential equation $(D^2 + 9)y = \cos 3x + \sin 2x$	Understand	CO 1	AHSB02.04
L				1

Part .	C (Problem Solving and Critical Thinking Questions)			
1	Find the Inverse of a matrix by using Gauss-Jordan method	Understand	CO 1	AHSB02.03
-		Chiefistand	001	1110202100
	$A = \begin{vmatrix} 1 & 3 & -3 \end{vmatrix}.$			
	$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$			
2	Find the Inverse of a matrix by using Gauss-Jordan method	Understand	CO 1	AHSB02.03
	$\begin{pmatrix} 2 & 3 & 4 \\ 4 & 2 & 4 \end{pmatrix}$			
	$\begin{pmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}$			
3		Understand	CO 1	AHSB02.02
	Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 0 & -1 \end{bmatrix}$ by Normal form.			
	$\begin{bmatrix} 2 & 3 & 1 & 4 \end{bmatrix}$			
4	$[2 \ 1 \ -3 \ -6]$	Understand	CO 1	AHSB02.03
'	Find the rank of the matrix A= $\begin{bmatrix} 2 & 3 & 1 & 4 \end{bmatrix}$ . by canonical	Chaerbana		1.1.5202.05
5	form	Understand	CO 1	AHSB02.03
5	0 1 2	Onderstand	COT	Alisb02.03
	Find the inverse of A if $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ by elementary row			
	3 1 1			
	operation.			
6	By using method of variation of parameters solve	Understand	CO 1	AHSB02.05
	$y'' + y = x \cos x .$			
7	Solve the differential equation	Understand	CO 1	AHSB02.04
	$(D^{3} - 4 D^{2} - D + 4) y = e^{3x} cos 2x$			
8		Understand	CO 1	AHSB02.05
0	Solve the differential equation $(D^2 + 3D + 2)y = e^{e^x}$ , By using	Understand	01	АПЗВ02.03
	method of variation of parameters			
9	Solve the differential equation	Understand	CO 1	AHSB02.04
10	$(D^{3} - 5D^{2} + 8D - 4)Y = e^{x} + 3e^{-x} + xe^{x}$ Apply the method of variation parameters to solve	TT. 1. 1	00.1	AUGD02.05
10	Apply the method of variation parameters to solve $\binom{1}{2}$	Understand	CO 1	AHSB02.05
	$(D^2 + a^2)y = \tan ax$			
	MODULE-II LINEAR TRANSFORMATIONS AND I		DAIS	
	LINEAR TRANSFORMATIONS AND I Part – A (Short Answer Questi		KALS	
1	State Cayley- Hamilton theorem.	Understand	CO 2	AHSB02.06
2		Understand	CO 2	AHSB02.06
	Find the sum of Eigen values of the matrix 1 3 1			
3	Show that the vectors $X_1=(1,1,2)$ , $X_2=(1,2,5)$ and $X_3=(5,3,4)$ are linearly dependent.	Understand	CO 2	AHSB02.09
4	$\begin{bmatrix} 6 & -2 & 2 \end{bmatrix}$	Remember	CO 2	AHSB02.06
	Find the characteristic equation of the matrix $A = \begin{vmatrix} -2 & 3 & -1 \end{vmatrix}$			

5		Understand	CO 2	AHSB02.06
5	$\begin{pmatrix} 2 & -1 & -1 \end{pmatrix}$	Onderstand	02	A115B02.00
	Find the Eigen values of the matrix $\begin{vmatrix} -1 & 2 & -1 \end{vmatrix}$			
	Find the Eigen values of the matrix $\begin{vmatrix} -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$			
6	Show that the vectors $X_1=(1,1,1)$ $X_2=(3,1,2)$ and $X_3=(2,1,4)$ are linearly independent.	Understand	CO 2	AHSB02.09
7	Define Modal and Spectral matrices.	Understand	CO 2	AHSB02.10
8	Define diaganalisation of a matrix.	Understand	CO 2	AHSB02.10
9	Find the Eigen values of the matrix $A^{-1}$ , $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ $\begin{pmatrix} 1 & 2 & -1 \end{pmatrix}$	Understand	CO 2	AHSB02.06
10	$(1 \ 2 \ -1)$	Understand	CO 2	AHSB02.06
	Find the eigen values $A^3$ of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$			
	$\begin{pmatrix} 2 & -2 & 1 \end{pmatrix}$			
11	Evaluate the double integral $\int_0^2 \int_0^x y dy dx$ .	Remember	CO 2	AHSB02.12
10		The desire 1	<u> </u>	
12	Evaluate the double integral $\int_0^{\pi} \int_0^{a \sin \theta} r dr d\theta$ .	Understand	CO 2	AHSB02.12
13	Evaluate the double integral $\int_0^3 \int_0^1 xy(x + y) dx dy$ .	Understand	CO 2	AHSB02.12
14	Find the value of double integral $\iint xy^2 dx dy$ .	Understand	CO 2	AHSB02.12
15	Evaluate the double integral $\int_{0}^{1} \int_{x}^{x^{2}} xy dx dy$	Understand	CO 2	AHSB02.12
16	Evaluate the double integral $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x+y) dx dy$	Remember	CO 2	AHSB02.12
17	Evaluate the double integral $\int_{0}^{1} \int_{1}^{2} xy dx dy$	Understand	CO 2	AHSB02.12
18	Evaluate the double integral $\int_0^{\infty} \int_0^{\frac{\pi}{2}} e^{-r^2} r  d\theta  dr$ .	Understand	CO 2	AHSB02.12
19	Evaluate the double integral $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r  dr  d\theta$ .	Understand	CO 2	AHSB02.12
20	State the formula to find area of the region using double integration in Cartesian form.	Remember	CO 2	AHSB02.12
	B (Long Answer Questions)	TT. 1 / 1	<u> </u>	
1	6 -2 2	Understand	CO 2	AHSB02.06
	Find the characteristic vectors of the matrix $A = \begin{bmatrix} -2 & 3 & -1 \end{bmatrix}$			
2	Diagonalisation of matrixA= $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 2 & 4 & -1 \end{bmatrix}$	Understand	CO 2	AHSB02.11
3	$\begin{bmatrix} 13 & -4 & 1 \end{bmatrix}$	Understand	CO 2	AHSB02.11
	Show that matrix $\begin{bmatrix} -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfying Cayley-Hamilton	Chiefbuild	202	1115202.11
	theorem and hence find its inverse, if its exists.	Undorstand	<u> </u>	
4	Use Cayley-Hamilton theorem to find $A^3$ , if $A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$	Understand	CO 2	AHSB02.11

5	Find the Eigen values and Eigen vectors of the matrix A and its $\begin{bmatrix} 1 & 3 & 4 \end{bmatrix}$	Understand	CO 2	AHSB02.06
	inverse, where $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$			
6	Find a matrix P such that $P^{-1}AP$ is a diagonal matrix, where A=	Understand	CO 2	AHSB02.10
	$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$			
7	Find the characteristic roots of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and the corresponding characteristic vectors	Understand	CO 2	AHSB02.06
8	corresponding characteristic vectors. Express A <sup>5</sup> -4A <sup>4</sup> -7A <sup>3</sup> +11A <sup>2</sup> -A-10I as a linear polynomial in A, where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$	Understand	CO 2	AHSB02.06
9	Verify Cayley-Hamilton theorem for A= $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and find A <sup>-1</sup> & A <sup>4</sup> .	Understand	CO 2	AHSB02.11
10	Diagonalize the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$ by linear transformation and hence find $A^4$ .	Understand	CO 2	AHSB02.10
11	Evaluate the double integral $\int_{0}^{\pi} \int_{0}^{a(1+\cos\theta)} r^2 \cos\theta dr d\theta.$	Understand	CO 2	AHSB02.12
12	Evaluate the double integral $\int_{0}^{1} \int_{x}^{\sqrt{x}} (x^{2} + y^{2}) dx dy.$	Understand	CO 2	AHSB02.12
13	Evaluate the double integral $\int_{0}^{5} \int_{0}^{x^2} x(x^2 + y^2) dx dy$ .	Understand	CO 2	AHSB02.12
14	Evaluate the double integral $\int_{0}^{5} \int_{0}^{x^{2}} x(x^{2} + y^{2}) dx dy.$ Evaluate the double integral $\int_{0}^{1} \int_{0}^{\frac{\pi}{2}} r \sin \theta d\theta dr.$	Understand	CO 2	AHSB02.12
15	By changing the order of integration evaluate the double integral $\int_0^1 \int_{x^2}^{2-x} xy  dx  dy.$	Understand	CO 2	AHSB02.13
16	By changing the order of integration Evaluate the double integral $\int_{0}^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy dy dx$	Understand	CO 2	AHSB02.13

17		Understand	CO 2	AHSB02.12
	Find the value of $\iint xydxdy$ taken over the positive quadrant of	Chaoistana	002	1110202112
	the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$			
			<u> </u>	
18	Evaluate the double integral using change of variables $\infty \infty$	Understand	CO 2	AHSB02.13
	$\iint e^{-(x^2+y^2)} dx dy.$			
19	0 0	Understand	CO 2	AHSB02.12
19	By transforming into polar coordinates Evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$	Understand	02	Апзво2.12
	over the annular region between the circles $x^2 + y^2 = a^2$ and			
	$x^2 + y^2 = b^2$ with $b > a$ .			
	Find the area of the region bounded by the parabola $y^2 = 4ax$ and	Understand	CO 2	AHSB02.12
Part -	x <sup>2</sup> =4ay. C (Problem Solving and Critical Thinking Questions)			
1	$\begin{bmatrix} i & 0 & 0 \end{bmatrix}$	Understand	CO 2	AHSB02.06
	Find Eigen values and Eigen vectors of $A = \begin{bmatrix} 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$			
2	Examine whether the vectors [2,-1,3,2], [1,3,4,2], [3,5,2,2] is linearly independent or dependent?	Understand	CO 2	AHSB02.07
3	Find Eigen values and corresponding Eigen vectors of the matrix	Understand	CO 2	AHSB02.06
	-1 5 -1			
	$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$			
4		Understand	CO 2	AHSB02.11
-	Verify Cayley-Hamilton theorem for If $A = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix}$	Understand	002	Alisbuz.11
5	[2 1 1]	Understand	CO 2	AHSB02.11
	Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$			
	Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$			
	and find $A^{-1}$ .			
_			~~ -	
6	Evaluate $\iint r^3 dr d\theta$ over the area included between the circles	Understand	CO 2	AHSB02.12
	$r = 2\sin\theta$ and $r = 4\sin\theta$ .			
7	Find the area of the cardioid $r = a(1+\cos\theta)$ .	Understand	CO 2	AHSB02.12
8	Find the area of the region bounded by the curves $y = x^3$ and $y = x$ .	Understand	CO 2	AHSB02.12
9	Evaluate $\iint xydxdy$ taken over the positive quadrant of the	Understand	CO 2	AHSB02.12
	ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .			
10	By changing the order of integration Evaluate the double integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$	Understand	CO 2	AHSB02.13

	MODULE-III				
	FUNCTIONS OF SINGLE VARIABLES AND TRIPLE INTEGRALS				
	Part - A (Short Answer Questi				
1	Discuss the applicability of Rolle's theorem for any function $f(x)$ in interval [a,b].	Understand	CO 3	AHSB02.14	
2	Discuss the applicability of Lagrange's mean value theorem for any function $f(x)$ in interval [a,b].	Understand	CO 3	AHSB02.14	
3	Discuss the applicability of Cauchy's mean value theorem for any function $f(x)$ in interval [a,b].	Understand	CO 3	AHSB02.14	
4	Interpret Rolle's theorem geometrically.	Understand	CO 3	AHSB02.14	
5	Interpret Lagrange's mean value theorem geometrically.	Remember	CO 3	AHSB02.14	
6	Given an example of function that is continuous on [-1, 1] and for which mean value theorem does not hold.	Understand	CO 3	AHSB02.14	
7	Using Lagrange's mean value theorem, find the value of c for $f(x) = \log x$ in (1, e).	Understand	CO 3	AHSB02.14	
8	Explain why mean value theorem does not hold for $f(x) = x^{2/3}$ in [-1,1]	Understand	CO 3	AHSB02.14	
9	Find the region in which $f(x) = 1 - 4x - x^2$ is increasing using mean value theorem.	Understand	CO 3	AHSB02.14	
10	If $f'(x) = 0$ throughout an interval [a, b], using mean value theorem show that $f(x)$ is constant.	Understand	CO 3	AHSB02.14	
11	Find the value of triple integral $\int_{-1}^{1} \int_{-2}^{2} \int_{-3}^{3} dx dy dz$ .	Understand	CO 3	AHSB02.15	
12	Find the volume of the tetrahedron bounded by the coordinate planes and the plane $x+y+z=1$ .	Understand	CO 3	AHSB02.15	
13	State the formula to find volume of the region using triple integration in Cartesian form.	Understand	CO 3	AHSB02.15	
14	Evaluate the triple integral $\int_0^2 \int_1^3 \int_1^2 xy^2 z  dz  dy  dx$	Understand	CO 3	AHSB02.15	
15	Evaluate the triple integral $\int_0^a \int_0^x \int_0^y xyz  dz  dy  dx$	Understand	CO 3	AHSB02.15	
16	Evaluate the triple integral $\int_0^a \int_0^x \int_0^y xyz  dz  dy  dx$ Evaluate the triple integral $\int_0^1 \int_0^2 \int_0^3 (x + y + z)  dz  dy  dx$	Understand	CO 3	AHSB02.15	
17	Evaluate the triple integral $\int_{0}^{1} \int_{0}^{1} \int_{0}^{1} xz  dz  dy  dx$	Understand	CO 3	AHSB02.15	
18	Evaluate the triple integral $\int_0^2 \int_0^1 \int_0^1 xz  dz  dy  dx$ Evaluate the triple integral $\int_{-2}^2 \int_{-3}^3 \int_{-1}^1 e^{x+y+z}  dz  dy  dx$	Understand	CO 3	AHSB02.15	
19	Evaluate the triple integral $\int_0^2 \int_0^3 \int_0^1 dz  dy  dx$	Understand	CO 3	AHSB02.15	
20	Evaluate the triple integral $\int_0^1 \int_{-1}^1 \int_{-1}^1 (x^2 + y^2 + z^2) dz  dy  dx$	Understand	CO 3	AHSB02.15	
Part 1		Understand	CO 3	AHSB02.14	
1	Verify Rolle's theorem for the function $f(x) = e^{-x} \sin x$ in the interval $[0, \pi]$ .	Onderstand	005	AIISD02.14	
2	Show that for any $x > 0, 1 + x < e^x < 1 + xe^x$	Understand	CO 3	AHSB02.14	
3	Verify Lagrange's mean value theorem for	Understand	CO 3	AHSB02.14	
	$f(x) = x^3 - x^2 - 5x + 3$ in the interval [0,4].				
4	If a <b, <math="" prove="" that="">\frac{b-a}{1+b^2} &lt; Tan^{-1}b - Tan^{-1}a &lt; \frac{b-a}{1+a^2} using</b,>	Understand	CO 3	AHSB02.14	
	Lagrange's Mean value theorem and hence deduce the following.				
	(i) $\frac{\pi}{4} + \frac{3}{25} < Tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$				
L	т <i>43 - 5</i> 4 0				

				1
	$\frac{5\pi + 4}{20} < Tan^{-1}2 < \frac{\pi + 2}{4}$			
	(ii) 20 4			
5	Analyze the value of c in the interval [3, 7] for the function	Understand	CO 3	AHSB02.14
	$f(x) = e^x, g(x) = e^{-x}$			
6	Find value of the C using Cauchy's mean value theorem for	Understand	CO 3	AHSB02.14
	$f(x) = \sqrt{x} \& g(x) = \frac{1}{\sqrt{x}}$ in [a,b] where $0 < a < b$			
7	Verify Cauchy's mean value theorem for $f(x) = x^2 \& g(x) = x^3$ in [1,2] and find the value of c.	Understand	CO 3	AHSB02.14
8	Verify Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$ where m, n are positive integers in [a, b].	Understand	CO 3	AHSB02.14
9	Using mean value theorem, for $0 < a < b$ , prove that	Understand	CO 3	AHSB02.14
	$\frac{1-\frac{a}{b} < \log \frac{b}{a} < \frac{b}{a} - 1 \text{ and hence show that } \frac{1}{6} < \log \frac{6}{5} < \frac{1}{5}.$ Find all numbers c between a and b b which satisfies lagranges mean			
10	Find all numbers c between a and b b which satisfies lagranges mean value theorem ,for the following function( $x$ )= ( $x$ -1)( $x$ -2)( $x$ -3) in [0 4]	Understand	CO 3	AHSB02.14
		<b>XX 1 1</b>	<b>GO 3</b>	
11	Evaluate the triple integral $\int_{0}^{1} \int_{0}^{1-z} \int_{0}^{1-y-z} xyz dx dy dz$ .	Understand	CO 3	AHSB02.15
12	Evaluate the triple integral $\int_{0}^{\log 2} \int_{0}^{x} \int_{0}^{x+\log y} e^{x+y+z} dx dy dz$ .	Understand	CO 3	AHSB02.15
13	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ .	Understand	CO 3	AHSB02.15
14	Find the volume of the tetrahedron bounded by the plane	Understand	CO 3	AHSB02.15
	x=0,y=0,z=0; and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes by triple integration.			
15	Using triple integration find the volume of the sphere $x^2+y^2+z^2=a^2$ .	Understand	CO 3	AHSB02.15
16	Evaluate $\iiint_{v} dxdydz$ where v is the finite region of space formed by	Understand	CO 3	AHSB02.15
	the planes $x=0,y=0,z=0$ and $2x+3y+4z=12$ .			
17	Evaluate $\iiint_{R} (x + y + z) dz dy dx$ where R is the region bounded by	Understand	CO 3	AHSB02.15
	the plane $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .			
18	the plane $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ . Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{x^2}{c^2} = 1$	Understand	CO 3	AHSB02.15
19	If R is the region bounded by the planes $x=0, y=0, z=1$ and the cylinder	Understand	CO 3	AHSB02.15
	$x^2 + y^2 = 1$ ,evaluate $\iiint_R xyzdxdydz$ .			
20	Evaluate $\int_{0}^{1} \int_{0}^{1} \int_{\sqrt{x^{2} + y^{2}}}^{2} xyz  dz  dy  dx$	Understand	CO 3	AHSB02.15
Part	- C (Problem Solving and Critical Thinking Questions)			·
1	Verify the hypothesis and conclusion of rolles thorem for the function defined below $f(x)=x^3 - 6x^2 + 11x - 6$ in [13]	Understand	CO 3	AHSB02.14
2	Verify the hypothesis and conclusion of rolles thorem for the function defined below $f(x) = \frac{\log \mathbb{E}[x^2 + ab]}{(a+b)x} in [a b]$	Understand	CO 3	AHSB02.14
				1

	Use lagranges mean value theorem to establish the following inequalities $x \le sin^{-1}x \le \frac{x}{\sqrt{1-x^2}}$ for $0 \le x \le 1$	Understand	CO 3	AHSB02.14
4	Calculate approximately $\sqrt[5]{245}$ by using L.M.V.T.	Understand	CO 3	AHSB02.14
	Verify Cauchy's mean value theorem for $f(x) = x^3 \& g(x) = 2-x$ in [0,9] and find the value of c.	Understand	CO 3	AHSB02.14
06	a sh co	Lin langt og 1	00.2	AUGD02.15
06	Evaluate $\int_0^a \int_0^b \int_0^c (x+y+z) dx dy dz$ .	Understand	CO 3	AHSB02.15
07	Evaluate $\int_{0}^{a} \int_{0}^{b} \int_{0}^{c} (x + y + z) dx dy dz$ . Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{0}^{\sqrt{1-x^{2}-y^{2}}} xyz dx dy dz$	Understand	CO 3	AHSB02.15
	Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z) dx dy dz$	Understand	CO 3	AHSB02.15
	Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ where D is the region bounded by the planes x=0,y=0,z=0,x+y+z=1	Understand	CO 3	AHSB02.15
10	Evaluate $\iiint xyz  dxdydz$ where D is the region bounded by the positive octant of the sphere $x^2+y^2+z^2=a^2$ .	Understand	CO 3	AHSB02.15
	MODULE-IV			
	FUNCTIONS OF SEVERAL VARIABLES AND EXT Part - A (Short Answer Question)		NCTION	
1	If $x = \frac{u^2}{v}$ , $y = \frac{v^2}{v}$ , find the value of $\frac{\partial(u, v)}{\partial(x, y)}$	Understand	CO 4	AHSB02.17
2	The stationary point of the function $f(x, y) = x^{2} + y^{2} + xy + x - 4y + 5$	Understand	CO 4	AHSB02.18
3	If $x = u(1-v)$ , $y = uv$ , find the value of $J'$ .	Understand	CO 4	AHSB02.17
4	Calculate $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ if $u = \frac{2yz}{x}, v = \frac{3zx}{y}w = \frac{4xy}{z}$	Understand	CO 4	AHSB02.17
5	If $x = u(1+v)$ , $y = v(1+u)$ then find the value of $\frac{\partial(x, y)}{\partial(u, v)}$	Understand	CO 4	AHSB02.17
6	Write the condition for the function $f(x,y)$ to be functionally dependent.	Understand	CO 4	AHSB02.17
7	Define jacobian of a function.	Understand	CO 4	AHSB02.17
8	Define a saddle point for the function of $f(x, y)$ .	Understand	CO 4	AHSB02.17
9	Write the condition for the function $f(x,y)$ to be functionally independent	Understand	CO 4	AHSB02.17
10	independent. Define a extreme point for the function of f(x, y).	Understand	CO 4	AHSB02.18
- •	· · · · · · · · · · · · · · · · · · ·			
11	Define <u>Stationary points</u>	Understand	CO 4	AHSB02.18
12	Define maxium function ?	Understand	CO 4	AHSB02.18
13	Define a minimum function?	Understand	CO 4	AHSB02.18
14	If u and v are functions of x and y then prove that $J J' = 1$	Understand	CO 4	AHSB02.18
15	$X = r\cos\theta, Y = r\sin\theta \text{ find J}$	Understand	CO 4	AHSB02.17
16	If $X = \log(x \tan^{-1} y)$ then $f_{xy}$ is equal to zero	Understand	CO 4	AHSB02.16
17	If $f(x,y,z)=0$ then the values $\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\delta z}{\delta x}=-1$	Understand	CO 4	AHSB02.16

18	Prove that if the function u,v,w of three independent variables x,y,z	Understand	CO 4	AHSB02.17
	are not independent ,then the Jacobian of u,v,w w.r.t x,y,z is			
10	always equals to zero.	<b>XX 1</b> . 1	<b>GO</b> 4	
19	If $z = \cos(\frac{x}{y}) + \sin(\frac{x}{y})$ , then Show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial x} = 0$	Understand	CO 4	AHSB02.20
20	1 1	Understand	CO 4	AHSB02.20
Dort	conditions . - B (Long Answer Questions)			
<b>rar</b>	i) If $x = u(1 - v)$ , $y = uv$ then prove that JJ'=1.	Understand	CO 4	AHSB02.18
1	i) If $x + y^2 = u$ , $y + z^2 = v$ , $z + x^2 = w$ find the value of	Chiderstand	001	1115002.10
	$\frac{\partial(x, y, z)}{\partial(u, v, w)}.$			
2	If $u = x^2 - y^2$ , $v = 2xy$ where $x = r\cos\theta$ , $y = r\sin\theta$ then	Understand	CO 4	AHSB02.18
_		Chaelstand	001	1115202.10
	show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$			
	$\partial(r, heta)$			
3	If $x = e^r \sec \theta$ , $y = e^r \tan \theta$ Prove that $\frac{\partial(x, y)}{\partial(r, \theta)} \frac{\partial(r, \theta)}{\partial(x, y)} = 1$ .	Understand	CO 4	AHSB02.18
	$\frac{\partial}{\partial r,\theta} \frac{\partial}{\partial r,y} = c  \text{target revenue}  \frac{\partial}{\partial r,\theta} \frac{\partial}{\partial r,y} = 1.$			
4	If $ux = yz$ , $vy = zx$ , $wz = xy$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	Understand	CO 4	AHSB02.18
5	$u^2 = v^2$	Understand	CO 4	AHSB02.18
	If $x = \frac{u^2}{v}$ , $y = \frac{v^2}{u}$ then find the Jacobian of the function u and v with			
	$V \qquad u$ respect to x and y			
6	Show that the functions	Understand	CO 4	AHSB02.19
	$u = x + y + z, v = x^{2} + y^{2} + z^{2} - 2xy - 2yz - 2xz$ and			
	$w = x^3 + y^3 + z^3 - 3xyz$ are functionally related.			
7		Understand	CO 4	AHSB02.18
	If x = u, y = tanv, z = w then prove that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u \sec^2 v$			
8		Understand	CO 4	AHSB02.18
0	Show that the functions $u = e^x \sin y$ , $v = e^x \cos y$ are not	Chaelstand	001	1115002.10
9	functionally related.	Understand	CO 4	AHSB02.18
	Prove that $u = x + y + z$ , $v = xy + yz + zx$ , $w = x^{2} + y^{2} + z^{2}$ are	Onderstand	04	AII5D02.10
10	functionally dependent. If $u = x + y + z$ , $uv = y + z$ , $z = uvw$	Understand	CO 4	AHSB02.18
10	Prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$	Onderstand	04	AII5D02.18
	$\frac{1}{\partial(u,v,w)} = u v$			
11	Find the maximum value of the function xyz when $x + y + z = a$ .	Understand	CO 4	AHSB02.19
12	Find the maximum value of the function $xyz$ when $x + y + z = a$ . Find the maxima and minima of the function $f(x, y) = x^3y^2$ (1-x-y).	Understand	CO 4	AHSB02.20
13	Find the maximum and minimum of the function	Understand	CO 4	AHSB02.20
	$f(x, y) = \sin x + \sin y + \sin (x + y)$			
14	Find the maximum and minimum values of	Understand	CO 4	AHSB02.20
	$f(x, y) = x^{3} + 3xy^{2} - 3x^{2} - 3y^{2} + 4$			
15	Find the shortest distance from the origin to the surface $xyz^2 = 2$	Understand	CO 4	AHSB02.18
16	Find the minimum value of $x^2 + y^2$ , subjects to the condition	Understand	CO 4	AHSB02.18
	ax+by=c using lagranges multipliers method.			
17	Find the points on the surface $z^2 = xy + 1$ nearest to the origin.	Understand	CO 4	AHSB02.20
18	Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube using lagranges multipliers method	Understand	CO 4	AHSB02.20

19	Find the value of the largest rectangular parallelepiped that can be	Understand	CO 4	AHSB02.20
	inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$			
	$a^2 b^2 c^2$			
20	Find the stationary points of $U(x, y) = \sin x \sin y \sin (x + y)$ where	Understand	CO 4	AHSB02.20
	$0 < x < \pi, 0 < y < \pi$ and find the maximum value of the function U.			
Part	<ul> <li>C (Problem Solving and Critical Thinking)</li> </ul>			
1	If $u = x + 3y^2 + z^3$ , $v = 4x^2yz$ , $w = 2z^2 - xy$ then find	Understand	CO 4	AHSB02.17
	$\frac{\partial(u, v, w)}{\partial(x, y, z)} $ at (1,-1,0).			
	$\partial(x, y, z)$			
2	If $u = e^{xyz}$ , show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$	Understand	CO 4	AHSB02.16
3	If	Understand	CO 4	AHSB02.16
-	$u = \log(x^2 + y^2 + z^2)$ , prove that			
	$(x^{2} + y^{2} + z^{2})\left(\frac{\partial^{2}u}{\partial x^{2}} + \frac{\partial^{2}u}{\partial x^{2}} + \frac{\partial^{2}u}{\partial x^{2}}\right) = 2$			
	$(x + y + 2) \left( \frac{\partial x^2}{\partial x^2} + \frac{\partial x^2}{\partial x^2} + \frac{\partial x^2}{\partial x^2} \right) = 2$			
4	Determine whether the following functions are functionally	Understand	CO 4	AHSB02.17
	dependent or not .If functionally dependent, find the relation			
	between them . $x = \frac{x-y}{x} = \frac{x+z}{x+z}$			
	$u = \frac{x - y}{x + z}, v = \frac{x + z}{y + z}$		~~ (	
5	Determine whether the following functions are functionally	Understand	CO 4	AHSB02.18
	dependent or not .If functionally dependent, find the relation between them .			
	$u = \frac{x+y}{1-xy}, v = tan^{-1}x + tan^{-1}y$			
	$u = \frac{1}{1-xy}, v = tun x + tun y$			
6	Find the maxima value of $\mu = r^2 v^3 z^4$ with the constrain condition	Understand	CO 4	AHSB02.18
6	Find the maxima value of $u = x^2 y^3 z^4$ with the constrain condition 2x + 3y + 4z = a	Understand	CO 4	AHSB02.18
	2x + 3y + 4z = a			
6 7	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the	Understand Understand	CO 4 CO 4	AHSB02.18 AHSB02.20
7	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the origin.	Understand	CO 4	AHSB02.20
	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first,			
7	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum.	Understand Understand	CO 4 CO 4	AHSB02.20 AHSB02.18
7 8	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first,	Understand	CO 4	AHSB02.20
7 8	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. Find three positive numbers whose sum is 100 and whose product is	Understand Understand	CO 4 CO 4	AHSB02.20 AHSB02.18
7 8 9	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. Find three positive numbers whose sum is 100 and whose product is maximum.	Understand Understand Understand	CO 4 CO 4 CO 4	AHSB02.20 AHSB02.18 AHSB02.18
7 8 9	2x + 3y + 4z = a Find the point of the plane $x + 2y + 3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. Find three positive numbers whose sum is 100 and whose product is maximum. A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> . Find the dimensions of the box requiring least material for its construction.	Understand Understand Understand	CO 4 CO 4 CO 4	AHSB02.20 AHSB02.18 AHSB02.18
7 8 9	2x + 3y + 4z = a Find the point of the plane $x + 2y + 3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. Find three positive numbers whose sum is 100 and whose product is maximum. A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> . Find the dimensions of the box requiring least material for its construction. MODULE-V	Understand Understand Understand	CO 4 CO 4 CO 4	AHSB02.20 AHSB02.18 AHSB02.18
7 8 9	2x + 3y + 4z = a Find the point of the plane $x + 2y + 3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. Find three positive numbers whose sum is 100 and whose product is maximum. A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> . Find the dimensions of the box requiring least material for its construction. MODULE-V VECTOR CALCULUS	Understand Understand Understand Understand	CO 4 CO 4 CO 4	AHSB02.20 AHSB02.18 AHSB02.18
7 8 9	2x+3y+4z = a Find the point of the plane $x+2y+3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. Find three positive numbers whose sum is 100 and whose product is maximum. A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> . Find the dimensions of the box requiring least material for its construction. MODULE-V VECTOR CALCULUS Part - A (Short Answer Question)	Understand Understand Understand Understand	CO 4 CO 4 CO 4	AHSB02.20 AHSB02.18 AHSB02.18
7 8 9 10	2x + 3y + 4z = a Find the point of the plane $x + 2y + 3z = 4$ that is closed to the origin. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. Find three positive numbers whose sum is 100 and whose product is maximum. A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> . Find the dimensions of the box requiring least material for its construction. MODULE-V VECTOR CALCULUS	Understand Understand Understand Understand	CO 4 CO 4 CO 4 CO 4 CO 4 CO 5 CO 5	AHSB02.20 AHSB02.18 AHSB02.18 AHSB02.18
7 8 9 10 1 2 3	2x + 3y + 4z = aFind the point of the plane $x + 2y + 3z = 4$ that is closed to the origin.Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum.Find three positive numbers whose sum is 100 and whose product is maximum.A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> .Find the dimensions of the box requiring least material for its construction.MODULE-V VECTOR CALCULUS Part - A (Short Answer Question)Define gradient of scalar point function.Define divergence of vector point function.Define curl of vector point function.	Understand Understand Understand Understand Understand Remember Remember Remember Remember	CO 4 CO 4 CO 4 CO 4 CO 4 CO 5 CO 5 CO 5 CO 5	AHSB02.20 AHSB02.18 AHSB02.18 AHSB02.18 AHSB02.18 AHSB02.19 AHSB02.19 AHSB02.19
7 8 9 10 1 2 3 4	2x + 3y + 4z = aFind the point of the plane $x + 2y + 3z = 4$ that is closed to the origin.Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum.Find three positive numbers whose sum is 100 and whose product is maximum.A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> .Find the dimensions of the box requiring least material for its construction.MODULE-V VECTOR CALCULUSPart - A (Short Answer Questi)Define gradient of scalar point function.Define curl of vector point function.Define curl of vector point function.State Laplacian operator.	Understand Understand Understand Understand Understand Remember Remember Remember Understand	CO 4 CO 4 CO 4 CO 4 CO 4 CO 5 CO 5 CO 5 CO 5 CO 5	AHSB02.20 AHSB02.18 AHSB02.18 AHSB02.18 AHSB02.18 AHSB02.19 AHSB02.19 AHSB02.19 AHSB02.19
7 8 9 10 10 1 2 3 4 5	$2x + 3y + 4z = a$ Find the point of the plane $x + 2y + 3z = 4$ that is closed to the origin.Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum.Find three positive numbers whose sum is 100 and whose product is maximum.A rectangular box open at the top is to have volume of 32 cubic <i>ft</i> .Find the dimensions of the box requiring least material for its construction.MODULE-V VECTOR CALCULUS Part - A (Short Answer Questing Define gradient of scalar point function.Define divergence of vector point function.Define curl of vector point function.State Laplacian operator.Find curl $\overline{f}$ where $\overline{f} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$ .	Understand Understand Understand Understand Understand Remember Remember Remember Understand Understand	CO 4 CO 4 CO 4 CO 4 CO 4 CO 5 CO 5 CO 5 CO 5 CO 5 CO 5	AHSB02.20 AHSB02.18 AHSB02.18 AHSB02.18 AHSB02.18 AHSB02.19 AHSB02.19 AHSB02.19 AHSB02.19 AHSB02.19 AHSB02.19
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10	$\overline{r}$	Understand	CO 5	AHSB02.19
	Show that $\nabla(f(r)) = \frac{\overline{r}}{r} f'(r).$			
11	Prove that $f = yzi + zxj + xyk$ is irrotational vector.	Understand	CO 5	AHSB02.19
12	Show that $(x+3y)i+(y-2z)j+(x-2z)k$ is solenoidal.	Understand	CO 5	AHSB02.20
13	Define work done by a force, circulation.	Understand	<u>CO 5</u>	AHSB02.20
14	State Stokes theorem of transformation between line integral and surface integral.	Understand	CO 5	AHSB02.22
15	Prove that div curl $\bar{f}$ =0 where $\overline{f} = f_1 \bar{i} + f_2 \bar{j} + f_3 \bar{k}$ .	Understand	CO 5	AHSB02.20
16	Define line integral on vector point function.	Remember	CO 5	AHSB02.21
17	Define surface integral of vector point function $\overline{F}$ .	Remember	CO 5	AHSB02.21
18	Define volume integral on closed surface S of volume V.	Remember	CO 5	AHSB02.21
19	State Green's theorem of transformation between line integral and double integral.	Understand	CO 5	AHSB02.22
20	State Gauss divergence theorem of transformation between surface integral and volume integral.	Understand	CO 5	AHSB02.22
	Part - B (Long Answer Question			·
1	Evaluate $\int_{C} \overline{f} \cdot d\overline{r}$ where $\overline{f} = 3xyi - y^2j$ and C is the parabola $y=2x^2$	Understand	CO 5	AHSB02.21
	from points (0, 0) to (1, 2).			
2	from points (0, 0) to (1, 2). Evaluate $\iint_{S} \overline{F} \cdot d\overline{s}$ if $\overline{F} = yzi + 2y^2j + xz^2k$ and S is the Surface of	Understand	CO 5	AHSB02.21
	the cylinder $x^2+y^2=9$ contained in the first octant between the planes z = 0 and z = 2.			
3	Find the work done in moving a particle in the force field	Understand	CO 5	AHSB02.21
	$\overline{F} = (3x^2)i + (2zx - y)j + zk$ along the straight line from(0,0,0)			
4	to (2,1,3). Find the circulation of	Understand	CO 5	AHSB02.21
4	$\overline{F} = (2x - y + 2z)\overline{i} + (x + y - z)\overline{j} + (3x - 2y - 5z)\overline{k}$ along the	Understand	05	Ansb02.21
	circle $x^2 + y^2 = 4$ in the xy plane.			
5	Verify Gauss divergence theorem for the vector point function $\sum_{i=1}^{n} \left( \frac{3}{2} \right)^{i} = 2 $ is 2 have the base holds.	Understand	CO 5	AHSB02.22
	$F = (x^3-yz)i - 2yxj + 2zk$ over the cube bounded by $x = y = z = 0$ and $x = y = z = a$ .			
6	Verify Gauss divergence theorem for $2x^2yi - y^2j + 4xz^2k$ taken over	Understand	CO 5	AHSB02.22
	the region of first octant of the cylinder $y^2 + z^2 = 9$ and $x = 2$ .			
7	Verify Green's theorem in the plane for	Understand	CO 5	AHSB02.22
	$\int_{C} (x^2 - xy^3) dx + (y^2 - 2xy) dy$ where C is a square with vertices			
	(0,0),(2,0),(2,2),(0,2).			
8	Applying Green's theorem evaluate $\iint_{x \to y} (y - \sin x) dx + \cos x dy$ where C	Understand	CO 5	AHSB02.22
	is the plane triangle enclosed by $y = 0$ , $y = \frac{2x}{\pi}$ , and $x = \frac{\pi}{2}$ .			
9	Apply Green's Theorem in the plane for	Understand	CO 5	AHSB02.22
	$\int (2x^2 - y^2) dx + (x^2 + y^2) dy$ where C is a is the boundary of the area			
	enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$ .			

10	Verify Stokes theorem for $f = (2x - y)i - yz^2 j - y^2 zk$ where S	Understand	CO 5	AHSB02.22
	is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by			
	the projection of the xy plane.			
11	Verify Stokes theorem for $\overline{f} = (x^2 - y^2)\overline{i} + 2xy\overline{j}$ over the box bounded	Understand	CO 5	AHSB02.24
12	by the planes x=0, x=a, y=0,y=b.	Lin densten d	CO 5	AUSD02.21
12	Find the directional derivative of the function $\phi = xy^2 + yz^3$ at the	Understand	CO 5	AHSB02.21
	point P(1,-2,-1) in the direction to the surface $x \log z - y^2 = -4 at (-1,2,1)$			
13	$\frac{-4 \text{ at } (-1,2,1)}{\text{ If } \overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k} \text{ evaluate } \int \overline{F}.\overline{n}ds \text{ where S is the surface of } $	Understand	CO 5	AHSB02.20
	s			
	the cube x = 0, x = a, y = 0, y = a, z = 0, z = a. If $\overline{f} = (5xy - 6x^2)\overline{i} + (2y - 4x)\overline{j}$ evaluate $\int_{0}^{1} \overline{f} \cdot d\overline{r}$ along the			
14	If $f = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ evaluate $\int_{0}^{1} \vec{f} \cdot d\vec{r}$ along the	Understand	CO 5	AHSB02.21
	curve C in xy-plane $y = x^3$ from (1,1) to (2,8).			
15	curve C in xy-plane $y = x^3$ from (1,1) to (2,8). Evaluate the line integral $\int (x^2 + xy)dx + (x^2 + y^{2})dy$ where C is	Understand	CO 5	AHSB02.21
	the square formed by lines $x = \pm 1$ , $y = \pm 1$ .			
16	If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ show that $\nabla r^n = nr^{n-2}\vec{r}$ .	Understand	CO 5	AHSB02.19
17	Evaluate by Stokes theorem $\int (e^x dx + 2y dy - dz)$ where c is the	Understand	CO 5	AHSB02.22
	$\int_{c} \frac{1}{c} \frac{dx + 2y dy}{dz} \frac{dz}{dz} \frac{dz}{dz}$			
	curve $x^2+y^2=9$ and $z=2$ .			
18		Understand	CO 5	AHSB02.22
10	Verify Stokes theorem for the function $x^2 i + xy j$ integrated round the	Understand	05	Alisbuz.zz
	square in the plane $z=0$ whose sides are along the line $x=0,y=0,x=a,$ y=a.			
19	Evaluate by Stokes theorem $\int (x+y)dx + (2x-z)dy + (y+z)dz$	Understand	CO 5	AHSB02.22
	C C			
	where C is the boundary of the triangle with vertices $(0,0,0),(1,0,0),(1,1,0)$ .			
20	Verify Green's theorem in the plane for	Understand	CO 5	AHSB02.22
	$\int (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is a region bounded by			
	$y = \sqrt{x}$ and $y = x^2$ .			
	Part – C (Problem Solving and Critica	U,		
1	Verify Gauss divergence theorem for $\bar{f} = x^2 \bar{i} + y^2 \bar{j} + z^2 \bar{k}$ taken	Understand	CO 5	AHSB02.22
	over the cube bounded by x=0,x=a, y=0,y=b, z=0,z=c.			
2	Find the work done in moving a particle in the force field $$	Understand	CO 5	AHSB02.21
	$\overline{F} = (3x^2)i + (2zx - y)j + zk$ along the curve defined by			
	$x^2 = 4y, 3x^3 = 8z$ from x=0 and x=2.			
3	Show that the force field given by	Understand	CO 5	AHSB02.20
3	Show that the force field given by $\overline{F} = 2xyz^3i + x^2z^3j + 3x^2yz^2k$ is conservative. Find the work	Understand	005	ANSDU2.20
	$F = 2xyz^{-1} + x^{-}z^{-}J + 3x^{-}yz^{-}k$ is conservative. Find the work done in moving a particle from (1,-1,2) to (3,2,-1) in this force			
	field.			
4	Show that the vector $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is	Understand	CO 5	AHSB02.21
	irrotational and find its scalar potential function.			

5	Using Gauss divergence theorem evaluate $\iint_{s} \vec{F}.d\bar{s}$ , for the $\vec{F} = y\vec{i} + x\vec{j} + z^{2}\vec{k}$ for the cylinder region S given by $x^{2} + y^{2} = a^{2}, z = 0$ and $z = b$ .	Understand	CO 5	AHSB02.22
6	Find the directional derivative of $\phi(x, y, z) = x^2 yz + 4xz^2$ at the point(1,-2,-1) in the direction of the normal to the surface f(x, y, z) = x logz - y <sup>2</sup> at (-1,2,1).	Understand	CO 5	AHSB02.20
7	Using Green's theorem in the plane evaluate $\int_{c} (2xy - x^{2})dx + (x^{2} + y^{2})dy$ where C is the region bounded by $y = x^{2}$ and $y^{2} = x$ .	Understand	CO 5	AHSB02.22
8	Applying Green's theorem evaluate $\int_{c} (xy + y^2) dx + x^2 dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$ .	Understand	CO 5	AHSB02.22
9	Verify Green's Theorem in the plane for $\int_{C} (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is the region bounded by x=0, y=0 and x + y=1.	Understand	CO 5	AHSB02.22
10	Verify Stokes theorem for $\overline{F} = (y - z + 2)i + (yz + 4)j - xzk$ where S is the surface of the cube x=0, y=0, z=0 and x=2,y=2,z=2 above the xy-plane.	Understand	CO 5	AHSB02.22

**Prepared by:** Ms. P Rajani, Assistant Professor

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