INFORMATION TECHNOLOGY
TUTORIAL QUESTION BANK

| Course Title | LINEAR ALGEBRA AND CALCULUS |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Course Code | AHSB02 |  |  |  |  |
| Programme | B.Tech |  |  |  |  |
| Semester | AE \| CSE | IT | ECE | EEE | ME | CE |  |  |  |  |
| Course Type | Foundation |  |  |  |  |
| Regulation | IARE - R18 |  |  |  |  |
| Course Structure | Theory |  |  | Practical |  |
|  | Lectures | Tutorials | Credits | Laboratory | Credits |
|  | 3 | 1 | 4 | - | - |
| Chief Coordinator | Ms. P Rajani, Assistant Professor |  |  |  |  |
| Course Faculty | Dr. M Anita, Professor <br> Dr. S Jagadha, Professor <br> Dr. J Suresh Goud, Assistant Professor <br> Ms. L Indira, Assistant Professor <br> Mr. Ch Somashekar, Assistant Professor <br> Ms. P Srilatha, Assistant Professor <br> Ms. C Rachana, Assistant Professor <br> Ms. V Subba Laxmi, Assistant Professor <br> Ms. B Praveena, Assistant Professor |  |  |  |  |

## COURSE OBJECTIVES:

| The course should enable the students to: |  |
| :---: | :--- |
| I | Determine rank of a matrix and solve linear differential equations of second order. |
| II | Determine the characteristic roots and apply double integrals to evaluate area. |
| III | Apply mean value theorems and apply triple integrals to evaluate volume. |
| IV | Determine the functional dependence and extremum value of a function |
| V | Analyze gradient, divergence, curl and evaluate line, surface, volume integrals over a vector field. |

## COURSE OUTCOMES (COs):

| CO 1 | Determine rank by reducing the matrix to Echelon and Normal forms. Determine inverse of the <br> matrix by Gauss Jordon Method and Solving Second and higher order differential equations with <br> constant coefficients. |
| :---: | :--- |
| CO 2 | Determine a modal matrix, and reducing a matrix to diagonal form. Evaluate inverse and powers of <br> matrices by using Cayley-Hamiltontheorem. Evaluate double integral. Utilize the concept of change <br> order of integration and change of variables to evaluate double integrals. Determine the area. |
| CO 3 | Apply the Mean value theorems for the single variable functions. <br> Apply triple integrals to evaluate volume. |


| CO 4 | Determine the maxima and minima for a function of several variable with and without constraints. |
| :---: | :--- |
| CO 5 | Analyze scalar and vector fields and compute the gradient, divergence and curl. Evaluate line, surface <br> and volume integral of vectors. Use Vector integral theorems to facilitate vector integration. |

COURSE LEARNING OUTCOMES (CLOs):
Students, who complete the course, will have demonstrated the ability to do the following:

| AHSB02.01 | Demonstrate knowledge of matrix calculation as an elegant and powerful mathematical <br> language in connection with rank of a matrix. |
| :---: | :--- |
| AHSB02.02 | Determine rank by reducing the matrix to Echelon and Normal forms. |
| AHSB02.03 | Determine inverse of the matrix by Gauss Jordon Method. |
| AHSB02.04 | Find the complete solution of a non-homogeneous differential equation as a linear combination <br> of the complementary function and a particular solution. |
| AHSB02.05 | Solving Second and higher order differential equations with constant coefficients. |
| AHSB02.06 | Interpret the Eigen values and Eigen vectors of matrix for a linear transformation and use <br> properties of Eigen values |
| AHSB02.07 | Understand the concept of Eigen values in real-world problems of control field where they are <br> pole of closed loop system. |
| AHSB02.08 | Apply the concept of Eigen values in real-world problems of mechanical systems where Eigen <br> values are natural frequency and mode shape. |
| AHSB02.09 | Use the system of linear equations and matrix to determine the dependency and independency. |
| AHSB02.10 | Determine a modal matrix, and reducing a matrix to diagonal form. |
| AHSB02.11 | Evaluate inverse and powers of matrices by using Cayley-Hamiltontheorem. |
| AHSB02.12 | Apply double integrals to evaluate area of a given function. |
| AHSB02.13 | Utilize the concept of change order of integration and change of variables to evaluate double <br> integrals. |
| AHSB02.14 | Apply the Mean value theorems for the single variable functions. |
| AHSB02.15 | Apply triple integrals to evaluate volume of a given function. |
| AHSB02.16 | Find partial derivatives numerically and symbolically and use them to analyze and interpret the <br> way a function varies. |
| AHSB02.17 | Understand the techniques of multidimensional change of variables to transform the coordinates <br> by utilizing the Jacobian. Determine Jacobian for the coordinate transformation. |
| AHSB02.18 | Apply maxima and minima for functions of several variable's and Lagrange's method of <br> multipliers. |
| AHSB02.19 | Analyze scalar and vector fields and compute the gradient, divergence and curl. |
| AHSB02.20 | Understand integration of vector function with given initial conditions. |
| AHSB02.21 | Evaluate line, surface and volume integral of vectors. |
| AHSB02.22 | Use Vector integral theorems to facilitate vector integration. |

## TUTORIAL QUESTION BANK

## MODULE - I

| MODULE - I |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| THEORY OF MATRICES AND LINEAR TRANSFORMATIONS |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |
| S No | Questions | $\begin{gathered} \text { Blooms } \\ \text { Taxonomy } \\ \text { Level } \end{gathered}$ | Course Outcomes | Course <br> Learning <br> Outcomes <br> (CLOs) |
| 1 | Define Orthogonal matrix. | Remember | CO 1 | AHSB02.01 |
| 2 | Find the value of k such that the rank of $\left[\begin{array}{cccc}1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1\end{array}\right]$ is 2. | Remember | CO 1 | AHSB02.01 |
| 3 | Prove that $\frac{1}{2}\left[\begin{array}{cc}1+i & -1+i \\ 1+i & 1-i\end{array}\right]$ is a unitary matrix. | Understand | CO 1 | AHSB02.01 |
| 4 | Find the value of k such that rank of $\left[\begin{array}{ccc}1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10\end{array}\right]$ is 2 | Understand | CO 1 | AHSB02.01 |
| 5 | Find the Skew-symmetric part of the matrix $\left[\begin{array}{ccc}1 & 1 & 2 \\ -1 & 1 & 1 \\ 3 & -1 & 2\end{array}\right]$. | Understand | CO 1 | AHSB02.01 |
| 6 | Define Rank of a matrix and Skew-Hermitian matrix., Unitary matrix. | Remember | CO 1 | AHSB02.01 |
| 7 | If $A=\left[\begin{array}{ccc}3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5\end{array}\right]$ is symmetric, then find the values of $a$ and <br> b. | Understand | CO 1 | AHSB02.01 |
| 8 | Define orthogonal matrix .Prove that $\mathrm{A}=\left[\begin{array}{cc}\cos \theta & \sin \theta \\ -\sin \theta & \cos \theta\end{array}\right]$ is orthogonal. | Understand | CO 1 | AHSB02.01 |
| 9 | Determine the values of $\mathrm{a}, \mathrm{b}, \mathrm{c}$ when the matrix $\left[\begin{array}{ccc}0 & 2 b & c \\ a & b & -c \\ a & -b & c\end{array}\right]$ is orthogonal. | Remember | CO 1 | AHSB02.01 |
| 10 | Express the matrix A as sum of symmetric and Skew-symmetric matrices. where $A=\left[\begin{array}{ccc}3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0\end{array}\right]$ | Understand | CO 1 | AHSB02.01 |
| 11 | Write the solution of the $\frac{d^{3} y}{d x^{3}}-3 \frac{d y}{d x}+2 y=0$ | Understand | CO 1 | AHSB02.04 |
| 12 | Write the solution of the ( $\left.4 \mathrm{D}^{2}-4 \mathrm{D}+1\right) \mathrm{y}=100$ | Understand | CO 1 | AHSB02.04 |


| 13 | Find the particular integral of $\frac{1}{\left(D^{2}-1\right)} x$ | Understand | CO 1 | AHSB02.04 |
| :---: | :---: | :---: | :---: | :---: |
| 14 | Solve the differential equation $\frac{d^{3} y}{d x^{3}}+y=0$ | Remember | CO 1 | AHSB02.04 |
| 15 | Solve the differential equation $\left(D^{2}+a^{2}\right) y=0$ | Understand | CO 1 | AHSB02.04 |
| 16 | Find the particular value of $\frac{1}{(D-3)} x$ | Understand | CO 1 | AHSB02.04 |
| 17 | Find the particular integral of $\left(D^{3}-D^{2}+4 D-4\right) y=e^{x}$ | Understand | CO 1 | AHSB02.04 |
| 18 | Solve the differential equation $\frac{1}{(D+1)(D-1)} e^{-x}$ | Understand | CO 1 | AHSB02.04 |
| 19 | Solve the differential equation $\left(D^{3}+D\right) y=0$ | Understand | CO 1 | AHSB02.04 |
| 20 | Solve the differential equation $\left(D^{6}-64\right) y=0$ | Remember | CO 1 | AHSB02.04 |
| Part - B (Long Answer Questions) |  |  |  |  |
| 1 | By reducing the matrix $\left[\begin{array}{ccc}-1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3\end{array}\right]$ into normal form, find its rank. | Understand | CO 1 | AHSB02.02 |
| 2 | Find the values of $a$ and $b$ such that rank of the matrix $\left[\begin{array}{cccc}1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b\end{array}\right]$ is 3 . | Understand | CO 1 | AHSB02.02 |
| 3 | Find the rank of the matrix $\mathrm{A}=\left[\begin{array}{cccc}2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1\end{array}\right]$ by reducing to echelon form. | Understand | CO 1 | AHSB02.02 |
| 4 | Reduce the matrix to its normal form where $A=\left[\begin{array}{cccc} -1 & -3 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{array}\right]$ | Understand | CO 1 | AHSB02.03 |
| 5 | Find the Inverse of a matrix by using Gauss-Jordan method $A=\left[\begin{array}{cccc} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{array}\right]$ | Understand | CO 1 | AHSB02.02 |


| 6 | Reduce the matrix A to its normal form where $A=\left[\begin{array}{cccc}0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1\end{array}\right]$ and hence find the rank | Understand | CO 1 | AHSB02.02 |
| :---: | :---: | :---: | :---: | :---: |
| 7 | Find value of K such that the matrix $\left(\begin{array}{cccc}4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3\end{array}\right)$ has rank 3 | Understand | CO 1 | AHSB02.02 |
| 8 | Find the rank of the matrix $\left[\begin{array}{llll}1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5\end{array}\right]$ by reducing to normal | Understand | CO 1 | AHSB02.02 |
| 9 | Find the rank of the matrix, by reducing it to the echelon form $\left[\begin{array}{ccccc} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{array}\right]$ | Understand | CO 1 | AHSB02.02 |
| 10 | Find the rank of the $A^{T}$ matrix if $\mathrm{A}=\left[\begin{array}{llll}4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4\end{array}\right]$ by Echelon form. | Understand | CO 1 | AHSB02.02 |
| 11 | Solve the differential equation $\left(D^{2}+1\right) y=\operatorname{cosec} x$ using variation of parameter. | Understand | CO 1 | AHSB02.04 |
| 12 | Solve the differential equation $D^{2}\left(D^{2}+4\right) y=96 x^{2}+\sin 2 x-k$ | Understand | CO 1 | AHSB02.04 |
| 13 | Solve the differential equation $\left(D^{2}+6 D+9\right) y=\sin 3 x$ | Understand | CO 1 | AHSB02.04 |
| 14 | Solve the differential equation $\left(D^{2}+2 D+1\right) y=x^{2}$ | Understand | CO 1 | AHSB02.04 |
| 15 | Solve the differential equation $\left(D^{3}-6 D^{2}+11 D-6\right) y=e^{-2 x}+e^{-3 x}$ | Understand | CO 1 | AHSB02.04 |
| 16 | Solve the differential equation $\left(D^{2}+1\right) y=\sin x \sin 2 x+e^{x} x^{2}$ | Understand | CO 1 | AHSB02.04 |
| 17 | Solve the differential equation $\left(D^{3}+1\right) y=3+5 e^{x}$ | Understand | CO 1 | AHSB02.04 |
| 18 | Solve the differential equation $\left(D^{2}-3 D+2\right) y=\cos h x$ | Understand | CO 1 | AHSB02.04 |
| 19 | Solve the differential equation $\left(D^{2}+4\right) y=x \cos x$ | Understand | CO 1 | AHSB02.04 |
| 20 | Solve the differential equation $\left(D^{2}+9\right) y=\cos 3 x+\sin 2 x$ | Understand | CO 1 | AHSB02.04 |


| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Find the Inverse of a matrix by using Gauss-Jordan method $A=\left[\begin{array}{ccc} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{array}\right] .$ | Understand | CO 1 | AHSB02.03 |
| 2 | Find the Inverse of a matrix by using Gauss-Jordan method $\left(\begin{array}{lll} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{array}\right)$ | Understand | CO 1 | AHSB02.03 |
| 3 | Find the rank of the matrix $\left[\begin{array}{llll}4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4\end{array}\right]$ by Normal form. | Understand | CO 1 | AHSB02.02 |
| 4 | Find the rank of the matrix $\mathrm{A}=\left[\begin{array}{cccc}2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2\end{array}\right]$. by canonical form | Understand | CO 1 | AHSB02.03 |
| 5 | Find the inverse of $A$ if $A=\left[\begin{array}{lll}0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1\end{array}\right]$ by elementary row operation. | Understand | CO 1 | AHSB02.03 |
| 6 | By using method of variation of parameters solve $y^{\prime \prime}+y=x \cos x$. | Understand | CO 1 | AHSB02.05 |
| 7 | Solve the differential equation $\left(D^{3}-4 D^{2}-D+4\right) y=e^{3 x} \cos 2 x$ | Understand | CO 1 | AHSB02.04 |
| 8 | Solve the differential equation $\left(D^{2}+3 D+2\right) y=e^{e^{x}}$, By using method of variation of parameters | Understand | CO 1 | AHSB02.05 |
| 9 | Solve the differential equation $\left(D^{3}-5 D^{2}+8 D-4\right) Y=e^{x}+3 e^{-x}+\mathrm{x} e^{x}$ | Understand | CO 1 | AHSB02.04 |
| 10 | Apply the method of variation parameters to solve $\left(D^{2}+a^{2}\right) y=\tan a x$ | Understand | CO 1 | AHSB02.05 |
| MODULE-II |  |  |  |  |
| LINEAR TRANSFORMATIONS AND DOUBLE INTEGRALS |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |
| 1 | State Cayley- Hamilton theorem. | Understand | CO 2 | AHSB02.06 |
| 2 | Find the sum of Eigen values of the matrix $\left[\begin{array}{lll}2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2\end{array}\right]$ | Understand | CO 2 | AHSB02.06 |
| 3 | Show that the vectors $X_{1}=(1,1,2), X_{2}=(1,2,5)$ and $X_{3}=(5,3,4)$ are linearly dependent. | Understand | CO 2 | AHSB02.09 |
| 4 | Find the characteristic equation of the matrix $A=\left[\begin{array}{ccc}6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3\end{array}\right]$ | Remember | CO 2 | AHSB02.06 |


| 5 | Find the Eigen values of the matrix $\left(\begin{array}{ccc}2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2\end{array}\right)$ | Understand | CO 2 | AHSB02.06 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | Show that the vectors $X_{1}=(1,1,1) X_{2}=(3,1,2)$ and $X_{3}=(2,1,4)$ are linearly independent. | Understand | CO 2 | AHSB02.09 |
| 7 | Define Modal and Spectral matrices. | Understand | CO 2 | AHSB02.10 |
| 8 | Define diaganalisation of a matrix. | Understand | CO2 | AHSB02.10 |
| 9 | Find the Eigen values of the matrix $A^{-1}, \mathrm{~A}=\left[\begin{array}{ccc}8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1\end{array}\right]$ | Understand | CO2 | AHSB02.06 |
| 10 | Find the eigen values $A^{3}$ of the matrix $\mathrm{A}=\left(\begin{array}{ccc}1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1\end{array}\right)$ | Understand | CO 2 | AHSB02.06 |
| 11 | Evaluate the double integral $\int_{0}^{2} \int_{0}^{x} y d y d x$. | Remember | CO 2 | AHSB02.12 |
| 12 | Evaluate the double integral $\int_{0}^{\pi} \int_{0}^{a \sin \theta} r d r d \theta$. | Understand | CO2 | AHSB02.12 |
| 13 | Evaluate the double integral $\int_{0}^{3} \int_{0}^{1} x y(x+y) d x d y$ | Understand | CO 2 | AHSB02.12 |
| 14 | Find the value of double integral $\int_{1}^{2} \int_{1}^{3} x y^{2} d x d y$. | Understand | CO 2 | AHSB02.12 |
| 15 | Evaluate the double integral $\int_{0}^{1} \int_{x}^{x^{2}} \mathrm{xydxdy}$ | Understand | CO 2 | AHSB02.12 |
| 16 | Evaluate the double integral $\int_{0}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{2}} \sin (x+y) d x d y$ | Remember | CO2 | AHSB02.12 |
| 17 | Evaluate the double integral $1 \int_{0}^{1} \int_{1}^{2} \mathrm{xydxdy}$ | Understand | CO 2 | AHSB02.12 |
| 18 | Evaluate the double integral $\int_{0}^{\infty} \int_{0}^{\pi / 2} e^{-r^{2}} r d \theta d r$. | Understand | CO2 | AHSB02.12 |
| 19 | Evaluate the double integral $\int_{0}^{\pi} \int_{0}^{a(1+\cos \theta)} r d r d \theta$. | Understand | CO 2 | AHSB02.12 |
| 20 | State the formula to find area of the region using double integration in Cartesian form. | Remember | CO2 | AHSB02.12 |
| Part | B (Long Answer Questions) |  |  |  |
| 1 | Find the characteristic vectors of the matrix $A=\left[\begin{array}{ccc}6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3\end{array}\right]$ | Understand | CO 2 | AHSB02.06 |
| 2 | Diagonalisation of matrix $\mathrm{A}=\left[\begin{array}{ccc}8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1\end{array}\right]$ | Understand | CO 2 | AHSB02.11 |
| 3 | Show that matrix $\left[\begin{array}{ccc}0 & c & -b \\ -c & 0 & a \\ b & -a & 0\end{array}\right]$ satisfying Cayley-Hamilton theorem and hence find its inverse, if its exists. | Understand | CO 2 | AHSB02.11 |
| 4 | Use Cayley-Hamilton theorem to find $A^{3}$, if $\mathrm{A}=\left[\begin{array}{ll}2 & 4 \\ 1 & 1\end{array}\right]$ | Understand | CO 2 | AHSB02.11 |


| 5 | Find the Eigen values and Eigen vectors of the matrix A and its inverse, where $\mathrm{A}=\left[\begin{array}{lll}1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3\end{array}\right]$ | Understand | CO 2 | AHSB02.06 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | Find a matrix P such that $\mathrm{P}^{-1} \mathrm{AP}$ is a diagonal matrix, where $\mathrm{A}=$ $\left[\begin{array}{ccc} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{array}\right]$ | Understand | CO 2 | AHSB02.10 |
| 7 | Find the characteristic roots of the matrix $\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$ and the corresponding characteristic vectors. | Understand | CO 2 | AHSB02.06 |
| 8 | Express $\mathrm{A}^{5}-4 \mathrm{~A}^{4}-7 \mathrm{~A}^{3}+11 \mathrm{~A}^{2}-\mathrm{A}-10 \mathrm{I}$ as a linear polynomial in A , where $A=\left[\begin{array}{ll}1 & 4 \\ 2 & 3\end{array}\right]$ | Understand | CO 2 | AHSB02.06 |
| 9 | Verify Cayley-Hamilton theorem for $\mathrm{A}=\left(\begin{array}{ccc}1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1\end{array}\right)$ and find $\mathrm{A}^{-1} \& \mathrm{~A}^{4}$. | Understand | CO 2 | AHSB02.11 |
| 10 | Diagonalize the matrix $\mathrm{A}=\left(\begin{array}{ccc}1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3\end{array}\right)$ by linear transformation and hence find $\mathrm{A}^{4}$. | Understand | CO 2 | AHSB02.10 |
| 11 | Evaluate the double integral $\int_{0}^{\pi a(1+\cos \theta)} \int_{0}^{2} \cos \theta d r d \theta$. | Understand | CO 2 | AHSB02.12 |
| 12 | Evaluate the double integral $\int_{0}^{1} \int_{x}^{\sqrt{x}}\left(x^{2}+y^{2}\right) d x d y$. | Understand | CO 2 | AHSB02.12 |
| 13 | Evaluate the double integral $\int_{0}^{5} \int_{0}^{x^{2}} x\left(x^{2}+y^{2}\right) d x d y$. | Understand | CO 2 | AHSB02.12 |
| 14 | Evaluate the double integral $\int_{0}^{1 / 2} \int_{0}^{\pi / 2} r \sin \theta d \theta d r$. | Understand | CO 2 | AHSB02.12 |
| 15 | By changing the order of integration evaluate the double integral $\int_{0}^{1} \int_{x^{2}}^{2-x} x y d x d y$ | Understand | CO 2 | AHSB02.13 |
| 16 | By changing the order of integration Evaluate the double integral $\int_{0}^{4 a} \int_{\frac{x^{2}}{4 a}}^{2 \sqrt{a x}} x y d y d x$ | Understand | CO 2 | AHSB02.13 |


| 17 | Find the value of $\iint x y d x d y$ taken over the positive quadrant of the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$. | Understand | CO2 | AHSB02.12 |
| :---: | :---: | :---: | :---: | :---: |
| 18 | Evaluate the double integral using change of variables $\int_{0}^{\infty} \int_{0}^{\infty} e^{-\left(x^{2}+y^{2}\right)} d x d y$. | Understand | CO 2 | AHSB02.13 |
| 19 | By transforming into polar coordinates Evaluate $\iint \frac{x^{2} y^{2}}{x^{2}+y^{2}} d x d y$ over the annular region between the circles $x^{2}+y^{2}=a^{2}$ and $x^{2}+y^{2}=b^{2}$ with $b>a$. | Understand | CO 2 | AHSB02.12 |
| 20 | Find the area of the region bounded by the parabola $\mathrm{y}^{2}=4 \mathrm{ax}$ and $x^{2}=4 a y$. | Understand | CO 2 | AHSB02.12 |
| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |  |
| 1 | Find Eigen values and Eigen vectors of $A=\left[\begin{array}{ccc}i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0\end{array}\right]$ | Understand | CO 2 | AHSB02.06 |
| 2 | Examine whether the vectors [2,-1,3,2], [1,3,4,2], [3,5,2,2] is linearly independent or dependent? | Understand | CO 2 | AHSB02.07 |
| 3 | Find Eigen values and corresponding Eigen vectors of the matrix $\left[\begin{array}{ccc} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{array}\right]$ | Understand | CO 2 | AHSB02.06 |
| 4 | Verify Cayley-Hamilton theorem for If $A=\left[\begin{array}{ccc}8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3\end{array}\right]$, | Understand | CO 2 | AHSB02.11 |
| 5 | Verify Cayley-Hamilton theorem for $A=\left[\begin{array}{lll}2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2\end{array}\right]$ and find $\mathrm{A}^{-1}$. | Understand | CO 2 | AHSB02.11 |
| 6 | Evaluate $\iint r^{3} d r d \theta$ over the area included between the circles $r=2 \sin \theta$ and $r=4 \sin \theta$. | Understand | CO 2 | AHSB02.12 |
| 7 | Find the area of the cardioid $\mathrm{r}=\mathrm{a}(1+\cos \theta)$. | Understand | CO 2 | AHSB02.12 |
| 8 | Find the area of the region bounded by the curves $\mathrm{y}=\mathrm{x}^{3}$ and $\mathrm{y}=\mathrm{x}$. | Understand | CO 2 | AHSB02.12 |
| 9 | Evaluate $\iint x y d x d y$ taken over the positive quadrant of the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$. | Understand | CO 2 | AHSB02.12 |
| 10 | By changing the order of integration Evaluate the double integral $\int_{0}^{\infty} \int_{x}^{\infty} \frac{e^{-y}}{y} d y d x$ | Understand | CO 2 | AHSB02.13 |

## MODULE-III

FUNCTIONS OF SINGLE VARIABLES AND TRIPLE INTEGRALS
Part - A (Short Answer Questions)

| 1 | Discuss the applicability of Rolle's theorem for any function $f(x)$ in interval [a,b]. | Understand | CO 3 | AHSB02.14 |
| :---: | :---: | :---: | :---: | :---: |
| 2 | Discuss the applicability of Lagrange's mean value theorem for any function $\mathrm{f}(\mathrm{x})$ in interval [a,b]. | Understand | CO 3 | AHSB02.14 |
| 3 | Discuss the applicability of Cauchy's mean value theorem for any function $\mathrm{f}(\mathrm{x})$ in interval [a,b]. | Understand | CO 3 | AHSB02.14 |
| 4 | Interpret Rolle's theorem geometrically. | Understand | CO 3 | AHSB02.14 |
| 5 | Interpret Lagrange's mean value theorem geometrically. | Remember | CO 3 | AHSB02.14 |
| 6 | Given an example of function that is continuous on $[-1,1]$ and for which mean value theorem does not hold. | Understand | CO 3 | AHSB02.14 |
| 7 | Using Lagrange's mean value theorem, find the value of c for $\mathrm{f}(\mathrm{x})=\log \mathrm{x}$ in $(1, \mathrm{e})$. | Understand | CO 3 | AHSB02.14 |
| 8 | Explain why mean value theorem does not hold for $f(x)=x^{2 / 3}$ in [-1,1] | Understand | CO 3 | AHSB02.14 |
| 9 | Find the region in which $f(x)=1-4 x-x^{2}$ is increasing using mean value theorem. | Understand | CO 3 | AHSB02.14 |
| 10 | If $f^{\prime}(x)=0$ throughout an interval $[\mathrm{a}, \mathrm{b}]$, using mean value theorem show that $\mathrm{f}(\mathrm{x})$ is constant. | Understand | CO 3 | AHSB02.14 |
| 11 | Find the value of triple integral $\int_{-1}^{1} \int_{-2}^{2} \int_{-3}^{3} d x d y d z$ | Understand | CO 3 | AHSB02.15 |
| 12 | Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\mathrm{x}+\mathrm{y}+\mathrm{z}=1$. | Understand | CO 3 | AHSB02.15 |
| 13 | State the formula to find volume of the region using triple integration in Cartesian form. | Understand | CO 3 | AHSB02.15 |
| 14 | Evaluate the triple integral $\int_{0}^{2} \int_{1}^{3} \int_{1}^{2} x y^{2} z d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| 15 | Evaluate the triple integral $\int_{0}^{a} \int_{0}^{x} \int_{0}^{y} x y z d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| 16 | Evaluate the triple integral $\int_{0}^{1} \int_{0}^{2} \int_{0}^{3}(x+y+z) d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| 17 | Evaluate the triple integral $\int_{0}^{1} \int_{0}^{1} \int_{0}^{1} x z d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| 18 | Evaluate the triple integral $\int_{-2}^{2} \int_{-3}^{3} \int_{-1}^{1} e^{x+y+z} d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| 19 | Evaluate the triple integral $\int_{0}^{2} \int_{0}^{3} \int_{0}^{1} d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| 20 | Evaluate the triple integral $\int_{0}^{1} \int_{-1}^{1} \int_{-1}^{1}\left(x^{2}+y^{2}+z^{2}\right) d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| Par | - B (Long Answer Questions) |  |  |  |
| 1 | Verify Rolle's theorem for the function $f(x)=e^{-x} \sin x$ in the interval $[0, \pi]$. | Understand | CO 3 | AHSB02.14 |
| 2 | Show that for any $x>0,1+x<e^{x}<1+x e^{x}$ | Understand | CO 3 | AHSB02.14 |
| 3 | Verify Lagrange's mean value theorem for $f(x)=x^{3}-x^{2}-5 x+3$ in the interval $[0,4]$. | Understand | CO 3 | AHSB02.14 |
| 4 | If $\mathrm{a}<\mathrm{b}$, prove that $\frac{b-a}{1+b^{2}}<\operatorname{Tan}^{-1} b-\operatorname{Tan}^{-1} a<\frac{b-a}{1+a^{2}}$ using Lagrange's Mean value theorem and hence deduce the following. <br> (i) $\frac{\pi}{4}+\frac{3}{25}<\operatorname{Tan}^{-1} \frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$ | Understand | CO 3 | AHSB02.14 |


|  | (ii) $\frac{5 \pi+4}{20}<\operatorname{Tan}^{-1} 2<\frac{\pi+2}{4}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 | Analyze the value of c in the interval [3, 7] for the function $f(x)=e^{x}, g(x)=e^{-x}$ | Understand | CO 3 | AHSB02.14 |
| 6 | Find value of the C using Cauchy's mean value theorem for $f(x)=\sqrt{x} \& g(x)=\frac{1}{\sqrt{x}}$ in $[\mathrm{a}, \mathrm{b}]$ where $0<\mathrm{a}<\mathrm{b}$ | Understand | CO 3 | AHSB02.14 |
| 7 | Verify Cauchy's mean value theorem for $f(x)=x^{2} \& g(x)=x^{3}$ in $[1,2]$ and find the value of c . | Understand | CO 3 | AHSB02.14 |
| 8 | Verify Rolle's theorem for the function $f(x)=(x-a)^{m}(x-b)^{n}$ where $\mathrm{m}, \mathrm{n}$ are positive integers in $[\mathrm{a}, \mathrm{b}]$. | Understand | CO 3 | AHSB02.14 |
| 9 | Using mean value theorem, for $0<a<b$, prove that $1-\frac{a}{b}<\log \frac{b}{a}<\frac{b}{a}-1$ and hence show that $\frac{1}{6}<\log \frac{6}{5}<\frac{1}{5}$. | Understand | CO 3 | AHSB02.14 |
| 10 | Find all numbers c between a and $\mathrm{b} b$ which satisfies lagranges mean value theorem ,for the following function $(x)=(x-1)(x-2)(x-3)$ in [0 4] | Understand | CO 3 | AHSB02.14 |
| 11 | Evaluate the triple integral $\int_{0}^{1} \int_{0}^{1-z} \int_{0}^{1-y-z} x y z d x d y d z$. | Understand | CO 3 | AHSB02.15 |
| 12 | Evaluate the triple integral $\int_{0}^{\log 2} \int_{0}^{x} \int_{0}^{x+\log y} e^{x+y+z} d x d y d z$. | Understand | CO 3 | AHSB02.15 |
| 13 | Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{0}^{\sqrt{1-x^{2}-y^{2}}} \frac{d x d y d z}{\sqrt{1-x^{2}-y^{2}-z^{2}}}$. | Understand | CO 3 | AHSB02.15 |
| 14 | Find the volume of the tetrahedron bounded by the plane $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0$; and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate planes by triple integration. | Understand | CO 3 | AHSB02.15 |
| 15 | Using triple integration find the volume of the sphere $\mathrm{x}^{2}+\mathrm{y}^{2}+\mathrm{z}^{2}=\mathrm{a}^{2}$. | Understand | CO 3 | AHSB02.15 |
| 16 | Evaluate $\iiint_{v} d x d y d z$ where v is the finite region of space formed by the planes $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0$ and $2 \mathrm{x}+3 \mathrm{y}+4 \mathrm{z}=12$. | Understand | CO 3 | AHSB02.15 |
| 17 | Evaluate $\iiint_{R}(x+y+z) d z d y d x$ where R is the region bounded by the plane $x=0, x=1, y=0, y=1, z=0, z=1$. | Understand | CO 3 | AHSB02.15 |
| 18 | Find the volume of the ellipsoid $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{x^{2}}{c^{2}}=1$ | Understand | CO 3 | AHSB02.15 |
| 19 | If R is the region bounded by the planes $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=1$ and the cylinder $x^{2}+y^{2}=1$, evaluate $\iiint_{R} x y z d x d y d z$. | Understand | CO 3 | AHSB02.15 |
| 20 | Evaluate $\int_{0}^{1} \int_{0}^{1} \int_{\sqrt{x^{2}+y^{2}}}^{2} x y z d z d y d x$ | Understand | CO 3 | AHSB02.15 |
| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |  |
| 1 | Verify the hypothesis and conclusion of rolles thorem for the function defined below $\mathrm{f}(\mathrm{x})=x^{3}-6 x^{2}+11 x-6$ in [1 3] | Understand | CO 3 | AHSB02.14 |
| 2 | Verify the hypothesis and conclusion of rolles thorem for the function defined below $\mathrm{f}(\mathrm{x})=\frac{\log \left(\left(\mathrm{x} \mathrm{x}^{2}+a b\right)\right.}{(a+b) x}$ in $[a b]$ | Understand | CO 3 | AHSB02.14 |
| 3 | Use lagranges mean value theorem to establish the following | Understand | CO 3 | AHSB02.14 |


|  | inequalities $x \leq \sin ^{-1} x \leq \frac{x}{\sqrt{1-x^{2}}}$ for $0 \leq x \leq 1$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 | Calculate approximately $\sqrt[5]{245}$ by using L.M.V.T. | Understand | CO 3 | AHSB02.14 |
| 5 | Verify Cauchy's mean value theorem for $f(x)=x^{3} \& g(x)=2-x$ in $[0,9]$ and find the value of $c$. | Understand | CO 3 | AHSB02.14 |
| 06 | Evaluate $\int_{0}^{a} \int_{0}^{b} \int_{0}^{c}(x+y+z) d x d y d z$ | Understand | CO 3 | AHSB02.15 |
| 07 | Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{0}^{\sqrt{1-x^{2}-y^{2}}} x y z d x d y d z$ | Understand | CO 3 | AHSB02.15 |
| 08 | Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z}(x+y+z) d x d y d z$ | Understand | CO 3 | AHSB02.15 |
| 09 | Evaluate $\iiint \frac{d x d y d z}{(x+y+z+1)^{3}}$. where D is the region bounded by the planes $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0, \mathrm{x}+\mathrm{y}+\mathrm{z}=1$ | Understand | CO 3 | AHSB02.15 |
| 10 | Evaluate $\iiint x y z d x d y d z$ where D is the region bounded by the positive octant of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$. | Understand | CO 3 | AHSB02.15 |
| MODULE-IV |  |  |  |  |
| FUNCTIONS OF SEVERAL VARIABLES AND EXTREMA OF A FUNCTION |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |
| 1 | If $x=\frac{u^{2}}{v}, y=\frac{v^{2}}{v}$, find the value of $\frac{\partial(u, v)}{\partial(x, y)}$ | Understand | CO 4 | AHSB02.17 |
| 2 | The stationary point of the function $f(x, y)=x^{2}+y^{2}+x y+x-4 y+5$ | Understand | CO 4 | AHSB02.18 |
| 3 | If $x=u(1-v), y=u v$, find the value of $J^{\prime}$. | Understand | CO 4 | AHSB02.17 |
| 4 | Calculate $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ if $u=\frac{2 y z}{x}, v=\frac{3 z x}{y} w=\frac{4 x y}{z}$ | Understand | CO 4 | AHSB02.17 |
| 5 | If $x=u(1+v), y=v(1+u)$ then find the value of $\frac{\partial(x, y)}{\partial(u, v)}$ | Understand | CO 4 | AHSB02.17 |
| 6 | Write the condition for the function $\mathrm{f}(\mathrm{x}, \mathrm{y})$ to be functionally dependent. | Understand | CO 4 | AHSB02.17 |
| 7 | Define jacobian of a function. | Understand | CO 4 | AHSB02.17 |
| 8 | Define a saddle point for the function of $f(x, y)$. | Understand | CO 4 | AHSB02.17 |
| 9 | Write the condition for the function $f(x, y)$ to be functionally independent. | Understand | CO 4 | AHSB02.17 |
| 10 | Define a extreme point for the function of $\mathrm{f}(\mathrm{x}, \mathrm{y})$. | Understand | CO 4 | AHSB02.18 |
| 11 | Define Stationary points | Understand | CO 4 | AHSB02.18 |
| 12 | Define maxium function? | Understand | CO 4 | AHSB02.18 |
| 13 | Define a minimum function? | Understand | CO 4 | AHSB02.18 |
| 14 | If u and v are functions of x and y then prove that $\mathrm{J} J^{\prime}=1$ | Understand | CO 4 | AHSB02.18 |
| 15 | $\mathrm{X}=\mathrm{rcos} \theta, Y=r \sin \theta$ find J | Understand | CO 4 | AHSB02.17 |
| 16 | If $\mathrm{X}=\log \left(\mathrm{x} \tan ^{-1} \mathrm{y}\right)$ then $f_{x y}$ is equal to zero | Understand | CO 4 | AHSB02.16 |
| 17 | If $\mathrm{f}(\mathrm{x}, \mathrm{y}, \mathrm{z})=0$ then the values $\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\delta z}{\delta x}=-1$ | Understand | CO 4 | AHSB02.16 |
| 18 | Prove that if the function $\mathrm{u}, \mathrm{v}, \mathrm{w}$ of three independent variables $\mathrm{x}, \mathrm{y}, \mathrm{z}$ | Understand | CO 4 | AHSB02.17 |


|  | are not independent ,then the Jacobian of $u, v, w$ w.r.t $x, y, z$ is always equals to zero. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 19 | If $\mathrm{z}=\cos \left(\frac{x}{v}\right)+\sin \left(7_{x}^{x}\right)$,then Show that $x \frac{\partial z}{\partial x}+y \frac{\partial z}{\partial x}=0$ | Understand | CO 4 | AHSB02.20 |
| 20 | Write the properties of maxima and minima under the various conditions. | Understand | CO 4 | AHSB02.20 |
| Part - B (Long Answer Questions) |  |  |  |  |
| 1 | i) If $\mathrm{x}=\mathrm{u}(1-\mathrm{v}), \mathrm{y}=\mathrm{uv}$ then prove that $\mathrm{JJ}^{\prime}=1$. <br> ii) If $x+y^{2}=u, y+z^{2}=v, z+x^{2}=w$ find the value of $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. | Understand | CO 4 | AHSB02.18 |
| 2 | If $u=x^{2}-y^{2}, v=2 x y$ where $x=r \cos \theta, y=r \sin \theta$ then show that $\frac{\partial(u, v)}{\partial(r, \theta)}=4 r^{3}$ | Understand | CO 4 | AHSB02.18 |
| 3 | If $x=e^{r} \sec \theta, y=e^{r} \tan \theta$ Prove that $\frac{\partial(x, y)}{\partial(r, \theta)} \frac{\partial(r, \theta)}{\partial(x, y)}=1$. | Understand | CO 4 | AHSB02.18 |
| 4 | If $u x=y z, v y=z x, w z=x y$ find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ | Understand | CO 4 | AHSB02.18 |
| 5 | If $\mathrm{x}=\frac{u^{2}}{v}, \mathrm{y}=\frac{v^{2}}{u}$ then find the Jacobian of the function u and v with respect to x and y | Understand | CO 4 | AHSB02.18 |
| 6 | Show that the functions $u=x+y+z, v=x^{2}+y^{2}+z^{2}-2 x y-2 y z-2 x z$ and $w=x^{3}+y^{3}+z^{3}-3 x y z$ are functionally related. | Understand | CO 4 | AHSB02.19 |
| 7 | If $\mathrm{x}=\mathrm{u}, \mathrm{y}=\operatorname{tanv}, \mathrm{z}=\mathrm{w}$ then prove that $\frac{\partial(\mathrm{x}, \mathrm{y}, \mathrm{z})}{\partial(\mathrm{u}, \mathrm{v}, \mathrm{w})}=\mathrm{u} \sec ^{2} \mathrm{v}$ | Understand | CO 4 | AHSB02.18 |
| 8 | Show that the functions $u=e^{x} \sin y, v=e^{x} \cos y$ are not functionally related. | Understand | CO 4 | AHSB02.18 |
| 9 | Prove that $u=x+y+z, v=x y+y z+z x, w=x^{2}+y^{2}+z^{2}$ are functionally dependent. | Understand | CO 4 | AHSB02.18 |
| 10 | If $u=x+y+z, u v=y+z, z=u v w$ Prove that $\frac{\partial(x, y, z)}{\partial(u, v, w)}=u^{2} v$ | Understand | CO 4 | AHSB02.18 |
| 11 | Find the maximum value of the function xyz when $\mathrm{x}+\mathrm{y}+\mathrm{z}=\mathrm{a}$. | Understand | CO 4 | AHSB02.19 |
| 12 | Find the maxima and minima of the function $f(x, y)=x^{3} y^{2}(1-x-y)$. | Understand | CO 4 | AHSB02.20 |
| 13 | Find the maximum and minimum of the function $f(x, y)=\sin x+\sin y+\sin (x+y)$ | Understand | CO 4 | AHSB02.20 |
| 14 | Find the maximum and minimum values of $f(x, y)=x^{3}+3 x y^{2}-3 x^{2}-3 y^{2}+4$ | Understand | CO 4 | AHSB02.20 |
| 15 | Find the shortest distance from the origin to the surface $x y z^{2}=2$ | Understand | CO 4 | AHSB02.18 |
| 16 | Find the minimum value of $x^{2}+y^{2}$, subjects to the condition $\mathrm{ax}+\mathrm{by}=\mathrm{c}$ using lagranges multipliers method. | Understand | CO 4 | AHSB02.18 |
| 17 | Find the points on the surface $z^{2}=x y+1$ nearest to the origin. | Understand | CO 4 | AHSB02.20 |
| 18 | Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube using lagranges multipliers method | Understand | CO 4 | AHSB02.20 |
| 19 | Find the value of the largest rectangular parallelepiped that can be | Understand | CO 4 | AHSB02.20 |


|  | inscribed in the ellipsoid $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 20 | Find the stationary points of $\mathrm{U}(\mathrm{x}, \mathrm{y})=\sin \mathrm{x} \sin \mathrm{y} \sin (\mathrm{x}+\mathrm{y})$ where $0<x<\pi, 0<y<\pi$ and find the maximum value of the function $U$. | Understand | CO 4 | AHSB02.20 |
| Part - C (Problem Solving and Critical Thinking) |  |  |  |  |
| 1 | If $u=x+3 y^{2}+z^{3}, v=4 x^{2} y z, w=2 z^{2}-x y$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1,-1,0)$. | Understand | CO 4 | AHSB02.17 |
| 2 | If $u=e^{x y z}$, show that $\frac{\partial^{3} u}{\partial x \partial y \partial z}=\left(1+3 x y z+x^{2} y^{2} z^{2}\right) e^{x y z}$ | Understand | CO 4 | AHSB02.16 |
| 3 | If $\begin{gathered} u=\log \left(x^{2}+y^{2}+z^{2}\right), \text { prove that } \\ \left(x^{2}+y^{2}+z^{2}\right)\left(\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial x^{2}}\right)=2 \end{gathered}$ | Understand | CO 4 | AHSB02.16 |
| 4 | Determine whether the following functions are functionally dependent or not .If functionally dependent, find the relation between them . $u=\frac{x-y}{x+z}, v=\frac{x+z}{y+z}$ | Understand | CO 4 | AHSB02.17 |
| 5 | Determine whether the following functions are functionally dependent or not .If functionally dependent, find the relation between them . $u=\frac{x+y}{1-x y}, v=\tan ^{-1} x+\tan ^{-1} y$ | Understand | CO 4 | AHSB02.18 |
| 6 | Find the maxima value of $u=x^{2} y^{3} z^{4}$ with the constrain condition $2 x+3 y+4 z=a$ | Understand | CO 4 | AHSB02.18 |
| 7 | Find the point of the plane $x+2 y+3 z=4$ that is closed to the origin. | Understand | CO 4 | AHSB02.20 |
| 8 | Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is maximum. | Understand | CO 4 | AHSB02.18 |
| 9 | Find three positive numbers whose sum is 100 and whose product is maximum. | Understand | CO 4 | AHSB02.18 |
| 10 | A rectangular box open at the top is to have volume of 32 cubic $f t$. Find the dimensions of the box requiring least material for its construction. | Understand | CO 4 | AHSB02.18 |
| MODULE-V |  |  |  |  |
| VECTOR CALCULUS |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |
| 1 | Define gradient of scalar point function. | Remember | CO 5 | AHSB02.19 |
| 2 | Define divergence of vector point function. | Remember | CO 5 | AHSB02.19 |
| 3 | Define curl of vector point function. | Remember | CO 5 | AHSB02.19 |
| 4 | State Laplacian operator. | Understand | CO 5 | AHSB02.19 |
| 5 | Find curl $\bar{f}$ where $\bar{f}=\operatorname{grad}\left(x^{3}+y^{3}+z^{3}-3 x y z\right)$. | Understand | CO 5 | AHSB02.19 |
| 6 | Find the angle between the normal to the surface $\mathrm{xy}=\mathrm{z}^{2}$ at the points $(4,1,2)$ and $(3,3,-3)$. | Understand | CO 5 | AHSB02.19 |
| 7 | Find a unit normal vector to the given surface $x^{2} y+2 x z=4$ at the point (2,-2,3). | Understand | CO 5 | AHSB02.19 |
| 8 | If $\bar{a}$ is a vector then prove that $\operatorname{grad}(\bar{a} \cdot \bar{r})=\bar{a}$. | Understand | CO 5 | AHSB02.19 |
| 9 | Define irrotational vector and solenoidal vector of vector point function. | Remember | CO 5 | AHSB02.19 |


| 10 | Show that $\nabla(f(r))=\frac{\bar{r}}{r} f^{\prime}(r)$. | Understand | CO 5 | AHSB02.19 |
| :---: | :---: | :---: | :---: | :---: |
| 11 | Prove that $\mathrm{f}=y z i+z x j+x y k$ is irrotational vector. | Understand | CO 5 | AHSB02.19 |
| 12 | Show that ( $\mathrm{x}+3 \mathrm{y}$ ) $\mathrm{i}+(\mathrm{y}-2 \mathrm{z}) \mathrm{j}+(\mathrm{x}-2 \mathrm{z}) \mathrm{k}$ i solenoidal. | Understand | CO 5 | AHSB02.20 |
| 13 | Define work done by a force, circulation. | Understand | CO 5 | AHSB02.20 |
| 14 | State Stokes theorem of transformation between line integral and surface integral. | Understand | CO 5 | AHSB02.22 |
| 15 | Prove that div $\operatorname{curl} \bar{f}=0$ where $\bar{f}=f_{1} \bar{i}+f_{2} \bar{j}+f_{3} \bar{k}$ | Understand | CO 5 | AHSB02.20 |
| 16 | Define line integral on vector point function. | Remember | CO 5 | AHSB02.21 |
| 17 | Define surface integral of vector point function $\bar{F}$ | Remember | CO 5 | AHSB02.21 |
| 18 | Define volume integral on closed surface S of volume V . | Remember | CO 5 | AHSB02.21 |
| 19 | State Green's theorem of transformation between line integral and double integral. | Understand | CO 5 | AHSB02.22 |
| 20 | State Gauss divergence theorem of transformation between surface integral and volume integral. | Understand | CO 5 | AHSB02.22 |
| Part - B (Long Answer Questions) |  |  |  |  |
| 1 | Evaluate $\int_{\mathrm{c}} \overline{\mathrm{f}}$. $\mathrm{d} \overline{\mathrm{r}}$ where $\bar{f}=3 x y i-y^{2} j$ and C is the parabola $\mathrm{y}=2 \mathrm{x}^{2}$ from points $(0,0)$ to $(1,2)$. | Understand | CO 5 | AHSB02.21 |
| 2 | Evaluate $\iint_{\mathrm{S}} \mathrm{F} . \mathrm{d}$ if $\bar{F}=y z i+2 y^{2} j+x z^{2} k$ and S is the Surface of the cylinder $x^{2}+y^{2}=9$ contained in the first octant between the planes $z$ $=0$ and $\mathrm{z}=2$. | Understand | CO 5 | AHSB02.21 |
| 3 | Find the work done in moving a particle in the force field $\bar{F}=\left(3 x^{2}\right) i+(2 z x-y) j+z k$ along the straight line from $(0,0,0)$ to $(2,1,3)$. | Understand | CO 5 | AHSB02.21 |
| 4 | Find the circulation of $\bar{F}=(2 x-y+2 z) \bar{i}+(x+y-z) \bar{j}+(3 x-2 y-5 z) \bar{k}$ along the circle $x^{2}+y^{2}=4$ in the xy plane. | Understand | CO 5 | AHSB02.21 |
| 5 | Verify Gauss divergence theorem for the vector point function $\mathrm{F}=\left(\mathrm{x}^{3}-\mathrm{yz}\right) \mathrm{i}-2 \mathrm{yxj}+2 \mathrm{zk}$ over the cube bounded by $\mathrm{x}=\mathrm{y}=\mathrm{z}=0$ and $x=y=z=a$. | Understand | CO 5 | AHSB02.22 |
| 6 | Verify Gauss divergence theorem for $2 x^{2} y i-y^{2} j+4 x z^{2} k$ taken over the region of first octant of the cylinder $y^{2}+z^{2}=9$ and $x=2$. | Understand | CO 5 | AHSB02.22 |
| 7 | Verify Green's theorem in the plane for $\int_{C}\left(x^{2}-x y^{3}\right) d x+\left(y^{2}-2 x y\right) d y$ where C is a square with vertices $(0,0),(2,0),(2,2),(0,2)$. | Understand | CO 5 | AHSB02.22 |
| 8 | Applying Green's theorem evaluate $\int_{C}(y-\sin x) d x+\cos x d y$ where C is the plane triangle enclosed by $y=0, y=\frac{2 x}{\pi}$, and $x=\frac{\pi}{2}$. | Understand | CO 5 | AHSB02.22 |
| 9 | Apply Green's Theorem in the plane for $\int\left(2 x^{2}-y^{2}\right) d x+\left(x^{2}+y^{2}\right) d y$ where C is a is the boundary of the area enclosed by the x -axis and upper half of the circle $x^{2}+y^{2}=a^{2}$. | Understand | CO 5 | AHSB02.22 |


| 10 | Verify Stokes theorem for $f=(2 x-y) i-y z^{2} j-y^{2} z k$ where S is the upper half surface of the sphere $x^{2}+y^{2}+z^{2}=1$ bounded by the projection of the xy plane. | Understand | CO 5 | AHSB02.22 |
| :---: | :---: | :---: | :---: | :---: |
| 11 | Verify Stokes theorem for $\bar{f}=\left(x^{2}-y^{2}\right) \bar{i}+2 x y \bar{j}$ over the box bounded by the planes $x=0, x=a, y=0, y=b$. | Understand | CO 5 | AHSB02.24 |
| 12 | Find the directional derivative of the function $\phi=x y^{2}+y z^{3}$ at the point $\mathrm{P}(1,-2,-1)$ in the direction to the surface $x \log z-y^{2}=$ -4 at $(-1,2,1)$. | Understand | CO 5 | AHSB02.21 |
| 13 | If $\bar{F}=4 x z \bar{i}-y^{2} \bar{j}+y z \bar{k}$ evaluate $\int_{s} \bar{F} . \bar{n} d s$ where $S$ is the surface of the cube $\mathrm{x}=0, \mathrm{x}=\mathrm{a}, \mathrm{y}=0, \mathrm{y}=\mathrm{a}, \mathrm{z}=0, \mathrm{z}=\mathrm{a}$. | Understand | CO 5 | AHSB02.20 |
| 14 | If $\bar{f}=\left(5 x y-6 x^{2}\right) \bar{i}+(2 y-4 x) \bar{j}$ evaluate $\int_{\mathrm{c}} \overline{\mathrm{f}} . \mathrm{d} \overline{\mathrm{r}}$ along the curve C in xy -plane $\mathrm{y}=\mathrm{x}^{3}$ from $(1,1)$ to $(2,8)$. | Understand | CO 5 | AHSB02.21 |
| 15 | Evaluate the line integral $\int_{c}\left(x^{2}+x y\right) d x+\left(x^{2}+y^{2)} d y\right.$ where C is the square formed by lines $x= \pm 1, y= \pm 1$. | Understand | CO 5 | AHSB02.21 |
| 16 | If $\bar{r}=x \bar{i}+y \bar{j}+z \bar{k}$ show that $\quad \nabla r^{n}=n r^{n-2} \bar{r}$. | Understand | CO 5 | AHSB02.19 |
| 17 | Evaluate by Stokes theorem $\int_{c}\left(e^{x} d x+2 y d y-d z\right)$ where c is the curve $x^{2}+y^{2}=9$ and $z=2$. | Understand | CO 5 | AHSB02.22 |
| 18 | Verify Stokes theorem for the function $x^{2} \bar{i}+x y \bar{j}$ integrated round the square in the plane $\mathrm{z}=0$ whose sides are along the line $\mathrm{x}=0, \mathrm{y}=0, \mathrm{x}=\mathrm{a}$, $y=a$. | Understand | CO 5 | AHSB02.22 |
| 19 | Evaluate by Stokes theorem $\int_{c}(x+y) d x+(2 x-z) d y+(y+z) d z$ where C is the boundary of the triangle with vertices ( $0,0,0$ ), ( $1,0,0$ ), ( $1,1,0$ ). | Understand | CO 5 | AHSB02.22 |
| 20 | Verify Green's theorem in the plane for $\int_{C}\left(3 x^{2}-8 y^{2}\right) d x+(4 y-6 x y) d y$ where C is a region bounded by $y=\sqrt{x}$ and $y=x^{2}$. | Understand | CO 5 | AHSB02.22 |
| Part - C (Problem Solving and Critical Thinking) |  |  |  |  |
| 1 | Verify Gauss divergence theorem for $\bar{f}=x^{2} \bar{i}+y^{2} \bar{j}+z^{2} \bar{k}$ taken over the cube bounded by $x=0, x=a, y=0, y=b, z=0, z=c$. | Understand | CO 5 | AHSB02.22 |
| 2 | Find the work done in moving a particle in the force field $\bar{F}=\left(3 x^{2}\right) i+(2 z x-y) j+z k$ along the curve defined by $x^{2}=4 y, 3 x^{3}=8 z$ from $\mathrm{x}=0$ and $\mathrm{x}=2$. | Understand | CO 5 | AHSB02.21 |
| 3 | Show that the force field given by $\bar{F}=2 x y z^{3} i+x^{2} z^{3} j+3 x^{2} y z^{2} k$ is conservative. Find the work done in moving a particle from $(1,-1,2)$ to $(3,2,-1)$ in this force field. | Understand | CO 5 | AHSB02.20 |
| 4 | Show that the vector $\left(x^{2}-y z\right) i+\left(y^{2}-z x\right) j+\left(z^{2}-x y\right) k$ is irrotational and find its scalar potential function. | Understand | CO 5 | AHSB02.21 |


| 5 | Using Gauss divergence theorem evaluate $\iint_{\mathrm{S}} \overline{\mathrm{F}} . \mathrm{d} \bar{s}$,for the $\bar{F}=y \vec{i}+x \vec{j}+z^{2} \vec{k}$ for the cylinder region S given by $\mathrm{x}^{2}+\mathrm{y}^{2}=$ $\mathrm{a}^{2}, \mathrm{z}=0$ and $\mathrm{z}=\mathrm{b}$. | Understand | CO 5 | AHSB02.22 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | Find the directional derivative of $\phi(x, y, z)=x^{2} y z+4 x z^{2}$ at the point $(1,-2,-1)$ in the direction of the normal to the surface $\mathrm{f}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ $=x \log z-y^{2}$ at $(-1,2,1)$. | Understand | CO 5 | AHSB02.20 |
| 7 | Using Green's theorem in the plane evaluate $\int_{C}\left(2 x y-x^{2}\right) d x+\left(x^{2}+y^{2}\right) d y$ where C is the region bounded by $y=x^{2}$ and $y^{2}=x$. | Understand | CO 5 | AHSB02.22 |
| 8 | Applying Green's theorem evaluate $\int_{c}\left(x y+y^{2}\right) d x+x^{2} d y$ where C is the region bounded by $y=\sqrt{x}$ and $y=x^{2}$. | Understand | CO 5 | AHSB02.22 |
| 9 | Verify Green's Theorem in the plane for $\int_{C}\left(3 x^{2}-8 y^{2}\right) d x+(4 y-6 x y) d y$ where C is the region bounded by $\mathrm{x}=0, \mathrm{y}=0$ and $\mathrm{x}+\mathrm{y}=1$. | Understand | CO 5 | AHSB02.22 |
| 10 | Verify Stokes theorem for $\bar{F}=(y-z+2) i+(y z+4) j-x z k$ where S is the surface of the cube $\mathrm{x}=0, \mathrm{y}=0, \mathrm{z}=0$ and $\mathrm{x}=2, \mathrm{y}=2, \mathrm{z}=2$ above the xy-plane. | Understand | CO 5 | AHSB02.22 |

## Prepared by:

Ms. P Rajani, Assistant Professor

