# IARE TO LIBERTY

# INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad-500043

## **MECHANICAL ENGINEERING**

## TUTORIAL QUESTION BANK

| Course Title          | LINEAR     | LINEAR ALGEBRA AND CALCULUS           |                   |         |            |         |  |
|-----------------------|------------|---------------------------------------|-------------------|---------|------------|---------|--|
| Course Code           | AHSB02     | AHSB02                                |                   |         |            |         |  |
| Programme             | B.Tech     | B.Tech                                |                   |         |            |         |  |
| Semester              | I AE       | I AE   CSE   IT   ECE   EEE   ME   CE |                   |         |            |         |  |
| Course Type           | Foundati   | on                                    |                   |         |            |         |  |
| Regulation            | IARE - R   | 18                                    |                   |         |            |         |  |
|                       |            | 7                                     | Theory            |         | Pract      | tical   |  |
| Course Structure      | Lecti      | ires                                  | Tutorials         | Credits | Laboratory | Credits |  |
|                       | 3          |                                       | 1                 | 4       | -          | -       |  |
| Chief Coordinator     | Ms. P Raj  | ani, Assi                             | stant Professor   |         |            |         |  |
| <b>Course Faculty</b> | Dr. M An   | ita, Profe                            | ssor              |         |            |         |  |
|                       | Dr. S Jaga | dha, Pro                              | fessor            |         |            |         |  |
|                       | Dr. J Sure | sh Goud,                              | Assistant Profe   | ssor    |            |         |  |
|                       | Ms. L Ind  | ira, Assis                            | stant Professor   |         |            |         |  |
|                       | Mr. Ch So  | masheka                               | r, Assistant Pro  | fessor  |            |         |  |
|                       | Ms. P Sril | atha, Ass                             | sistant Professor |         |            |         |  |
|                       | Ms. C Rad  | Ms. C Rachana, Assistant Professor    |                   |         |            |         |  |
|                       | Ms. V Sul  | ba Laxn                               | ni, Assistant Pro | fessor  |            |         |  |
|                       | Ms. B Pra  | veena, A                              | ssistant Professo | or      |            |         |  |

#### **COURSE OBJECTIVES:**

| The cours | The course should enable the students to:  |  |  |  |  |  |
|-----------|--|--|--|--|--|--|
| I         | Determine rank of a matrix and solve linear differential equations of second order.                  |  |  |  |  |  |
| II        | II Determine the characteristic roots and apply double integrals to evaluate area.                   |  |  |  |  |  |
| III       | Apply mean value theorems and apply triple integrals to evaluate volume.                             |  |  |  |  |  |
| IV        | Determine the functional dependence and extremum value of a function                                 |  |  |  |  |  |
| V         | Analyze gradient, divergence, curl and evaluate line, surface, volume integrals over a vector field. |  |  |  |  |  |

#### **COURSE OUTCOMES (COs):**

| CO 1 | Determine rank by reducing the matrix to Echelon and Normal forms. Determine inverse of the        |
|------|--|
| COT  |  |
|      | matrix by Gauss Jordon Method and Solving Second and higher order differential equations with      |
|      | constant coefficients.   |
| CO 2 | Determine a modal matrix, and reducing a matrix to diagonal form. Evaluate inverse and powers of   |
|      | matrices by using Cayley-Hamilton theorem. Evaluate double integral. Utilize the concept of change |
|      | order of integration and change of variables to evaluate double integrals. Determine the area.     |
| CO 3 | Apply the Mean value theorems for the single variable functions.                                   |
|      | Apply triple integrals to evaluate volume.   |

| CO 4 | Determine the maxima and minima for a function of several variable with and without constraints.       |
|------|--|
| CO 5 | Analyze scalar and vector fields and compute the gradient, divergence and curl. Evaluate line, surface |
|      | and volume integral of vectors. Use Vector integral theorems to facilitate vector integration.         |

# **COURSE LEARNING OUTCOMES (CLOs):**

# Students, who complete the course, will have demonstrated the ability to do the following:

| AHSB02.01 | Demonstrate knowledge of matrix calculation as an elegant and powerful mathematical language in connection with rank of a matrix.   |
|-----------|---|
| AHSB02.02 | Determine rank by reducing the matrix to Echelon and Normal forms.  |
| AHSB02.03 | Determine inverse of the matrix by Gauss Jordon Method.   |
| AHSB02.04 | Find the complete solution of a non-homogeneous differential equation as a linear combination of the complementary function and a particular solution.                          |
| AHSB02.05 | Solving Second and higher order differential equations with constant coefficients.  |
| AHSB02.06 | Interpret the Eigen values and Eigen vectors of matrix for a linear transformation and use properties of Eigen values   |
| AHSB02.07 | Understand the concept of Eigen values in real-world problems of control field where they are pole of closed loop system.   |
| AHSB02.08 | Apply the concept of Eigen values in real-world problems of mechanical systems where Eigen values are natural frequency and mode shape.   |
| AHSB02.09 | Use the system of linear equations and matrix to determine the dependency and independency.   |
| AHSB02.10 | Determine a modal matrix, and reducing a matrix to diagonal form.   |
| AHSB02.11 | Evaluate inverse and powers of matrices by using Cayley-Hamilton theorem.   |
| AHSB02.12 | Apply double integrals to evaluate area of a given function.  |
| AHSB02.13 | Utilize the concept of change order of integration and change of variables to evaluate double integrals.  |
| AHSB02.14 | Apply the Mean value theorems for the single variable functions.  |
| AHSB02.15 | Apply triple integrals to evaluate volume of a given function.  |
| AHSB02.16 | Find partial derivatives numerically and symbolically and use them to analyze and interpret the way a function varies.  |
| AHSB02.17 | Understand the techniques of multidimensional change of variables to transform the coordinates by utilizing the Jacobian. Determine Jacobian for the coordinate transformation. |
| AHSB02.18 | Apply maxima and minima for functions of several variable's and Lagrange's method of multipliers.   |
| AHSB02.19 | Analyze scalar and vector fields and compute the gradient, divergence and curl.   |
| AHSB02.20 | Understand integration of vector function with given initial conditions.  |
| AHSB02.21 | Evaluate line, surface and volume integral of vectors.  |
| AHSB02.22 | Use Vector integral theorems to facilitate vector integration.  |

# TUTORIAL QUESTION BANK

|      | MODULE - I  |                             |                    |  |  |
|------|---|-----------------------------|--------------------|--|--|
|      | THEORY OF MATRICES AND LINEAR TRA<br>Part - A (Short Answer Question  |                             | ONS                |  |  |
| S No | Questions Questions   | Blooms<br>Taxonomy<br>Level | Course<br>Outcomes | Course<br>Learning<br>Outcomes<br>(CLOs) |  |
| 1    | Define Orthogonal matrix.   | Remember                    | CO 1               | AHSB02.01                                |  |
| 2    | Find the value of k such that the rank of $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2.                                    | Remember                    | CO 1               | AHSB02.01                                |  |
| 3    | Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is a unitary matrix.  | Understand                  | CO 1               | AHSB02.01                                |  |
| 4    | Find the value of k such that rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2   | Understand                  | CO 1               | AHSB02.01                                |  |
| 5    | Find the Skew-symmetric part of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$ .  | Understand                  | CO 1               | AHSB02.01                                |  |
| 6    | Define Rank of a matrix and Skew-Hermitian matrix., Unitary matrix.   | Remember                    | CO 1               | AHSB02.01                                |  |
| 7    | If $A = \begin{bmatrix} 3 & a & b \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix}$ is symmetric, then find the values of a and b.  | Understand                  | CO 1               | AHSB02.01                                |  |
| 8    | Define orthogonal matrix .Prove that $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ is orthogonal.                       | Understand                  | CO 1               | AHSB02.01                                |  |
| 9    | Determine the values of a, b, c when the matrix $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$  | Remember                    | CO 1               | AHSB02.01                                |  |
| 10   | is orthogonal.  Express the matrix A as sum of symmetric and Skew-symmetric matrices. where $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ | Understand                  | CO 1               | AHSB02.01                                |  |
| 11   | Write the solution of the $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$   | Understand                  | CO 1               | AHSB02.04                                |  |
| 12   | Write the solution of the $(4D^2-4D+1)y=100$  | Understand                  | CO 1               | AHSB02.04                                |  |

| 13 | 1   | Understand | CO 1 | AHSB02.04 |
|----|---|------------|------|-----------|
| 13 | Find the particular integral of $\frac{1}{(D^2-1)}x$  | Onderstand | COT  | AHSB02.04 |
| 14 | Solve the differential equation $\frac{d^3y}{dx^3} + y = 0$<br>Solve the differential equation $(D^2 + a^2)y = 0$   | Remember   | CO 1 | AHSB02.04 |
| 15 | Solve the differential equation $(D^2 + a^2)y = 0$  | Understand | CO 1 | AHSB02.04 |
| 16 | Find the particular value of $\frac{1}{(D-3)}x$   | Understand | CO 1 | AHSB02.04 |
| 17 | Find the particular integral of $(D^3 - D^2 + 4D - 4)y = e^x$   | Understand | CO 1 | AHSB02.04 |
| 18 | Solve the differential equation $\frac{1}{(D+1)(D-1)}e^{-x}$  | Understand | CO 1 | AHSB02.04 |
| 19 | Solve the differential equation $(D^3 + D)y = 0$  | Understand | CO 1 | AHSB02.04 |
| 20 | Solve the differential equation $(D^6 - 64)y = 0$   | Remember   | CO 1 | AHSB02.04 |
|    | Part - B (Long Answer Questi  |            |      |           |
| 1  | By reducing the matrix $\begin{bmatrix} -1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$ into normal form, find its rank.  | Understand | CO 1 | AHSB02.02 |
| 2  | Find the values of a and b such that rank of the matrix $ \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix} $ is 3.                    | Understand | CO 1 | AHSB02.02 |
| 3  | Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ by reducing to                    | Understand | CO 1 | AHSB02.02 |
| 4  | echelon form.  Peduce the matrix to its normal form where   | Understand | CO 1 | AHSB02 03 |
| 4  | Reduce the matrix to its normal form where $ \begin{bmatrix} -1 & -3 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix} $                   | Understand | CO 1 | AHSB02.03 |
| 5  | Find the Inverse of a matrix by using Gauss-Jordan method $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}.$ | Understand | CO 1 | AHSB02.02 |

| 6  | Reduce the matrix A to its normal form where  | Understand | CO 1 | AHSB02.02 |
|----|---|------------|------|-----------|
|    | $\begin{bmatrix} 0 & 1 & 2 & -2 \end{bmatrix}$  |            |      |           |
|    | $A = \begin{bmatrix} 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ and hence find the rank  |            |      |           |
|    |   |            |      |           |
| 7  | (4 4 -3 1)  | Understand | CO 1 | AHSB02.02 |
|    | Find value of K such that the matrix $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ has rank 3      |            |      |           |
| 8  | Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing to normal form | Understand | CO 1 | AHSB02.02 |
| 9  | Find the rank of the matrix, by reducing it to the echelon form   | Understand | CO 1 | AHSB02.02 |
|    | $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$                                |            |      |           |
| 10 | $\begin{bmatrix} 4 & 0 & 2 & 1 \end{bmatrix}$   | Understand | CO 1 | AHSB02.02 |
|    | Find the rank of the $A^T$ matrix if $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ by Echelon form.                   |            |      |           |
| 11 | Solve the differential equation $(D^2 + 1)y = cosecx$ using variation   | Understand | CO 1 | AHSB02.04 |
|    | of parameter.   |            |      |           |
| 12 | Solve the differential equation $D^{2}(D^{2} + 4)y = 96x^{2} + \sin 2x - k$   | Understand | CO 1 | AHSB02.04 |
| 13 | Solve the differential equation $(D^2 + 6D + 9)y = sin3x$   | Understand | CO 1 | AHSB02.04 |
| 14 | Solve the differential equation $(D^2 + 2D + 1)y = x^2$   | Understand | CO 1 | AHSB02.04 |
| 15 | Solve the differential equation $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$   | Understand | CO 1 | AHSB02.04 |
| 16 | Solve the differential equation $(D^2 + 1)y = \sin x \sin 2x + e^x x^2$   | Understand | CO 1 | AHSB02.04 |
| 17 | Solve the differential equation $(D^3 + 1)y = 3 + 5e^x$   | Understand | CO 1 | AHSB02.04 |
| 18 | Solve the differential equation $(D^2 - 3D + 2)y = \cos hx$   | Understand | CO 1 | AHSB02.04 |
| 19 | Solve the differential equation $(D^2 + 4)y = x \cos x$   | Understand | CO 1 | AHSB02.04 |
| 20 | Solve the differential equation $(D^2 + 9)y = \cos 3x + \sin 2x$  | Understand | CO 1 | AHSB02.04 |
|    |   |            |      |           |

| Part - | C (Problem Solving and Critical Thinking Questions)  |            |      |             |
|--------|--|------------|------|-------------|
| 1      | Find the Inverse of a matrix by using Gauss-Jordan method  | Understand | CO 1 | AHSB02.03   |
|        |  |            |      |             |
|        | $\begin{bmatrix} A & 1 & 2 & 2 \end{bmatrix}$  |            |      |             |
|        | $A = \begin{bmatrix} 1 & 5 & -5 \end{bmatrix}$   |            |      |             |
|        | $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$   |            |      |             |
| 2      | Find the Inverse of a matrix by using Gauss-Jordan method  | Understand | CO 1 | AHSB02.03   |
|        | $\begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$  |            |      |             |
|        | $\begin{pmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}$  |            |      |             |
| 3      | $\begin{bmatrix} 1 & 2 & 4 \end{pmatrix}$  | Understand | CO 1 | AHSB02.02   |
|        |  |            |      |             |
|        | Find the rank of the matrix 2 1 3 4 by Normal form.  |            |      |             |
|        | 2 3 4 7 3  |            |      |             |
|        | 2 3 1 4  |            |      |             |
| 4      | $\begin{bmatrix} 2 & 3 & 1 & 4 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 & -3 & -6 \end{bmatrix}$                        | Understand | CO 1 | AHSB02.03   |
| 4      | Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ . by canonical     | Onderstand | CO 1 | AD3DU2.U3   |
|        | [1 1 1 2]  |            |      |             |
|        | form   | TT1        | 60.1 | AHGDOG OG   |
| 5      | Find the inverse of $A$ if $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary row | Understand | CO 1 | AHSB02.03   |
|        | Find the inverse of A if $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ by elementary row                             |            |      |             |
|        |  |            |      |             |
|        |  |            |      |             |
|        | operation.   |            |      |             |
| 6      | By using method of variation of parameters solve   | Understand | CO 1 | AHSB02.05   |
|        | $y'' + y = x \cos x.$  |            |      |             |
| 7      | Solve the differential equation  | Understand | CO 1 | AHSB02.04   |
|        | $\left(D^3 - 4D^2 - D + 4\right)y = e^{3x}\cos 2x$   |            |      |             |
| 0      | ,  | TT 1 . 1   | GO 1 | A 110D02.07 |
| 8      | Solve the differential equation $(D^2 + 3D + 2)y = e^{e^x}$ , By using   | Understand | CO 1 | AHSB02.05   |
|        | method of variation of parameters  |            |      |             |
| 9      | Solve the differential equation  | Understand | CO 1 | AHSB02.04   |
| 4.0    | $(D^3 - 5D^2 + 8D - 4)Y = e^x + 3e^{-x} + xe^x$<br>Apply the method of variation parameters to solve                 | **         | G0 : | ATTORNOS    |
| 10     |  | Understand | CO 1 | AHSB02.05   |
|        | $(D^2 + a^2)y = \tan ax$   |            |      |             |
|        | MODULE-II  |            | DAIG |             |
|        | LINEAR TRANSFORMATIONS AND Part – A (Short Answer Questi   |            | KALS |             |
| 1      | State Cayley- Hamilton theorem.  | Understand | CO 2 | AHSB02.06   |
| 2      | [2 2 1]  | Understand | CO 2 | AHSB02.06   |
|        |  |            |      |             |
|        | Find the sum of Eigen values of the matrix \ \begin{array}{c c c c c c c c c c c c c c c c c c c                     |            |      |             |
|        | [1 2 2]  |            |      |             |
| 3      | Show that the vectors $X_1$ =(1,1,2), $X_2$ =(1,2,5) and $X_3$ =(5,3,4) are  | Understand | CO 2 | AHSB02.09   |
|        | linearly dependent.  |            |      |             |
| 4      | [ 6 −2 2 ]   | Remember   | CO 2 | AHSB02.06   |
|        | Find the characteristic equation of the matrix $A = \begin{bmatrix} -2 & 3 & -1 \end{bmatrix}$                       |            |      |             |
|        | 2 -1 3   |            |      |             |
| 1      |  |            | 1    |             |

| 5      | Find the Eigen values of the matrix $ \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} $  | Understand | CO 2 | AHSB02.06 |
|--------|--|------------|------|-----------|
| 6      | Show that the vectors $X_1=(1,1,1)$ $X_2=(3,1,2)$ and $X_3=(2,1,4)$ are  | Understand | CO 2 | AHSB02.09 |
| 0      | linearly independent.  |            |      |           |
| 7      | Define Modal and Spectral matrices.  | Understand | CO 2 | AHSB02.10 |
| 8      | Define diaganalisation of a matrix.  | Understand | CO 2 | AHSB02.10 |
| 9      | Find the Eigen values of the matrix $A^{-1}$ , $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$  | Understand | CO 2 | AHSB02.06 |
| 10     | Find the eigen values $A^3$ of the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$   | Understand | CO 2 | AHSB02.06 |
| 11     | $\Gamma$   | Remember   | CO 2 | AHSB02.12 |
|        | Evaluate the double integral $\int_0^2 \int_0^x y dy dx$ .   |            |      |           |
| 12     | Evaluate the double integral $\int_0^{\pi} \int_0^{a \sin \theta} d\theta$ .   | Understand | CO 2 | AHSB02.12 |
| 13     | Evaluate the double integral $\int_0^3 \int_0^1 xy(x + y) dxdy$ .  | Understand | CO 2 | AHSB02.12 |
| 14     | Find the value of double integral $\int_{-\infty}^{2} \int_{-\infty}^{3} xy^2 dx dy$ .   | Understand | CO 2 | AHSB02.12 |
| 15     | Evaluate the double integral $\int_0^1 \int_x^{x^2} xy dx dy$  | Understand | CO 2 | AHSB02.12 |
| 16     | Evaluate the double integral $\int_{0}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{2}} \sin(x+y)  dx  dy$   | Remember   | CO 2 | AHSB02.12 |
| 17     | Evaluate the double integral $1 \int_0^1 \int_1^2 xy dx dy$  | Understand | CO 2 | AHSB02.12 |
| 18     | Evaluate the double integral $\int_0^\infty \int_0^{\pi/2} e^{-r^2} r  d\theta  dr$ .  | Understand | CO 2 | AHSB02.12 |
| 19     | Evaluate the double integral $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r  dr  d\theta$ .  | Understand | CO 2 | AHSB02.12 |
| 20     | State the formula to find area of the region using double integration in Cartesian form.   | Remember   | CO 2 | AHSB02.12 |
| Part - | B (Long Answer Questions)  | Understand | CO 2 | AHSB02.06 |
| 1      | Find the characteristic vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  | Understand | CO 2 | An3b02.00 |
| 2      | Diagonalisation of matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 2 & 4 & 1 \end{bmatrix}$  | Understand | CO 2 | AHSB02.11 |
| 3      | Show that matrix $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfying Cayley-Hamilton theorem <i>and</i> hence find its inverse, if its exists. | Understand | CO 2 | AHSB02.11 |
| 4      | Use Cayley-Hamilton theorem to find $A^3$ , if $A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$  | Understand | CO 2 | AHSB02.11 |
|        |  |            |      |           |

| 5  | Find the Eigen values and Eigen vectors of the matrix A and its $\begin{bmatrix} 1 & 3 & 4 \end{bmatrix}$  | Understand                       | CO 2      | AHSB02.06                       |
|----|--|----------------------------------|-----------|---------------------------------|
|    | inverse, where $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$   |                                  |           |                                 |
| 6  | Find a matrix P such that P <sup>-1</sup> AP is a diagonal matrix, where A=  | Understand                       | CO 2      | AHSB02.10                       |
|    | $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$   |                                  |           |                                 |
| 7  | Find the characteristic roots of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and the  | Understand                       | CO 2      | AHSB02.06                       |
|    |  |                                  |           |                                 |
| 8  | corresponding characteristic vectors.  Express A <sup>5</sup> -4A <sup>4</sup> -7A <sup>3</sup> +11A <sup>2</sup> -A-10I as a linear polynomial in A, where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$   | Understand                       | CO 2      | AHSB02.06                       |
| 9  |  | Understand                       | CO 2      | AHSB02.11                       |
|    | Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and find $A^{-1} \& A^{4}$ .   |                                  |           |                                 |
| 10 | Diagonalize the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$ by linear transformation   | Understand                       | CO 2      | AHSB02.10                       |
|    | and hence find A <sup>4</sup> .  |                                  |           |                                 |
|    |  |                                  |           |                                 |
| 11 | Evaluate the double integral $\int_{0}^{\pi} \int_{0}^{a(1+\cos\theta)} r^{2} \cos\theta dr d\theta.$  | Understand                       | CO 2      | AHSB02.12                       |
| 12 | Evaluate the double integral $\int_{0}^{1} \int_{0}^{x^{2}} r^{2} \cos \theta dr d\theta.$ Evaluate the double integral $\int_{0}^{1} \int_{0}^{\sqrt{x}} (x^{2} + y^{2}) dx dy.$  | Understand Understand            | CO 2      | AHSB02.12  AHSB02.12            |
| 12 | Evaluate the double integral $\int_{0}^{1} \int_{0}^{x^{2}} r^{2} \cos \theta dr d\theta.$ Evaluate the double integral $\int_{0}^{1} \int_{0}^{\sqrt{x}} (x^{2} + y^{2}) dx dy.$  |                                  |           |                                 |
| 12 | Evaluate the double integral $\int_{0}^{1} \int_{0}^{x^{2}} r^{2} \cos \theta dr d\theta.$ Evaluate the double integral $\int_{0}^{1} \int_{0}^{\sqrt{x}} (x^{2} + y^{2}) dx dy.$  | Understand                       | CO 2      | AHSB02.12                       |
| 12 | Evaluate the double integral $\int_{0}^{1} \int_{0}^{\sqrt{x}} (x^2 + y^2) dx dy$ .  Evaluate the double integral $\int_{0}^{5} \int_{x}^{x^2} x(x^2 + y^2) dx dy$ .  Evaluate the double integral $\int_{0}^{5} \int_{0}^{x^2} x(x^2 + y^2) dx dy$ .  Evaluate the double integral $\int_{0}^{1} \int_{0}^{\pi/2} r \sin \theta d\theta dr$ .  By changing the order of integration evaluate the double integral $\int_{0}^{1} \int_{x^2}^{2-x} xy \ dx \ dy$ . | Understand Understand            | CO 2      | AHSB02.12<br>AHSB02.12          |
| 12 | Evaluate the double integral $\int_{0}^{1} \int_{0}^{\sqrt{x}} r^2 \cos\theta dr d\theta$ .  Evaluate the double integral $\int_{0}^{1} \int_{x}^{\sqrt{x}} (x^2 + y^2) dx dy$ .  Evaluate the double integral $\int_{0}^{5} \int_{0}^{x^2} x (x^2 + y^2) dx dy$ .  Evaluate the double integral $\int_{0}^{1} \int_{0}^{\pi/2} r \sin\theta d\theta dr$ .  By changing the order of integration evaluate the double integral                                    | Understand Understand Understand | CO 2 CO 2 | AHSB02.12  AHSB02.12  AHSB02.12 |

| 17     | Find the value of $\iint xydxdy$ taken over the positive quadrant of  | Understand | CO 2 | AHSB02.12 |
|--------|---|------------|------|-----------|
|        | the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .  Evaluate the double integral using change of variables                           |            |      |           |
| 18     | Evaluate the double integral using change of variables  | Understand | CO 2 | AHSB02.13 |
|        | $\int_{0}^{\infty} \int_{0}^{\infty} e^{-(x^2+y^2)} dx dy.$   |            |      |           |
| 19     | By transforming into polar coordinates Evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$   | Understand | CO 2 | AHSB02.12 |
|        | over the annular region between the circles $x^2 + y^2 = a^2$ and   |            |      |           |
|        | $x^2 + y^2 = b^2 \text{ with } b > a$   |            |      |           |
| 20     | Find the area of the region bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$ .   | Understand | CO 2 | AHSB02.12 |
| Part - | C (Problem Solving and Critical Thinking Questions)   |            |      |           |
| 1      | Find Eigen values and Eigen vectors of $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$                          | Understand | CO 2 | AHSB02.06 |
| 2      | Examine whether the vectors [2,-1,3,2], [1,3,4,2], [3,5,2,2] is linearly independent or dependent?                                      | Understand | CO 2 | AHSB02.07 |
| 3      | Find Eigen values and corresponding Eigen vectors of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ | Understand | CO 2 | AHSB02.06 |
| 4      | Verify Cayley-Hamilton theorem for If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$                       | Understand | CO 2 | AHSB02.11 |
| 5      | Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and find $A^{-1}$ .          | Understand | CO 2 | AHSB02.11 |
|        |   |            |      |           |
| 6      | Evaluate $\iint r^3 dr d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$ .                    | Understand | CO 2 | AHSB02.12 |
| 7      | Find the area of the cardioid $r = a(1+\cos\theta)$ .   | Understand | CO 2 | AHSB02.12 |
| 8      | Find the area of the region bounded by the curves $y = x^3$ and $y = x$ .   | Understand | CO 2 | AHSB02.12 |
| 9      | Evaluate $\iint xydxdy$ taken over the positive quadrant of the   | Understand | CO 2 | AHSB02.12 |
|        | ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .   |            |      |           |
| 10     | By changing the order of integration Evaluate the double integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}  dy dx$                 | Understand | CO 2 | AHSB02.13 |
|        |   |            |      |           |

|           | MODULE-III  |            |      |            |  |  |
|-----------|---|------------|------|------------|--|--|
|           | FUNCTIONS OF SINGLE VARIABLES AND TRIPLE INTEGRALS  |            |      |            |  |  |
| 1         | Part - A (Short Answer Question   |            | CO 2 | A HCD02 14 |  |  |
| 1         | Discuss the applicability of Rolle's theorem for any function f(x) in interval [a,b].   | Understand | CO 3 | AHSB02.14  |  |  |
| 2         | Discuss the applicability of Lagrange's mean value theorem for any function f(x) in interval [a,b].   | Understand | CO 3 | AHSB02.14  |  |  |
| 3         | Discuss the applicability of Cauchy's mean value theorem for any function f(x) in interval [a,b].   | Understand | CO 3 | AHSB02.14  |  |  |
| 4         | Interpret Rolle's theorem geometrically.  | Understand | CO 3 | AHSB02.14  |  |  |
| 5         | Interpret Lagrange's mean value theorem geometrically.  | Remember   | CO 3 | AHSB02.14  |  |  |
| 6         | Given an example of function that is continuous on [-1, 1] and for which mean value theorem does not hold.  | Understand | CO 3 | AHSB02.14  |  |  |
| 7         | Using Lagrange's mean value theorem, find the value of c for $f(x) = \log x$ in $(1, e)$ .  | Understand | CO 3 | AHSB02.14  |  |  |
| 8         | Explain why mean value theorem does not hold for $f(x) = x^{2/3}$ in [-1,1]   | Understand | CO 3 | AHSB02.14  |  |  |
| 9         | Find the region in which $f(x) = 1 - 4x - x^2$ is increasing using mean value theorem.  | Understand | CO 3 | AHSB02.14  |  |  |
| 10        | If $f'(x) = 0$ throughout an interval [a, b], using mean value theorem show that $f(x)$ is constant.  | Understand | CO 3 | AHSB02.14  |  |  |
|           | show that I(x) is constant.   |            |      |            |  |  |
| 11        | Find the value of triple integral $\int_{-1}^{1} \int_{-2}^{2} \int_{-3}^{3} dx dy dz$ .  | Understand | CO 3 | AHSB02.15  |  |  |
| 12        | Find the volume of the tetrahedron bounded by the coordinate planes and the plane x+y+z=1.  | Understand | CO 3 | AHSB02.15  |  |  |
| 13        | State the formula to find volume of the region using triple integration in Cartesian form.  | Understand | CO 3 | AHSB02.15  |  |  |
| 14        | Evaluate the triple integral $\int_0^2 \int_1^3 \int_1^2 xy^2z \ dz \ dy \ dx$  | Understand | CO 3 | AHSB02.15  |  |  |
| 15        | Evaluate the triple integral $\int_0^a \int_0^x \int_0^y xyz \ dz \ dy \ dx$  | Understand | CO 3 | AHSB02.15  |  |  |
| 16        | Evaluate the triple integral $\int_0^1 \int_0^2 \int_0^3 (x+y+z) dz dy dx$  | Understand | CO 3 | AHSB02.15  |  |  |
| 17        | Evaluate the triple integral $\int_0^1 \int_0^1 \int_0^1 xz \ dz \ dy \ dx$   | Understand | CO 3 | AHSB02.15  |  |  |
| 18        | Evaluate the triple integral $\int_{-2}^{2} \int_{-3}^{3} \int_{-1}^{1} e^{x+y+z} dz dy dx$   | Understand | CO 3 | AHSB02.15  |  |  |
| 19        | Evaluate the triple integral $\int_0^2 \int_0^3 \int_0^1 dz \ dy \ dx$  | Understand | CO 3 | AHSB02.15  |  |  |
| 20        | Evaluate the triple integral $\int_0^1 \int_{-1}^1 \int_{-1}^1 (x^2 + y^2 + z^2) dz \ dy \ dx$  | Understand | CO 3 | AHSB02.15  |  |  |
| Part<br>1 | - B (Long Answer Questions)   | Understand | CO 3 | AHSB02.14  |  |  |
|           | Verify Rolle's theorem for the function $f(x) = e^{-x} \sin x$ in the interval $[0, \pi]$ .   | Onderstand | CO 3 | Alisbuz.14 |  |  |
| 2         | Show that for any $x > 0, 1 + x < e^x < 1 + xe^x$   | Understand | CO 3 | AHSB02.14  |  |  |
| 3         | Verify Lagrange's mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in the interval [0,4].   | Understand | CO 3 | AHSB02.14  |  |  |
| 4         | If a <b, <math="" prove="" that="">\frac{b-a}{1+b^2} &lt; Tan^{-1}b - Tan^{-1}a &lt; \frac{b-a}{1+a^2} using Lagrange's Mean value theorem and hence deduce the following.</b,> | Understand | CO 3 | AHSB02.14  |  |  |
|           | (i) $\frac{\pi}{4} + \frac{3}{25} < Tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$   |            |      |            |  |  |

|      |   |               |      | T            |
|------|---|---------------|------|--------------|
|      | $\frac{5\pi + 4}{20} < Tan^{-1}2 < \frac{\pi + 2}{4}$   |               |      |              |
|      | (ii) 20 Tun 2 4   |               |      |              |
| 5    | Analyze the value of c in the interval [3, 7] for the function  | Understand    | CO 3 | AHSB02.14    |
|      |   | Chacistana    |      | 1110202111   |
|      | $f(x) = e^x, g(x) = e^{-x}$   |               |      |              |
| 6    | Find value of the C using Cauchy's mean value theorem for   | Understand    | CO 3 | AHSB02.14    |
|      | $f(r) = \sqrt{r} \Re g(r) = \frac{1}{r} \ln \left[ a h \right] $ where $0 < a < h$  |               |      |              |
|      | $f(x) = \sqrt{x} \& g(x) = \frac{1}{\sqrt{x}}$ in [a,b] where $0 < a < b$   |               |      |              |
| 7    | Verify Cauchy's mean value theorem for $f(x) = x^2 & g(x) = x^3 \text{ in } [1,2]$  | Understand    | CO 3 | AHSB02.14    |
|      |   |               |      |              |
| 8    | and find the value of c.<br>Verify Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$  | Understand    | CO 3 | AHSB02.14    |
| 0    | where m, n are positive integers in [a, b].   | Officerstatio | CO 3 | A113D02.14   |
| 9    | Using mean value theorem, for $0 < a < b$ , prove that  | Understand    | CO 3 | AHSB02.14    |
|      |   |               |      |              |
|      | $1 - \frac{\pi}{b} < \log \frac{\pi}{c} < \frac{\pi}{c} - 1$ and hence show that $\frac{\pi}{c} < \log \frac{\pi}{c} < \frac{\pi}{c}$ .   |               |      |              |
| 10   | $1 - \frac{a}{b} < \log \frac{b}{a} < \frac{b}{a} - 1 \text{ and hence show that } \frac{1}{6} < \log \frac{6}{5} < \frac{1}{5}.$ Find all numbers c between a and b b which satisfies lagranges mean | Understand    | CO 3 | AHSB02.14    |
| 10   | value theorem ,for the following function( $x$ )= ( $x$ -1)( $x$ -2)( $x$ -3) in [0 4]  | Onderstand    | CO 3 | 711151502.14 |
|      |   |               |      |              |
| 11   | 1 1-z1-y-z  | Understand    | CO 3 | AHSB02.15    |
|      | Evaluate the triple integral $\int \int xyzdxdydz$ .  |               |      |              |
|      |   |               |      |              |
| 12   | Evaluate the triple integral $\int_{0}^{1} \int_{0}^{1} \int_{0}^{xyz} dx dy dz$ .  Evaluate the triple integral $\int_{0}^{\log 2} \int_{0}^{x} \int_{0}^{x+\log y} e^{x+y+z} dx dy dz$ .            | Understand    | CO 3 | AHSB02.15    |
|      | Evaluate the triple integral $\int \int e^{x+y+z} dx dy dz$ .   |               |      |              |
|      | 0 0 0   |               |      |              |
| 13   | Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ .   | Understand    | CO 3 | AHSB02.15    |
|      | Evaluate $J_0 J_0 = J_0 = \frac{1}{\sqrt{1-x^2-y^2-z^2}}$ .   |               |      |              |
| 14   | Find the volume of the tetrahedron bounded by the plane   | Understand    | CO 3 | AHSB02.15    |
|      | y = 0 $y = 0$ and the coordinate planes by tainle   |               |      |              |
|      | x=0,y=0,z=0;and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes by triple   |               |      |              |
|      | integration.  |               |      |              |
| 15   | Using triple integration find the volume of the sphere $x^2+y^2+z^2=a^2$ .  | Understand    | CO 3 | AHSB02.15    |
| 16   | Evaluate $\iiint dxdydz$ where v is the finite region of space formed by  | Understand    | CO 3 | AHSB02.15    |
|      | Evaluate III and yaz, where v is the finite region of space formed by   |               |      |              |
|      | the planes $x=0,y=0,z=0$ and $2x+3y+4z=12$ .  |               |      |              |
| 17   | ***   | Understand    | CO 3 | AHSB02.15    |
|      | Evaluate $\iiint (x + y + z)dzdydx$ where R is the region bounded by  |               |      |              |
|      | R = 1 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 -   |               |      |              |
|      | the plane $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$   |               |      |              |
| 18   | the plane $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$<br>Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{x^2}{c^2} = 1$   | Understand    | CO 3 | AHSB02.15    |
| 19   | If R is the region bounded by the planes $x=0,y=0,z=1$ and the cylinder   | Understand    | CO 3 | AHSB02.15    |
|      | $x^2 + y^2 = 1$ , evaluate $\iiint xyzdxdydz$ .   |               |      |              |
|      | R JJJ Volumby 115   |               |      |              |
| 20   |   | Understand    | CO 3 | AHSB02.15    |
|      | Evaluate $\int_{0}^{1} \int_{0}^{1} \int_{x^{2}+y^{2}}^{2} xyz \ dz \ dy \ dx$  |               |      |              |
| Part | - C (Problem Solving and Critical Thinking Questions)   |               |      | I            |
| 1    | Verify the hypothesis and conclusion of rolles thorem for the   | Understand    | CO 3 | AHSB02.14    |
| _    | function defined below $f(x)=x^3-6x^2+11x-6$ in [13]  |               |      |              |
| 2    | function defined below $f(x)=x^3-6x^2+11x-6$ in [13]<br>Verify the hypothesis and conclusion of rolles thorem for the   | Understand    | CO 3 | AHSB02.14    |
|      | function defined below $f(x) = \frac{\log \mathbb{E}(x^2 + ab)}{(a+b)x} in [a b]$   |               |      |              |
|      | $(a+b)x \qquad (a+b)x$  |               |      |              |
| 3    | Use lagranges mean value theorem to establish the following   | Understand    | CO 3 | AHSB02.14    |
| J    | OSC lagranges mean value incorem to establish the following   | Understalld   | COS  | A113DU2.14   |

| i                      | nequalities $x \le \sin^{-1} x \le \frac{x}{\sqrt{1-x^2}} $ for $0 \le x \le 1$  |                |        |                     |
|------------------------|--|----------------|--------|---------------------|
|                        | Calculate approximately $\sqrt[5]{245}$ by using L.M.V.T.  | Understand     | CO 3   | AHSB02.14           |
| 5 <sub>\(\cdot\)</sub> | Verify Cauchy's mean value theorem for $f(x) = x^3 & g(x) = 2-x$ in  | Understand     | CO 3   | AHSB02.14           |
|                        | 0,9] and find the value of c.  |                |        |                     |
|                        |  |                |        |                     |
| $06 \mid E$            | Evaluate $\int_0^a \int_0^b \int_0^c (x+y+z) dx dy dz$ .   | Understand     | CO 3   | AHSB02.15           |
| 07<br>H                | Evaluate $\int_0^a \int_0^b \int_0^c (x+y+z) dx dy dz.$ Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz dx dy dz$ | Understand     | CO 3   | AHSB02.15           |
| 08                     | Evaluate   | Understand     | CO 3   | AHSB02.15           |
|                        | $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z) dx  dy  dz$   |                |        |                     |
|                        | Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ where D is the region bounded by the planes $z=0, y=0, z=0, x+y+z=1$                        | Understand     | CO 3   | AHSB02.15           |
| 10 E                   | Evaluate $\iiint xyz \ dxdydz$ where D is the region bounded by the positive octant of the sphere $x^2+y^2+z^2=a^2$ .                    | Understand     | CO 3   | AHSB02.15           |
|                        | MODULE-IV FUNCTIONS OF SEVERAL VARIABLES AND EXT   | PDEMA OF A FIL | NCTION |                     |
|                        | Part - A (Short Answer Question  |                | NCTION |                     |
| 1                      | If $x = \frac{u^2}{v}$ , $y = \frac{v^2}{v}$ , find the value of $\frac{\partial(u,v)}{\partial(x,y)}$                                   | Understand     | CO 4   | AHSB02.17           |
| 2                      | The stationary point of the function $f(x,y) = x^2 + y^2 + xy + x - 4y + 5$  | Understand     | CO 4   | AHSB02.18           |
| 3                      | If $x = u(1-v)$ , $y = uv$ , find the value of $J'$ .  | Understand     | CO 4   | AHSB02.17           |
| 4                      | Calculate $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ if $u = \frac{2yz}{x}, v = \frac{3zx}{y} w = \frac{4xy}{z}$                      | Understand     | CO 4   | AHSB02.17           |
| 5                      | If $x = u(1+v)$ , $y = v(1+u)$ then find the value of $\frac{\partial(x,y)}{\partial(u,v)}$  | Understand     | CO 4   | AHSB02.17           |
| 6                      | Write the condition for the function $f(x,y)$ to be functionally dependent.  | Understand     | CO 4   | AHSB02.17           |
| 7                      | Define jacobian of a function.   | Understand     | CO 4   | AHSB02.17           |
| 8                      | Define a saddle point for the function of $f(x, y)$ .  | Understand     | CO 4   | AHSB02.17           |
| 9                      | Write the condition for the function f(x,y) to be functionally independent.  | Understand     | CO 4   | AHSB02.17           |
| 10                     | Define a extreme point for the function of $f(x, y)$ .   | Understand     | CO 4   | AHSB02.18           |
| 11                     | Define Stationary points   | Understand     | CO 4   | AHSB02.18           |
| 12                     | Define maxium function ?   | Understand     | CO 4   | AHSB02.18 AHSB02.18 |
| 13                     | Define a minimum function?   | Understand     | CO 4   | AHSB02.18           |
| 14                     | If u and v are functions of x and y then prove that $JJ'=1$  | Understand     | CO 4   | AHSB02.18           |
| 15                     | $X = rcos\theta, Y = rsin\theta$ find J  | Understand     | CO 4   | AHSB02.17           |
| 16                     | If $X = \log(x \tan^{-1} y)$ then $f_{xy}$ is equal to zero  | Understand     | CO 4   | AHSB02.16           |
| 17                     | If $f(x,y,z)=0$ then the values $\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial x} = -1$         | Understand     | CO 4   | AHSB02.16           |
| 18                     | Prove that if the function u,v,w of three independent variables x,y,z  | Understand     | CO 4   | AHSB02.17           |

|      | are not independent ,then the Jacobian of u,v,w w.r.t x,y,z is always equals to zero.   |            |      |                   |
|------|---|------------|------|-------------------|
| 19   | If $z = \cos(\frac{x}{y}) + \sin(\frac{x}{y})$ , then Show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial x} = 0$                   | Understand | CO 4 | AHSB02.20         |
| 20   | Write the properties of maxima and minima under the various conditions.   | Understand | CO 4 | AHSB02.20         |
| Part | - B (Long Answer Questions)   |            |      |                   |
| 1    | i) If $x = u(1 - v)$ , $y = uv$ then prove that $JJ'=1$ .   | Understand | CO 4 | AHSB02.18         |
|      | ii) If $x + y^2 = u$ , $y + z^2 = v$ , $z + x^2 = w$ find the value of  |            |      |                   |
|      | $\partial(x,y,z)$   |            |      |                   |
|      | $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ .   |            |      |                   |
| 2    | If $u = x^2 - y^2$ , $v = 2xy$ where $x = r\cos\theta$ , $y = r\sin\theta$ then   | Understand | CO 4 | AHSB02.18         |
|      | show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$   |            |      |                   |
| 3    | If $x = e^r \sec \theta$ , $y = e^r \tan \theta$ Prove that $\frac{\partial(x, y)}{\partial(r, \theta)} \frac{\partial(r, \theta)}{\partial(x, y)} = 1$ . | Understand | CO 4 | AHSB02.18         |
| 4    | If $ux = yz, vy = zx, wz = xy$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$   | Understand | CO 4 | AHSB02.18         |
| 5    |   | Understand | CO 4 | AHSB02.18         |
|      | If $x = \frac{u^2}{v}$ , $y = \frac{v^2}{u}$ then find the Jacobian of the function u and v with respect to x and y                                       |            |      |                   |
| 6    | Show that the functions   | Understand | CO 4 | AHSB02.19         |
|      | $u = x + y + z, v = x^{2} + y^{2} + z^{2} - 2xy - 2yz - 2xz$ and  |            |      |                   |
|      | $w = x^3 + y^3 + z^3 - 3xyz$ are functionally related.  |            |      |                   |
| 7    | If $x = u$ , $y = tanv$ , $z = w$ then prove that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u \sec^2 v$  | Understand | CO 4 | AHSB02.18         |
| 8    | Show that the functions $u = e^x \sin y, v = e^x \cos y$ are not  | Understand | CO 4 | AHSB02.18         |
|      | functionally related.   | ** 1       | GO 1 | A *** G D O O O O |
| 9    | Prove that $u = x + y + z$ , $v = xy + yz + zx$ , $w = x^2 + y^2 + z^2$ are functionally dependent.   | Understand | CO 4 | AHSB02.18         |
| 10   | If $u = x + y + z$ , $uv = y + z$ , $z = uvw$   | Understand | CO 4 | AHSB02.18         |
|      | Prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$  |            |      |                   |
|      | $\sigma(u,v,w)$   |            |      |                   |
| 11   | Find the maximum value of the function xyz when $x + y + z = a$ .   | Understand | CO 4 | AHSB02.19         |
| 12   | Find the maxima and minima of the function $f(x, y) = x^3y^2$ (1-x-y).  | Understand | CO 4 | AHSB02.20         |
| 13   | Find the maximum and minimum of the function  | Understand | CO 4 | AHSB02.20         |
|      | $f(x,y) = \sin x + \sin y + \sin(x+y)$  |            |      |                   |
| 14   | Find the maximum and minimum values of  | Understand | CO 4 | AHSB02.20         |
|      | $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$  |            |      |                   |
| 15   | Find the shortest distance from the origin to the surface $xyz^2 = 2$   | Understand | CO 4 | AHSB02.18         |
| 16   | Find the minimum value of $x^2 + y^2$ , subjects to the condition $ax+by=c$ using lagranges multipliers method.   | Understand | CO 4 | AHSB02.18         |
| 17   | Find the points on the surface $z^2 = xy + 1$ nearest to the origin.  | Understand | CO 4 | AHSB02.20         |
| 18   | Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube using lagranges multipliers method                    | Understand | CO 4 | AHSB02.20         |
| 19   | Find the value of the largest rectangular parallelepiped that can be  | Understand | CO 4 | AHSB02.20         |
| /    | 1 ms ms , area of the largest resumbatar parameteriped that can be  | Chathana   |      | 11100002.20       |

|      | 2 2 2  |                      |       |                        |
|------|--|----------------------|-------|------------------------|
|      | inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .   |                      |       |                        |
|      | $a^2$ $b^2$ $c^2$  |                      |       |                        |
| 20   |  | XX 1 . 1             | GO 1  | 4 11GD 02 20           |
| 20   | Find the stationary points of $U(x,y) = \sin x \sin y \sin(x+y)$ where   | Understand           | CO 4  | AHSB02.20              |
|      | $0 < x < \pi, 0 < y < \pi$ and find the maximum value of the function U.   |                      |       |                        |
| Part | - C (Problem Solving and Critical Thinking)  |                      |       |                        |
| 1    | If $u = x + 3y^2 + z^3$ , $v = 4x^2yz$ , $w = 2z^2 - xy$ then find   | Understand           | CO 4  | AHSB02.17              |
|      |  |                      |       |                        |
|      | $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1,-1,0).   |                      |       |                        |
|      | $\partial(x,y,z)$  |                      |       |                        |
| 2    | If $u = e^{xyz}$ , show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$   | Understand           | CO 4  | AHSB02.16              |
| 3    | If   | Understand           | CO 4  | AHSB02.16              |
|      | $u = \log(x^2 + y^2 + z^2), prove that$  |                      |       |                        |
|      | $(x^{2} + y^{2} + z^{2}) \left( \frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial^{2} u}{\partial x^{2}} + \frac{\partial^{2} u}{\partial x^{2}} \right) = 2$ |                      |       |                        |
| 4    | $\langle v_n - v_n \rangle$  | TT1                  | CO. 1 | AHGDOC 17              |
| 4    | Determine whether the following functions are functionally dependent or not .If functionally dependent, find the relation  | Understand           | CO 4  | AHSB02.17              |
|      | between them.  |                      |       |                        |
|      | $u = \frac{x-y}{x+z}, v = \frac{x+z}{y+z}$   |                      |       |                        |
| 5    | x+z, $y+z$ Determine whether the following functions are functionally  | Understand           | CO 4  | AHSB02.18              |
| )    | dependent or not .If functionally dependent , find the relation  | Understand           | CO 4  | AHSB02.16              |
|      | between them.  |                      |       |                        |
|      | $u = \frac{x+y}{1-xy}, v = tan^{-1}x + tan^{-1}y$  |                      |       |                        |
|      | 1-xy   |                      |       |                        |
| 6    | Find the maxima value of $u = x^2 y^3 z^4$ with the constrain condition  | Understand           | CO 4  | AHSB02.18              |
|      | -  |                      |       |                        |
|      | 2x + 3y + 4z = a   |                      |       |                        |
| 7    | Find the point of the plane $x+2y+3z=4$ that is closed to the  | Understand           | CO 4  | AHSB02.20              |
|      | origin.  |                      |       |                        |
| 8    | Divide 24 into three parts such that the continued product of the first,   | Understand           | CO 4  | AHSB02.18              |
| 9    | square of the second and cube of the third is maximum.  Find three positive numbers whose sum is 100 and whose product is  | Understand           | CO 4  | AHSB02.18              |
|      | maximum.   | Officerstand         | CO 4  | Alisb02.16             |
| 10   | A rectangular box open at the top is to have volume of 32 cubic $ft$ .   | Understand           | CO 4  | AHSB02.18              |
|      | Find the dimensions of the box requiring least material for its  |                      |       |                        |
|      | construction.  |                      |       |                        |
|      | MODULE-V   |                      |       |                        |
|      | VECTOR CALCULUS  |                      |       |                        |
| 1 1  | Part - A (Short Answer Question  |                      | CO 5  | ALICDO2 10             |
| 2    | Define gradient of scalar point function.  Define divergence of vector point function.   | Remember<br>Remember | CO 5  | AHSB02.19<br>AHSB02.19 |
|      | Define curl of vector point function.  Define curl of vector point function.   | Remember             | CO 5  | AHSB02.19<br>AHSB02.19 |
| 4    | State Laplacian operator.  | Understand           | CO 5  | AHSB02.19              |
|      | Find curl $\bar{f}$ where $\bar{f} = \operatorname{grad}(x^3 + y^3 + z^3 - 3xyz)$ .  | Understand           | CO 5  | AHSB02.19              |
| 6    | Find the angle between the normal to the surface $xy = z^2$ at the points  | Understand           | CO 5  | AHSB02.19              |
| 7    | (4, 1, 2) and $(3,3,-3)$ .<br>Find a unit normal vector to the given surface $x^2y+2xz=4$ at the   | Understand           | CO 5  | AHSB02.19              |
|      | point(2,-2,3).   | Chacistana           |       | 7415002.19             |
|      | If $\bar{a}$ is a vector then prove that grad $(\bar{a}.\bar{r}) = \bar{a}$ .  | Understand           | CO 5  | AHSB02.19              |
|      |  | D 1                  | CO 5  | ATICDOS 10             |
|      | Define irrotational vector and solenoidal vector of vector point function.   | Remember             | CO 5  | AHSB02.19              |

| 10 | <del></del>  | Understand               | CO 5 | AHSB02.19              |
|----|--|--------------------------|------|------------------------|
| 10 | Show that $\nabla (f(r)) = \frac{\overline{r}}{r} f'(r)$ .   | Understand               | CO 3 | Ansbu2.19              |
|    | r  |                          |      |                        |
| 11 | Duovo that for any in many is impossional vector   | Understand               | CO 5 | AHSB02.19              |
|    | Prove that $f = yzi + zxj + xyk$ is irrotational vector.   |                          |      |                        |
| 12 | Show that (x+3y)i+(y-2z)j+(x-2z)k is solenoidal.  Define work done by a force, circulation.  | Understand<br>Understand | CO 5 | AHSB02.20<br>AHSB02.20 |
| 14 | State Stokes theorem of transformation between line integral and   | Understand               | CO 5 | AHSB02.22              |
|    | surface integral.  |                          |      |                        |
| 15 | Prove that div curl $\bar{f}$ = 0 where $\bar{f} = f_1 \bar{i} + f_2 \bar{j} + f_3 \bar{k}$ .  | Understand               | CO 5 | AHSB02.20              |
| 16 | Define line integral on vector point function.   | Remember                 | CO 5 | AHSB02.21              |
| 17 | Define surface integral of vector point function $F$ .   | Remember                 | CO 5 | AHSB02.21              |
| 18 | Define volume integral on closed surface S of volume V.  | Remember                 | CO 5 | AHSB02.21              |
| 19 | State Green's theorem of transformation between line integral and double integral.   | Understand               | CO 5 | AHSB02.22              |
| 20 | State Gauss divergence theorem of transformation between surface integral and volume integral.   | Understand               | CO 5 | AHSB02.22              |
|    | Part - B (Long Answer Question   | ons)                     |      |                        |
| 1  | Evaluate $\int_{C} \overline{f} . d\overline{r}$ where $\overline{f} = 3xyi - y^2j$ and C is the parabola $y=2x^2$                     | Understand               | CO 5 | AHSB02.21              |
|    | from points $(0,0)$ to $(1,2)$ .   |                          |      |                        |
| 2  | from points $(0, 0)$ to $(1, 2)$ .<br>Evaluate $\iint_{S} \bar{F}.d\bar{s}$ if $\bar{F} = yzi + 2y^2j + xz^2k$ and S is the Surface of | Understand               | CO 5 | AHSB02.21              |
|    | the cylinder $x^2+y^2=9$ contained in the first octant between the planes $z=0$ and $z=2$ .  |                          |      |                        |
| 3  | Find the work done in moving a particle in the force field   | Understand               | CO 5 | AHSB02.21              |
|    | $\overline{F} = (3x^2)i + (2zx - y)j + zk$ along the straight line from(0,0,0)   |                          |      |                        |
|    | to (2,1,3).  |                          |      |                        |
| 4  | Find the circulation of  | Understand               | CO 5 | AHSB02.21              |
|    | $\overline{F} = (2x - y + 2z)\overline{i} + (x + y - z)\overline{j} + (3x - 2y - 5z)\overline{k}$ along                                |                          |      |                        |
|    | the circle $x^2 + y^2 = 4$ in the xy plane.  |                          |      |                        |
| 5  | Verify Gauss divergence theorem for the vector point function  | Understand               | CO 5 | AHSB02.22              |
| 3  | F = $(x^3-yz)i - 2yxj + 2zk$ over the cube bounded by $x = y = z = 0$ and  | Understand               | CO 3 | Апэви2.22              |
|    | x = y = z = a.   |                          |      |                        |
| 6  | Verify Gauss divergence theorem for $2x^2yi - y^2j + 4xz^2k$ taken over  | Understand               | CO 5 | AHSB02.22              |
|    | the region of first octant of the cylinder $y^2 + z^2 = 9$ and $x = 2$ .   |                          |      |                        |
| 7  | Verify Green's theorem in the plane for  | Understand               | CO 5 | AHSB02.22              |
|    | $\int_{C} (x^2 - xy^3) dx + (y^2 - 2xy) dy$ where C is a square with vertices  |                          |      |                        |
|    | (0,0),(2,0),(2,2),(0,2).   |                          |      |                        |
| 8  | Applying Green's theorem evaluate $\iint (y - \sin x) dx + \cos x dy$ where C  | Understand               | CO 5 | AHSB02.22              |
|    | is the plane triangle enclosed by $y = 0$ , $y = \frac{2x}{\pi}$ , and $x = \frac{\pi}{2}$ .   |                          |      |                        |
| 9  | Apply Green's Theorem in the plane for   | Understand               | CO 5 | AHSB02.22              |
|    | $\int_{C}^{11} (2x^{2} - y^{2}) dx + (x^{2} + y^{2}) dy$ where C is a is the boundary of the area                                      |                          |      |                        |
|    | enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$ .  |                          |      |                        |
|    |  |                          |      |                        |
|    |  |                          |      |                        |

| 10 | Verify Stokes theorem for $f = (2x - y)i - yz^2j - y^2zk$ where S  | Understand   | CO 5  | AHSB02.22    |
|----|--|--------------|-------|--------------|
|    | is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by   |              |       |              |
|    | the projection of the xy plane.  |              |       |              |
| 11 | Verify Stokes theorem for $\overline{f} = (x^2 - y^2)\overline{i} + 2xy\overline{j}$ over the box bounded                                    | Understand   | CO 5  | AHSB02.24    |
| 11 | by the planes $x=0$ , $x=a$ , $y=0$ , $y=b$ .  | Chacistana   | 603   | 711151502.21 |
| 12 | Find the directional derivative of the function $\phi = xy^2 + yz^3$ at the  | Understand   | CO 5  | AHSB02.21    |
|    | point P(1,-2,-1) in the direction to the surface $x \log z - y^2 =$  |              |       |              |
| 12 | -4 at (-1,2,1).  | I Indonesa d | CO 5  | ALICDO2 20   |
| 13 | If $\overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k}$ evaluate $\int \overline{F}.\overline{n}ds$ where S is the surface of | Understand   | 003   | AHSB02.20    |
|    | the cube $x = 0$ , $x = a$ , $y = 0$ , $y = a$ , $z = 0$ , $z = a$ .   |              |       |              |
| 14 | If $\bar{f} = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$ evaluate $\int \bar{f} \cdot d\bar{r}$ along the                                       | Understand   | CO 5  | AHSB02.21    |
|    | C  |              |       |              |
| 15 | curve C in xy-plane $y = x^3$ from (1,1) to (2,8).<br>Evaluate the line integral $\int (x^2 + xy)dx + (x^2 + y^2)dy$ where C is              | Understand   | CO 5  | AHSB02.21    |
|    | c  |              |       |              |
|    | the square formed by lines $x = \pm 1$ , $y = \pm 1$ .   |              |       |              |
| 16 | If $r = x\overline{i} + y\overline{j} + z\overline{k}$ show that $\nabla r^n = nr^{n-2}\overline{r}$ .                                       | Understand   | CO 5  | AHSB02.19    |
| 17 | Evaluate by Stokes theorem $\int (e^x dx + 2y dy - dz)$ where c is the   | Understand   | CO 5  | AHSB02.22    |
|    | $\stackrel{\circ}{c}$  |              |       |              |
|    | curve $x^2+y^2=9$ and $z=2$ .  |              |       |              |
| 18 | Verify Stokes theorem for the function $x^2 \vec{i} + xy \vec{j}$ integrated round the   | Understand   | CO 5  | AHSB02.22    |
|    | square in the plane z=0 whose sides are along the line x=0,y=0,x=a,  |              |       |              |
| 19 | y=a.   | Understand   | CO 5  | AHSB02.22    |
| 19 | Evaluate by Stokes theorem $\int_{C} (x+y)dx + (2x-z)dy + (y+z)dz$   | Onderstand   | CO 3  | AHSB02.22    |
|    | where C is the boundary of the triangle with vertices  |              |       |              |
| 20 | (0,0,0),(1,0,0),(1,1,0).<br>Verify Green's theorem in the plane for  | Understand   | CO 5  | AHSB02.22    |
| 20 | $\int (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is a region bounded by   | Onderstand   | CO 3  | Alisbuz.zz   |
|    | $C = \begin{cases} \int_{C} (3x - 6y) dx + (4y - 6xy) dy \text{ where } C \text{ is a region sounded } 0y \\ C \end{cases}$                  |              |       |              |
|    | $y = \sqrt{x}$ and $y = x^2$ .   |              |       |              |
|    | Part – C (Problem Solving and Critica  |              |       |              |
| 1  | Verify Gauss divergence theorem for $\bar{f} = x^2 \bar{i} + y^2 \bar{j} + z^2 \bar{k}$ taken  | Understand   | CO 5  | AHSB02.22    |
|    | over the cube bounded by x=0,x=a, y=0,y=b, z=0,z=c.  | TT: 1        | CO. 5 | A HGD02 21   |
| 2  | Find the work done in moving a particle in the force field $\overline{E}$  | Understand   | CO 5  | AHSB02.21    |
|    | $\overline{F} = (3x^2)i + (2zx - y)j + zk$ along the curve defined by  |              |       |              |
|    | $x^2 = 4y$ , $3x^3 = 8z$ from x=0 and x=2.   |              |       |              |
| 3  | Show that the force field given by   | Understand   | CO 5  | AHSB02.20    |
|    | $\overline{F} = 2xyz^3i + x^2z^3j + 3x^2yz^2k$ is conservative. Find the work  |              |       |              |
|    | done in moving a particle from (1,-1,2) to (3,2,-1) in this force field.   |              |       |              |
| 4  | Show that the vector $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is  | Understand   | CO 5  | AHSB02.21    |
|    | irrotational and find its scalar potential function.   |              |       |              |
|    | <u> </u>   |              |       |              |

| 5  | Using Gauss divergence theorem evaluate $\iint_S \vec{F} \cdot d\vec{s}$ , for the $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylinder region S given by $x^2 + y^2 = a^2$ , $z = 0$ and $z = b$ . | Understand      | CO 5 | AHSB02.22 |
|----|--|-----------------|------|-----------|
|    |  | II. 1. material | CO 5 | AHCDO2 20 |
| 6  | Find the directional derivative of $\phi(x, y, z) = x^2 yz + 4xz^2$ at the   | Understand      | CO 5 | AHSB02.20 |
|    | point(1,-2,-1) in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2 at (-1,2,1)$ .   |                 |      |           |
| 7  | Using Green's theorem in the plane evaluate  | Understand      | CO 5 | AHSB02.22 |
|    | $\int_{C} (2xy - x^{2})dx + (x^{2} + y^{2})dy$ where C is the region bounded by $y = x^{2}$ and $y^{2} = x$ .  |                 |      |           |
| 8  | Applying Green's theorem evaluate $\int_{c} (xy + y^2)dx + x^2 dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$ .   | Understand      | CO 5 | AHSB02.22 |
| 9  | Verify Green's Theorem in the plane for  | Understand      | CO 5 | AHSB02.22 |
|    | $\int_{C} (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where C is the region bounded by x=0, y=0 and x + y=1.   |                 |      |           |
| 10 | Verify Stokes theorem for $\overline{F} = (y - z + 2)i + (yz + 4)j - xzk$ where  | Understand      | CO 5 | AHSB02.22 |
|    | S is the surface of the cube x=0, y=0, z=0 and x=2,y=2,z=2 above the xy-plane.   |                 |      |           |

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