LECTURE NOTES

ON

MODERN PHYSICS

B. Tech II semester

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UNIT – I

CRYSTALLOGRAPHY AND CRYSTAL STRUCTURES

Crystallography and crystal structures: Space lattice, unit cell, lattice parameters, crystal systems, Bravais lattices, directions and planes in crystals, Miller indices, interplanar spacing of orthogonal crystal systems, atomic radius, coordination number and packing factor of SC, BCC, FCC, NaCl and diamond structures.

UNIT – II
X-RAY DIFFRACTION AND DEFECTS IN CRYSTALS

X-ray diffraction: Bragg's law, Laue method, powder method and applications; Defects in crystals: Concepts of point defects, vacancies, substitutional, interstitial, frenkel, schottky defects, line defects and Burger's vector.

UNIT – III
LASERS AND SENSORS

Lasers: Characteristics of lasers, spontaneous and stimulated emission of radiation, metastable state, population inversion, lasing action, ruby laser, semiconductor diode laser and applications of lasers.

Sensors: Introduction, basic principles, sensor materials and applications: principle of pressure, optical, acoustic and thermal sensing.

UNIT – IV
FIBER OPTICS

Fiber optics: Principle and construction of an optical fiber, acceptance angle, numerical aperture, types of optical fibers (Single mode, multimode, step index, graded index), attenuation in optical fibers, application of optical fibers and optical fiber communication system with block diagram.

UNIT – V
INTERFERENCE AND DIFFRACTION

Interference: Phase difference, path difference, coherence, conditions for constructive and destructive interference, interference in thin films due to reflected light, Newton rings experiment. Diffraction: Introduction, differences between interference and diffraction, types of diffraction, Fraunhofer diffraction due to single slit, N-slits, diffraction grating experiment.

Textbooks:
1. Dr. K. Vijaya Kumar, Dr. S. Chandralingam, "Modern Engineering Physics", Chand & Co. New Delhi, 1st Edition, 2010.
2. V. Rajendran, "Engineering Physics", Tata Mc Graw Hill Book Publishers, 1st Edition, 2010.

Reference books:
1. P. K. Palanisamy, "Engineering Physics", Scitech Publishers, 4th Edition, 2014.
2. R. K. Gaur, S. L. Gupta, "Engineering Physics", Dhanpat Rai Publications, 8th Edition, 2001.
3. A. J. Dekker, " Solid State Physics", Macmillan India ltd, 1st Edition, 2000.
4. Hitendra K. Malik, A. K. Singh, "Engineering Physics", Mc Graw Hill Education, 1st Edition, 2009.

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UNIT-I
CRYSTALLOGRAPHY AND CRYSTAL STRUCTURES

Matter exists in three different states namely solids, liquids and gaseous states. In liquid and gaseous states the atoms, molecules or ions move from one place to another and there is no fixed position of atoms in the substance. In solids the position of atom or molecules are fixed and they are may or may not be present at regular intervals of distances in three dimensions.

If the atoms or molecules in a solid are periodically at regular intervals of distances in 3 dimensions then such a solids are known as *crystalline solids.*

If the atoms or molecules does not have such periodicity in the substances is known as *Amorphous solid.*
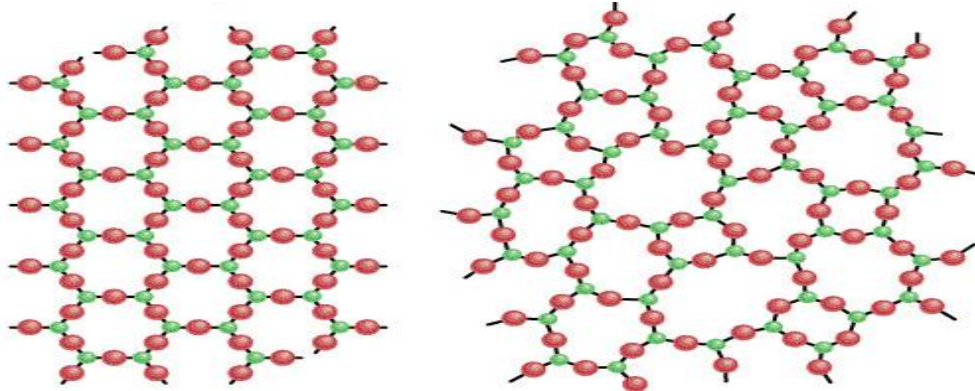

Fig: (a) crystalline solid   *(b) Amorphous solid*

If the periodicity of atoms or molecules are extended throughout the solid such a solid known as *single crystalline solid.*
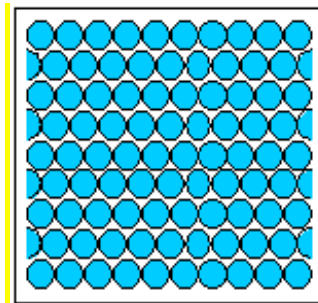

Fig: Single crystalline solid

If the periodicity of atoms or molecules are up to small regions called grains. And these grains are large in number of different sizes in the solid. Such a materials are known as *polycrystalline solids.*
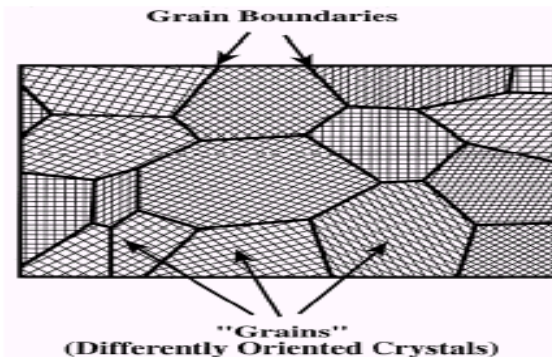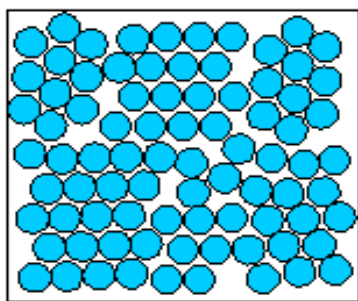
Fig: *polycrystalline solid.*

Crystalline Solids	Amorphous solids
The crystalline solids posses' regular arrangement of ions or atoms or molecules.	These solids posses complete random arrangement of ions or atoms or molecules.
The solids have different physical properties (thermal, electrical conductivity, refractive index) in different directions.	These solids have some physical properties in all the directions.
These are anisotropic in nature.	These are isotropic in nature.
These solids posses elasticity.	These solids posses plasticity except rubber.
These posses more density.	These posses less density.
Eg: calcite, quartz, gold, silver, Al etc...	Eg: glass, plastic etc.

Space lattice (or) Crystal lattice

The atomic arrangement in a crystal is called crystal structure. In crystalline materials, the atoms or molecules are arranged periodically in regular intervals of distances. To explain crystal symmetries easily, it is convenient to imagine a points in space about which these atoms are located such points in space are called lattice points and the totality of all such lattice points form *space lattice* or *crystal lattice.*

(or)

The geometric arrangement of lattice points, which describes the three dimensions arrangements of atoms or molecules or ions in a crystal, is called space lattice or *crystal lattice*.

If the points are arranged in a single or single column, then the lattice is solid to be *linear lattice.* If it is arranged in a two dimension then the lattice is called *planar lattice.*

Basis:

The crystal structure is formed by associating with every lattice point a set of assembly of atoms or molecules or ions identical in composition, arrangement, orientation is called as *Basis.*

(Or)

A group of atoms or molecules identical in composition is called *Basis.*

Lattice + basis   →   Crystal structures

Fig: Space lattice + basis = crystal structure
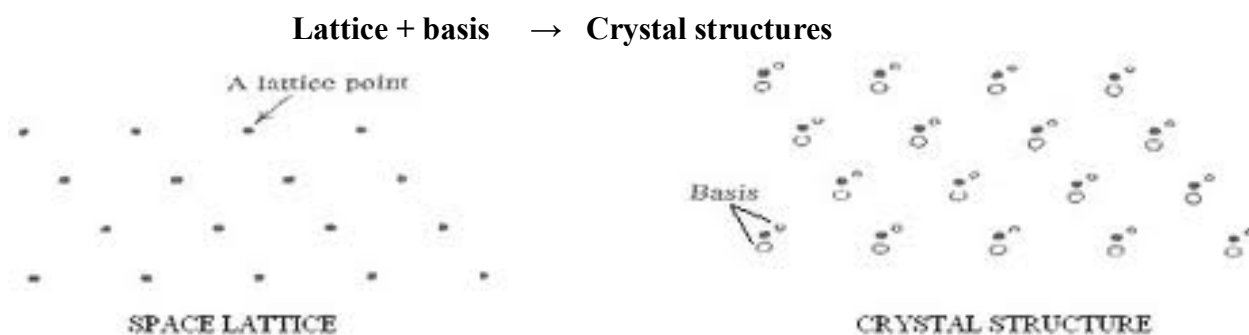
In crystalline solids like aluminium, sodium, copper the basis is single atom. In case of NaCl, NaBr, KCl, KBr the basis is diatomic and in CaF_2 the basis is triatomic.

Unit Cell

The smallest portion of space which can generate the complete crystal structure by repeating its own dimensions in various directions is called a "*Unit Cell*".

(Or)

In every crystal some fundamental grouping of particles is repeated. Such fundamental grouping of particles is called a "*Unit Cell*".

Primitive Unit Cell

It is defined as Unit cell which contains only one lattice point per unit cell.

Multiple Unit Cell (or) Non-Primitive Cell

The unit cell which contains more than one lattice point per unit then it is known as *Multiple Unit Cell (or) Non-Primitive Unit Cell.*
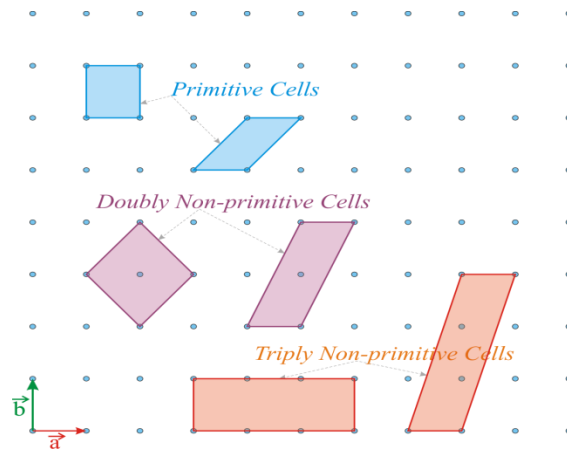
Fig: Unit cell

Lattice parameters of a Unit Cell

The basic lattice parameters are interfacial angles and primitives.

Primitives:

The intercepts on x,y,z-axis in a unit cell are called primitives . They are denoted by a,b,c. The primitives will give the knowledge of the size of the unit cell. The interfacial angles give the shape of the unit cell.

α β γ ------>   interfacial angles
a, b, c ------->   primitives
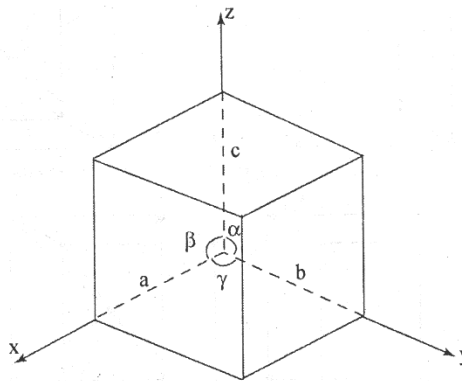*X,Y,Z -------> crystallographic axis*

Fig: Unit cell and its lattice parameters

Crystal Systems

The crystal systems are divided into seven typed based on the lattice parameters   i.e. primitives and interfacial angles,

The different crystal systems are

    1). Cubic system
    2). Tetragonal system
    3). Orthorhombic system

4). Monoclinic system
5). Triclinic system
6). Trigonal system
7). Hexagonal system

1) **Cubic system**:- In this crystal system all the three edges of the unit cell are equal and right angles to each other

a=b=c; $\alpha = \beta = \gamma = 90^0$
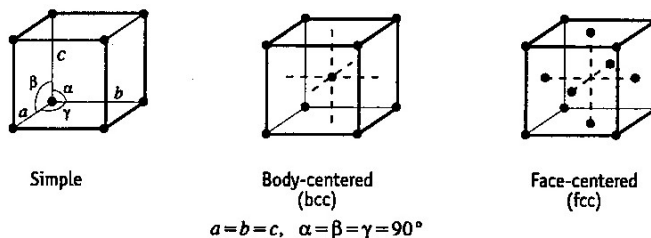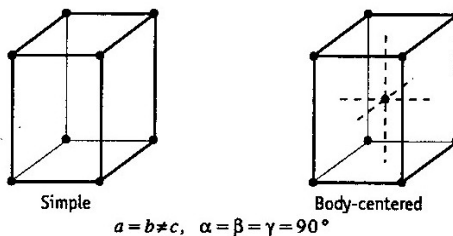
Eg :- NaCl , CaF_2 etc.

Fig: Cubic Crystal Systems

2) **Tetragonal system**: -- In this system all the two edges of the unit cell are equal while the third is different. The three axes are mutually perpendicular.

a=b≠c; $\alpha = \beta = \gamma = 90^0$

Eg: - TiO2, NiSO4, Sno_4 etc.

3).**Orthorhombic system**:--In this crystal system the three edges are different but three axis are perpendicular each other

a ≠ b≠ c ; $\alpha = \beta = \gamma = 90^0$

Eg: $BaSO_4$, KNO_3

4).**Monoclinic system**:-- In this system the edges of the unit cell are different , two axis at right angles and third axis is obliquely inclined.

a ≠ b ≠ c; $\alpha = \beta = \gamma = 90^0 \neq \gamma$

Eg: $Na_2B_4O_710H_2O$ (Borax), $CaSO_42H_2O$ (Gypsum)

8

Simple   Base-centered

$a \neq b \neq c, \quad \alpha = \gamma = 90° \neq \beta$

MONOCLINIC SYSTEM

5).**Triclinic system**: In this system the edges of unit cell are different and the 3 axis are obliquely inclined to each other

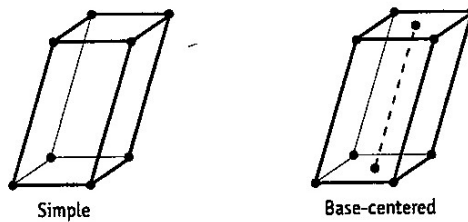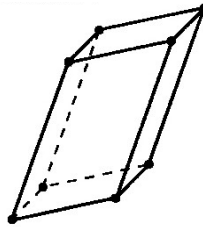$\quad$ $a \neq b \neq c; \quad \alpha \neq \beta \neq \gamma \neq 90^0$

$\quad$ Eg : $K_2Cr_2O_7, CuSO_4 5H_2O$

$a \neq b \neq c$
$\alpha \neq \beta \neq \gamma \neq 90°$
TRICLINIC SYSTEM

6).**Trigonal system**: In this crystal system all the 3 edges of the unit cell are equal and are equally inclined to each other at an angle other than 90^0.

$\quad$ $a = b = c; \quad\quad \alpha = \beta = \gamma \neq 90^0$

$\quad$ Eg: Bismuth, calcite.
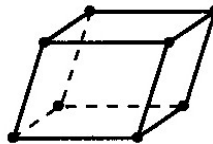
$a = b = c$
$\alpha = \beta = \gamma \neq 90°$
IOMBOHEDRAL (TRIGONAL) SYSTEM

7). **Hexagonal system**: In this system the two axes of the unit cell are equal in length lies in one plate at 120^0 with each other and the third axis is perpendicular to this plane .

$\quad$ $a=b\neq c; \quad\quad \alpha=\beta= 90^0; \gamma=120^0$

$\quad$ Eg: SiO_2, Mg, Zn
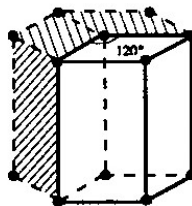
$a = b \neq c$
$\alpha = \beta = 90°, \gamma = 120°$
HEXAGONAL SYSTEM

Bravais Lattices

According to crystal system Bravais showed that there are 14 ways of arranging points in space lattice such that all the lattice points have exactly the same environment. these 14 different lattice types collectively called as Bravais lattices. The different space lattice formed by Primitive (P), Body centered (I), Face centered (F), Base centered(C).

S.No	Crystal system	Bravais lattice	No. of lattice system	Axial length	Interfacial angles	examples
1	Cubic	P,I,F	3	a=b=c	$\alpha=\beta=\gamma=90^0$	NaCl,
2	Tetragonal	P,I	2	a=b=c	$\alpha=\beta=\gamma=90^0$	TiO_2
3	Orthorhombic	P,I,F,C	4	a=b=c	$\alpha=\beta=\gamma=90^0$	KNo_3
4	Monoclinic	P,C	2	a=b=c	$\alpha=\beta=\gamma=90^0$	$CaSo_42H_2o$
5	Triclinic	P	1	a=b=c	$\alpha=\beta=\gamma=90^0$	$K_2Cr_2O_7$
6	Trigonal	P	1	a=b=c	$\alpha=\beta=\gamma=90^0$	calcite
7	Hexagonal	P	1	a=b=c	$\alpha=\beta=\gamma=90^0$	SiO_2

Simple Cubic (SC) Structure

The unit cell contains 8 atoms located at 8 corners is called simple cubic structure. In this structure the all atoms touch each other at their edges. Effective number of atoms in unit cell is 1. i.e. 8 corners have 8 atoms; each corner atom is shared by 8 unit cell. So the share of the each atom towards unit cell is 1/8.

Therefore   n = 8x1/8

N=1.

If we consider an atom at one corner as centre it is surrounded by 6 equidistant neighboring atoms. Hence the coordination number of simple cubic is 6.
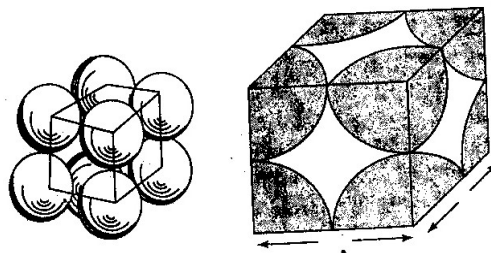
Nearest neighbor distance:

In this structure atoms touch each other at their edges. Hence a'=2r,
Where 'a' is Lattice constant.

Packing fraction:

It is defined as *the ratio of volume of all atoms in the unit cell to the volume of unit cell.*

$$\text{Therefore packing fraction} = \frac{\text{Volume of the all atoms in the unit cell}}{\text{Volume of the unit cell.}}$$

$$= n \times \frac{\frac{4}{3}\pi r^3}{a^3}$$

$$= 1 \times \frac{\frac{4}{3}\pi r^3}{8r^3}$$

$$= \frac{\pi}{6}$$

$$= 0.52$$

$$= 52\%$$

Eg: polonium.

Body Centered Cubic (BCC) Structure

The structure which 8 atoms are located at 8 corners and one atom is located at centre of body i.e. unit cell such a structure is called body centered cubic structure (BCC). In this structure the corner atoms may not touch each other. But they are in contract with the atom which is located at the centre of the unit cell.

Effective number of atoms in BCC is 2 . i.e. 8 atoms are located at 8 corners; one atom is located at centre of the body. So, share of each corner atom towards 8 unit cell is 1/8.

$$\text{Therefore } n = (8 \times \frac{1}{8}) + 1$$
$$= 2.$$
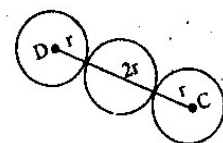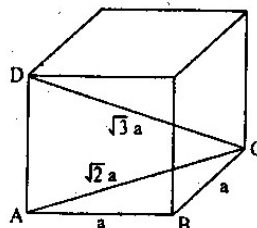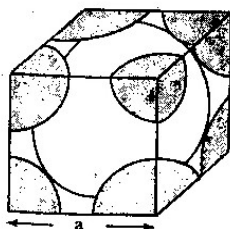
Coordination Number: -
equidistant neighboring atoms. Hence the coordination number is 8.

Nearest neibhor Distance (a):

From the Δ ABC,
$$(AC)^2 = (AB)^2 + (BC)^2$$
$$(AC)^2 = a^2 + a^2$$
$$(AC)^2 = 2a^2$$
From the Δ ACD,
$$(AD)^2 = (AC)^2 + (CD)^2$$
$$(4r)^2 = 2a^2 + a^2$$

$$16r^2 = 3a^2$$
$$4r = \sqrt{3}\,a$$
$$r = \sqrt{3}\,a/4$$
$$a = 4r/\sqrt{3}.$$

Packing fraction $= n \times \dfrac{\frac{4}{3}\pi r^3}{a^3}$

$$= 2 \times \dfrac{\frac{4}{3}\pi r^3}{\frac{64r^3}{3\sqrt{3}}}$$

$$= \sqrt{3}\,\dfrac{\pi}{8}$$

$$= 0.68$$

$$= 68\%.$$

Eg: Na, Fe etc.

Face Centered Cubic (FCC) structure

The structure in which 8 atoms are located at 8 corners, 6 atoms are located at the centre of the solids. Such a structure is called FCC. In this structure the corner atoms may not touch each other but they are in contract with the atoms located on its faces.

Effective Number in F.C.C is 4. i.e. 8 atoms are located at 8 corners, 6 atoms on sides shared by two sides. So they,

$$n = (8 \times \tfrac{1}{8}) + (6 \times \tfrac{1}{2}) = 4.$$
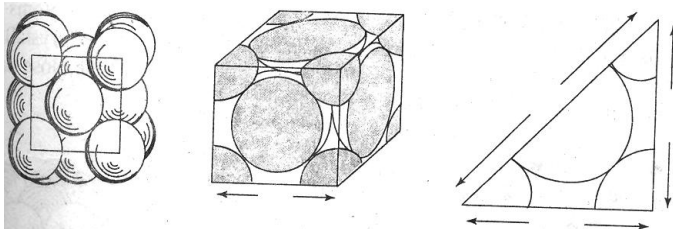
Coordination number is 12. If we consider one atom at a face there are 12 atoms of same distance. Therefore, Coordination Number in FCC is 12.

Nearest Distance (a):

From the Δ ABC,

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$16r^2 = a^2 + a^2$$
$$16r^2 = 2a^2$$
$$8r^2 = a^2$$
$$2\sqrt{2}r = a$$
$$r = a/2\sqrt{2}.$$

$\therefore$ Packing fraction $= n \times \dfrac{\frac{4}{3}\pi r^3}{a^3}$

$$= 4 \times \dfrac{\frac{4}{3}\pi r^3}{16\sqrt{2}r^3}$$

$$= \dfrac{\pi}{3\sqrt{2}}$$

$$= 0.74$$

12

= 74%.

Eg: Cu, Al, Ag   etc.

Miller Indices

It is possible for defining a system of parallel and equidistant planes which can be imagined to pass through the atoms in a space lattice, such that they include all the atoms in the crystal. Such a system of planes is called crystal planes. Many different systems of planes could be identified for a given space lattice.

The position of a crystal plane can be specified in terms of three integers called Miller indices.If these are enclosed in "( )" as (h, k, l) then it represent a plane. If they are enclosed in "[ ]" as [h, k, l] then it represent the direction of crystal.Steps to determine Miller Indices for a given plane as shown in the figure. Take a lattice point as origin 'o' of crystallographic axis x, y, z in a space lattice. Let A, B, C be the crystal plane intercepts there axis of 4a, 4b, 3c.

s

Step (1):   Determine the coordinate of intercepts made by the plan along the crystallographic axis

        x   y   z
        4a   4b   3c

In general

        x   y   z
        pa   qb   rc   (where   p=4, q=4, r=3).

Step (2):--   Divide the intercepts with lattice point translational distances along the axis.

        4a/a   4b/b   3c/c
        4   4   3

  In general

        Pa/a   qb/b   rc/c
        P   q   r

Step (3):--   Determine the reciprocals of these numbers

   i.e.   1/4   1/4   1/3

In general   1/p   1/q   1/r.

Step (4):-- Reduce the reciprocals to smallest integers and enclosed them in "( ) "[by multiplying with L.C.M]

        1/4× 12   1/4× 12   1/3× 12
         3   3   4
        (3   3   4)

Miller Indices may be defined as the reciprocals of the intercepts may by the plane on crystallographic axis when reduced to smallest numbers

Important features of Miller indices are,

1. When a plane is parallel to any axis, the intercept of the plane on that axis is infinity. Hence its Miller index for that axis is zero.

2. When the intercept of a plane on any axis is negative, a bar is put on the corresponding Miller index.
3. All equally spaced parallel planes have the same index number (h k l)

     Ex: The planes (1 1 2) and (2 2 4) are parallel to each other

Distance of Separation between Successive planes

     Let us consider a rectangular coordinate system with origin 'o' at one of the lattice point. Let (h k l) be the miller indices of a plane A,B,C which makes intercepts at OA, OB, OC along x, y, z axis respectively. Let OM be the normal passing through the origin and meet the plane A, B, C at N. such that ON = d, the normal makes an angles α, β, γ along x, y, z axis respectively.
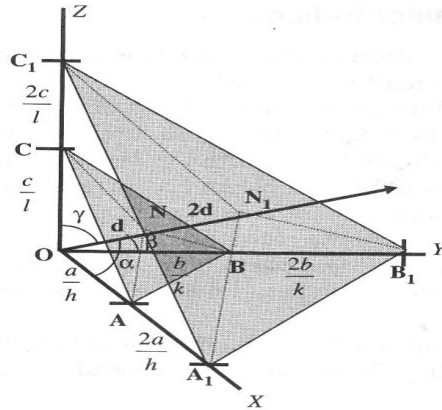
Fig: Inter planar Spacing

The intercepts   OA, OB, OC are such that OA = a/h, OB = b/k, OC = c/l →(1)

We have,

    Cos α = ON/OA = d_1/(a/h)

    Cos β = ON/OB = d_1/ (b/k)   → (2)

    Cos γ = ON/OC = d_1/(c/l)

We know that,

    $\text{Cos}^2 \alpha + \text{Cos}^2 \beta + \text{Cos}^2 \gamma = 1$ → (3)

Submit eq (2) in eq (3),

$$d_1{}^2 \left[\frac{1}{\left(\frac{a^2}{h^2}\right)+\left(\frac{b^2}{k^2}\right)+\left(\frac{c^2}{l^2}\right)} \right] = 1$$

$$d_1{}^2 \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right] = 1$$

$$d_1{}^2 = \frac{1}{[\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}]}$$

$$d_1 = \frac{1}{\sqrt{\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}}}$$

Let (h/2 k/2 l/2) be the miller indices of next plane A' B' C' making intercepts OA' OB' OC' along x, y, z axis respectively. The normal meets the 2nd plane A', B', C' at N' such that ON' = d_2. The extension of d_1 to d_2 the normal makes the same along α, β, γ with x, y, z axis respectively.

The intercepts of OA', OB', OC' are such that

$$OA' = \frac{a}{h/2} \qquad OB' = \frac{b}{k/2} \quad , \qquad OC' = \frac{c}{l/2}$$

$$\left. \begin{array}{l} \text{Cos } \alpha = ON'/OA' = hd_2/2 \\ \text{Cos } \beta = ON'/OB' = kd_2/2b \\ \text{Cos } \gamma = ON'/OC' = ld_2/2c \end{array} \right\} \qquad \rightarrow (4)$$

We know that,

$$\text{Cos}^2\, \alpha + \text{Cos}^2\, \beta + \text{Cos}^2\, \gamma = 1 \qquad \rightarrow (5)$$

$$d_2{}^2 \left[\frac{h^2}{4a^2} + \frac{k^2}{4b^2} + \frac{l^2}{4c^2} \right]$$

$$d_2 = \frac{2}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

Let the separation between the planes ABC & ABC be 'd '.

$$d = d_2 - d_1$$
$$= {}_{2d1} - d_1$$
$$= d_1$$

$$d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

For cubic system   a = b = c.

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

For Tetragonal system a = b ≠ c.

$$d = \frac{1}{\sqrt{\frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}}}$$

Structure of Sodium Chloride (NaCl)

Sodium chloride crystal is an ionic one. In the NaCl lattice, positively charged sodium ion and negatively charged chlorine ion are situated side by side. It consists of two FCC sub lattices. One of the chlorine ion has its origin at (0, 0, 0) point while sodium ion has its origin midway along the cube edge at (a/2, 0, 0) point as shown in figure. Due to the electro static force between sodium and chlorine ions, the two ions are attracted towards each other. When the two ions come closer their outer electron shells comes closer giving rise to strong forces of repulsion when attraction and repulsion balance, equilibrium is obtained.
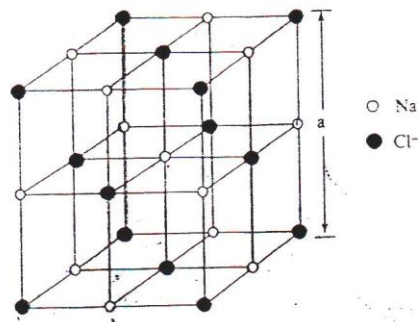
Fig : Structure of Sodium Chloride

Each unit cell contains eight corner ions, six face centered ions, twelve ions at halfway of the edges and one at the center of the body.

Number of Cl^- ions per unit cell is $(8 \times 1/8) + (6 \times 1/2) = 1+3 = 4$ ions

Number of Na^{+-} ions per unit cell is $(12 \times 1/4) + 1 = 3+1 = 4$ ions

Each unit cell has four NaCl molecules. Each Na^+ ion has six Cl^- and each Cl^- has six Na^+ ions. The coordination number of NaCl for opposite kind is six and same kind is twelve. The nearest neighbor's distance is $a/2$ for opposite kind and $a\sqrt{2}$ for the same kind. KCl, KBr, MgO, AgBr etc., have the similar structure as NaCl.

Diamond Structure

Diamond possesses a structure which is a combination of two interpenetrating FCC sub lattices along the body diagonal at a length of quarter of cube edge. One sub lattice, say x, has its origin at the point (0, 0, 0) and other sub lattice has its origin quarter of the way along the body diagonal i.e., at the point (a/4, a/4, a/4). The fractions such as ¼, ½, ¾ denote the height above the base since the coordination number is 4, the structure is loosely packed. In the unit cell, in addition to the eight corner atoms, there are six face centered atoms and four atoms are located inside the unit cell. Each corner atom is shared by eight adjacent unit cells and each face centered atom is shared by two unit cells.

Effective number of atoms per unit cell

In the unit cell in additional to the 8 corner atoms there are 6 face centered atoms and four more atoms located inside the unit cell. Each corner atom is shared by 8 adjacent unit cells and each face centered atom is shared by 2 cells hence the total number of atoms unit cell is

$$(8 \times \frac{1}{8}) + (6 \times \frac{1}{2}) + 4 = 8$$
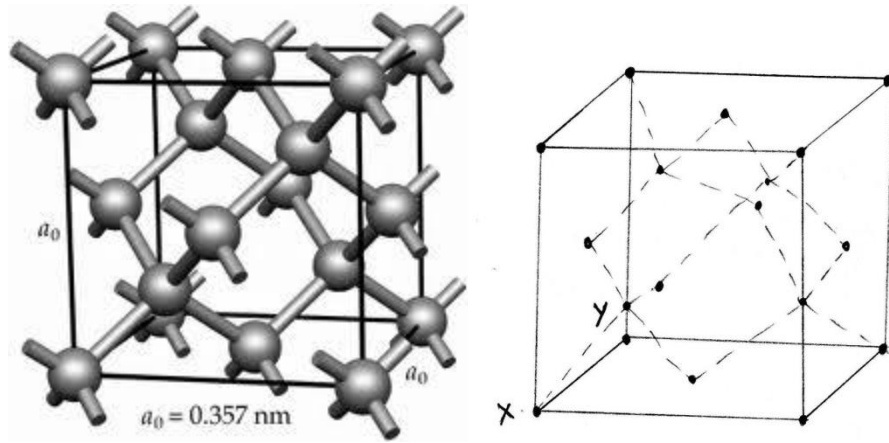
Fig: Diamond Structure

Coordination number: In this lattice each atoms has 4 nearest neighbors with it forms covalent bonds. Thus the coordination number of diamond crystal is 4.

Atomic Radius:

From the diagram

$$(XY)^2 = \frac{a^2}{16} + \frac{a^2}{16}$$

$$(XY)^2 = \frac{2a^2}{16} = \frac{a^2}{8}$$

$$(XZ)^2 = (XY)^2 + (YZ)^2$$

$$(XZ)^2 = \frac{a^2}{8} + \frac{a^2}{16}$$

$$4r^2 = \frac{3a^2}{16}$$

$$r^2 = \frac{3a^2}{64}$$

$$r = \frac{\sqrt{3}a}{8}$$

$$a = \frac{8r}{\sqrt{3}}$$

Nearest neighbor distance:

$$r = \frac{\sqrt{3}a}{8}$$

$$2r = \frac{\sqrt{3}a}{4}$$

Atomic packing fraction:

$$\text{Packing fraction} = \frac{n \times \frac{4}{3}\pi r^3}{a^3}$$

$$= \frac{8 \times \frac{4}{3}\pi r^3}{\frac{8 \times 8 \times 8 \times r^3}{3\sqrt{3}}}$$

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$$= \frac{\sqrt{3}\pi}{16} = 0.34 = 34\%$$

X-RAY DIFFRACTION & DEFECTS IN CRYSTALS

Introduction to X – ray Diffraction

X-Rays are electromagnetic waves like ordinary light, therefore, they should exhibit interference and diffraction .Diffraction occurs when waves pass across an object whose dimensions are of the order of their own wavelengths. The wavelength of X-rays is of the order of 0.1nm or 10 -8 cm so that ordinary devices such as ruled diffraction gratings do not produce observable effects with X-rays. Laue suggested that a crystal which consisted of a 3-dimensional array of regularly spaced atoms could serve the purpose of a grating. The crystal differs from ordinary grating in the sense that the diffracting centers in the crystal are not in one plane. Hence the crystal acts as a space grating rather than a plane grating. There are 3 main X-ray diffraction methods by which the crystal structure can be analyzed.

Laue Method – for single crystal

Powder Method- for finely divided crystalline or polycrystalline powder

Rotating crystal Method - for single crystal

Bragg′s Law

The monochromatic x-ray beam of wavelength λ is incident at an angle θ to Bragg's planes. Let the inter planer spacing of crystal planes is'd'. The dots in the planes represent positions of atoms in the crystal. Every atom in the crystal is a source of scattering for the X-ray incident on it.
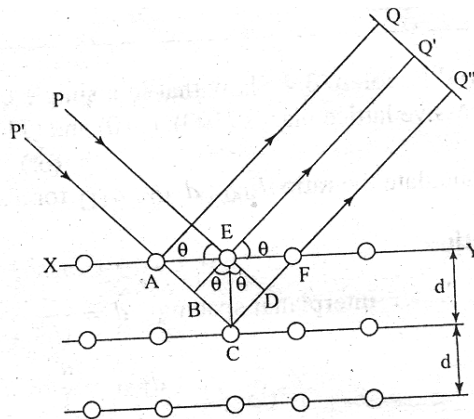
Fig: X-Ray Scattering by Crystal

Apart of the incident x-ray beam AB incident an atom at B in plane 1 is scattered along the direction BC. Similarly apart of incident x-ray DE fall on atom at E in plane 2 and is scattered in the direction EF and it is parallel to BC. Let the beam AB and DE make an angle θ with the Bragg's planes. This angle θ is called the angle of diffraction or glancing angle. If the path difference between the rays ABC & DEF is equal to nλ, than the rays reflected from constructive plane or phase. So constructive interference takes place among the reflected ray BC and EF, then the path difference between the rays is,

PE+QE= nλ

From the Δ^{le} PBE,

$$\sin\theta = \frac{PE}{BE} = \frac{PE}{d}$$

$$PE = d\sin\theta$$

From the Δ^{le} QBE,

$$\sin\theta = \frac{EQ}{EB} = \frac{EQ}{d}$$

$$EQ = d\sin\theta$$

For constructive interference,

$$PE + EQ = n\lambda$$

$$d\sin\theta + d\sin\theta = n\lambda$$

$$2d\sin\theta = n\lambda$$

The above equation is called Bragg's Law.

Laue method

S$_1$ & S2 are two lead screens in which two pin holes act as slits. X-ray beam from an X –ray tube is allowed to pass through these two slits S1 & S2 . The beam transmitted through S2 will be a narrow pencil of X – rays . The beam proceeds further to fall on a single crystal such that Zinc blended (ZnS) which is mounted suitably on a support . The single crystal acts as a 3 – dimensional diffraction grating to the incident beam. Thus, the beam undergoes diffraction in the crystal and then falls on the photographic film. The diffracted waves undergo constructive interference in certain directions, and fall on the photographic film with reinforced intensity. In all other directions, the interference will be destructive and the photographic film remains unaffected.

The resultant interference pattern due to diffraction through the crystal as a whole will be recorded on the photographic film (which requires many hours of exposure to the incident beam). When the film is developed, it reveals a pattern of fine spots, known as Laue spots.
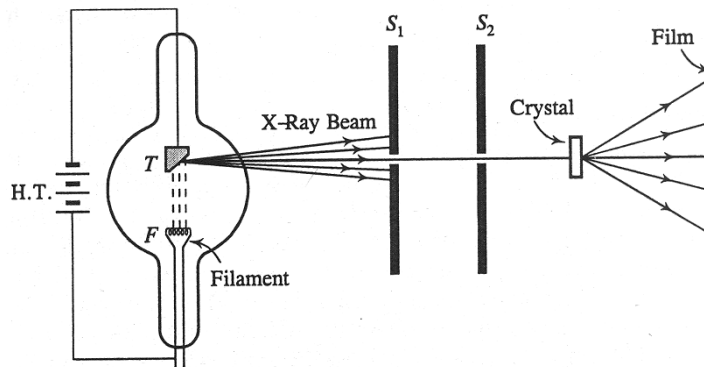
X–RAY DIFFRACTION (LAUE's METHOD)

The distribution spots follow a particular way of arrangement that is the characteristic of the specimen used in the form of crystal to diffract the beam. The Laue spot photograph obtained by diffracting the beam at several orientations of the crystal to the incident beam are used for determining the symmetry and orientations of the internal arrangement of atoms, molecules in the crystal lattice . it is also used to study the imperfections in the crystal .

Powder Method (or) Debye Scherrer method

X-ray powder method is usually carried out for polycrystalline materials. The given polycrystalline material is grounded to fine powder and this powder is taken in capillary tube. This tube is made up of non diffracting material, and fixed at the centre of cylindrical Debye Scherrer cylindrical camera as shown in figure.

The principle under this technique is that millions of tiny crystals in powder have completely random orientation all the possible diffraction phases are available for Bragg reflection to takes place. All the orientations are equally reflected ray will form a cone. whole axis likes along the direction of incident beam and whose semi vertical angle is twice the glancing angle for that particular planes.
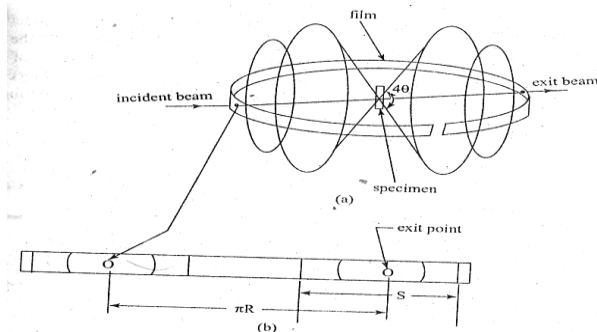
Fig: Powder method - apparatus

The different cones intercept the film in a series of concentric circular from the radial of these arcs. The angle can be calculated and hence the spacing between the atoms can be evaluated as shown in figure. The photographic film is in cylindrical shape, whose axis is perpendicular to the beam.

Let s be the distance of particular arc from the centre. Let R be the radius of camera. Then

$$4\theta = \frac{S}{R}$$

$$\theta = \frac{S}{4R}$$

If S_1, S_2, S_3, are the distances between symmetrical lines on the stretched film, then

$$\theta_1 = \frac{S1}{4R}$$

$$\theta_2 = \frac{S2}{4R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Using these values of θ_n in Bragg's equation

$$n\lambda = 2d \, Sin\theta_n.$$

Where,

n=1, 2, 3, … is the Order of diffraction

d= Interplanar spacing

θ_n = Angle of diffraction for n^{th} order.

The inter-planar spacing d can be calculated.

Introduction to Defects in crystals

The structural imperfections of crystal can be classified on the basis of their geometric structures as point defect, line defect, surface defect and volume defect.

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Point defects

As the name indicates, these defects are at same point in the crystal .so these are called as point defects point defects are also called as zero dimensional defects. The common types of point defect in a solid material are grouped into 3 categories.

1. Lattice site defect
   a) Vacancies (Schottky defect)
   b) Interstitial vacancies (Frenkel defect)
2. Compositional defect
   a) Substitutional impurity defect
   b) Interstitial impurity defect
3. Electronic defect

1. **Lattice site defects**

In this type of defect, some atoms may not present in their regular atomic sites (or) positions. They are,

a) **Vacancies**

As shown in the diagram at a point one or two or three atoms are missed. This is referred as single or double or treble vacancies. The vacancies are formed due to imperfect pattern during crystallization or due to thermal vibration at high temperature.
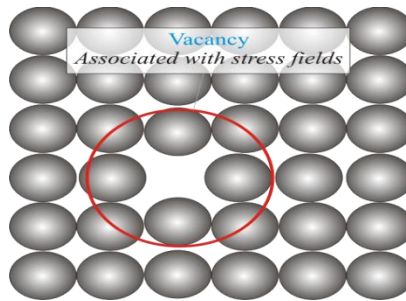

Fig: Vacancy

Schottky defect

In an ionic crystal if a action vacancy exist, then in the very nearby place. On anion vacancy will also exist that is moved to surface of the crystal. So that change neutrality is maintained in vacancy region as shown in diagram. This is known as Schottky defect.
Eg: Crystals such as NaCl, KCl, KBr etc,
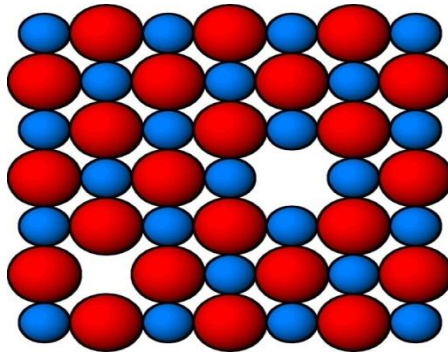
Fig: Schottky Defect

b) Interstitial vacancies

If an atom is moved to an interstitial space in the crystal then the defect is known as interstitial defect.

Frenkel defect

In an ionic crystal, if a cation (positive ion) moves to an interstitial space so that a vacancy is formed in its atomic position Here change neutrality is maintained in the defect region as shown in the diagram.

This type of defect is known as Frenkel defect.

Eg; crystal such as CaF_2, AgCl, AgI, AgBr etc.
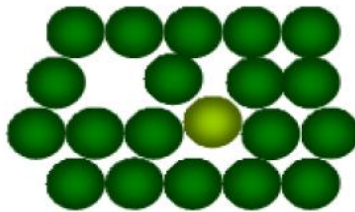
Frenkel defect

2. Compositional defect

The presence of impurity atoms in the crystal leads to compositional defects. Impurity atoms are present at the sites of regular parent atoms are in the sites of regular parent atoms are in the interstitial spaces. These defects are described as follows.

a) Substitutional Impurity defect:

As shown in diagram during crystallization free foreign atoms occupies the regular parental positions or atomic sites.

Eg: In extrinsic semiconductors either trivalent or pentavalent atoms he sites of silicon or germanium atoms.
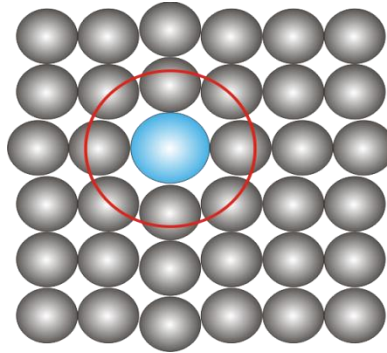
Fig: Substitutional impurity

b) Interstitial impurity defect

The spaces between the parents the parental atoms in the crystal are known as interstitial spaces small sized (lower atomic number) atoms, such as hydrogen etc. may fit in these interstitial spaces. These atoms are known as interstitial atoms and the defect formed due to presence of interstitial atoms is known as interstitial impurity defect.
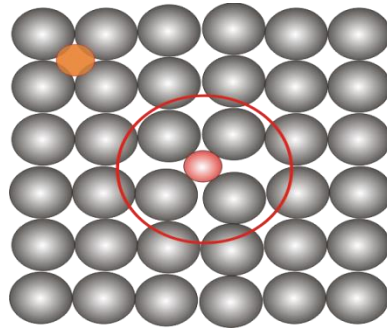
Fig: Interstitial impurity defect

3. Electronic defect

At absolute zero in a purely covalent crystal (silicon) the electrons are tightly bound to the core and all are set to be in the value band. Absolute zero sum of the electrons likely to occupy. So in the crystal of pure silicon some temperature .so in the crystal of pure of silicon some of the electrons from the covalent bonds get thermally released and become free to move. In this way the deficiency of an electron creates a hole. Then the electrons & holes give rise to electronic imperfection.

Line defects

If a crystal plane ends somewhere in the crystal, that along the edge of the incomplete. Plane produces defect in the crystal called line defect the line defect is of two types,

a) Edge dislocation
b) Screw dislocation

a) **Edge dislocation**

In a perfect crystal , the atoms arranged in both vertical and horizontal plane parallel to the side faces as shown in figure if one of these vertical planes does not extend to full length but ends in between within the crystal. Then it is called edge dislocation therefore in this defect, a line of atoms is not in proper removed.
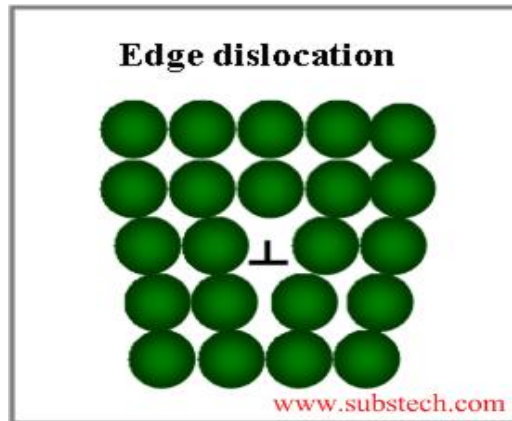
Figure: Edge dislocation

The following points are observed.
i)   In the perfect crystal, the atoms are in equilibrium position.
ii)   In the imperfect crystal, just above the discontinuity the atoms are prell hard and are in a stat of completion.
iii)   Just below the discontinuity, the atoms are pulled apart and are in a state of tension.
iv)   This distorted configuration extends all along the edge inside the crystal, perpendicular to the cross section.
v)   Edge dislocations are represented by perpendicular or T depending on whether the incomplete plane stale from the top or from the bottom of the crystal. These two configurations are referred as positive or negative edge dislocation respectively.

Burger's Vector

To understand the concept of burgers vector let us consider two crystal one perfect and the other edge dislocation as shown figure
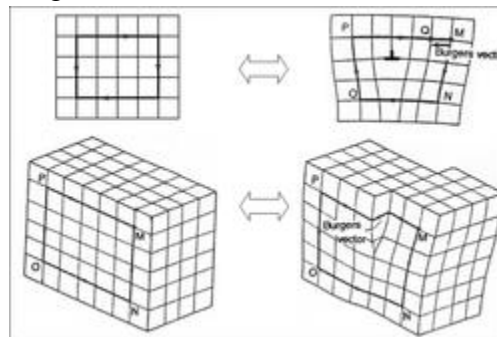
Figure: Burger's Vector

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The burgers vectors of a dislocation are determined by tracing a burgers circuit. Let us consider the burgers circuit in case of perfect crystal. The burgers circuit is realized by drawing a rectangular from point p. starting from point p, we go up by 6 steps to a point Q then move towards right by 6 steps to a point R, further move down by 6 steps to a point S.

Finally move towards left by 6 steps to reach the starting point p. This gives a closed circuit PQRS known as Burger circuit.

b) **Screw dislocation**

Figure shows when a part of the crystal is displaced relative to the rest of the crystal. It is important to mention here that the displacement terminates with the crystal. Screw dislocation is defined as displacement of the atoms in one part of a crystal relative to the rest of the crystal. The screw dislocation forms a ramp around the dislocation line. The Buggers vector is also in the figure. Buggers vector in parallel dislocation line. Normally the real dislocations in the crystal are mixture of edge quell screw dislocation.
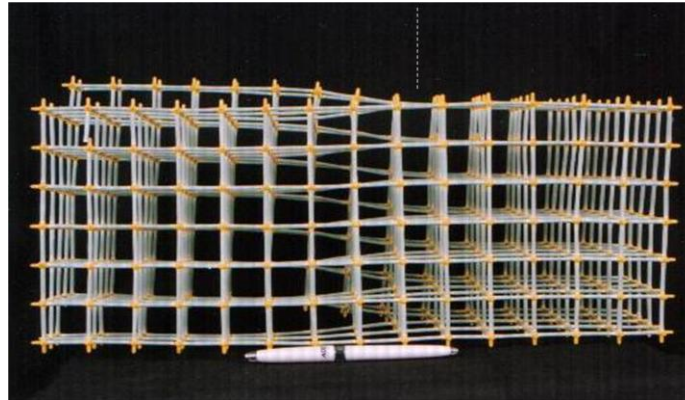
Figure: Screw dislocation

UNIT-III
LASERS & SENSORS

Introduction

LASER stands for light amplification by stimulated emission of radiation. It is different from conventional light (such as tube light or electric bulb), there is no coordination among different atoms emitting radiation. Laser is a device that emits light (electromagnetic radiation) through a process is called stimulated emission.

Spontaneous and stimulated emission

In lasers, the interaction between matter and light is of three different types. They are: absorption, spontaneous emission and stimulates emission .Let E_1 and E_2 be ground and excited states of an atom. The dot represents an atom. Transition between these states

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involves absorption and emission of a photon of energy $E_2-E_1=h\nu_{12}$. Where 'h' is Planck's constant.

(a) Absorption: As shown in fig (a), if a photon of energy $h\nu_{12}(E_2-E_1)$ collides with an atom present in the ground state of energy E_1 then the atom completely absorbs the incident photon and makes transition to excited state E_2.

(b) Spontaneous emission: As shown in fig (b), an atom initially present in the excited state makes transition voluntarily on its own. Without any aid of external stimulus or an agency to the ground. State and emits a photon of energy $h\nu_{12}(=E_2-E_1)$.this is called spontaneous emission. These are incoherent.

(c) Stimulated emission: As shown in fig (c), a photon having energy $h\nu_{12}(E_2-E_1)$impinges on an atom present in the excited state and the atom is stimulated to make transition to the ground state and gives off a photon of energy $h\nu_{12}$. The emitted photon is in phase with the incident photon. These are coherent. This type of emission is known as stimulated emission.
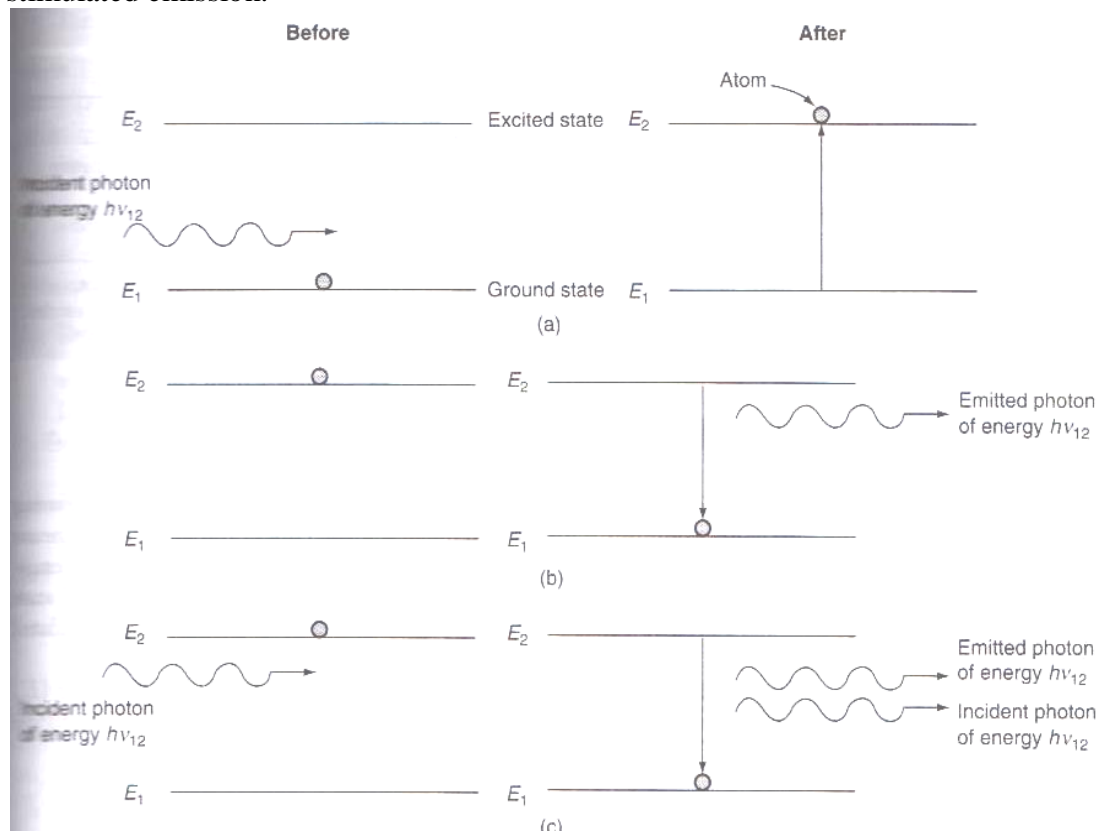
Fig. (a) Absorption ;(b) Spontaneous emission;(c) Stimulated emission

Differences between Spontaneous emission and stimulated emission of radiation

Spontaneous emission	Stimulated emission
1. Polychromatic radiation	1. Monochromatic radiation
2. Less intensity	2. High intensity
3. Less directionality, more angular spread during propagation	3. High directionality, so less angular spread during propagation.
4. Spatially and temporally in	4. Specially and temporally coherent

coherent radiation	radiation.
5. Spontaneous emission takes place when excited atoms make a transition to lower energy level voluntarily without any external stimulation.	**5.** Stimulated emission takes place when a photon of energy equal to $h\nu_{12}$ (=E_2-E_1) stimulates an excited atom to make transition to lower energy level.

Characteristics of Laser Light

(i). Coherence: Coherence is one of the unique properties of laser light. It arises from the stimulated emission process. Since a common stimulus triggers the emission events which provide the amplified light, the emitted photons are in step and have a definite phase relation to each other. This coherence is described in terms of temporal and spatial coherence.

(ii). Monochromaticity: A laser beam is more or less in single wave length. I.e. the line width of laser beams is extremely narrow. The wavelengths spread of conventional light sources is usually 1 in 10^6, where as in case of laser light it will be 1 in 10^5.I.e. if the frequency of radiation is 10^{15} Hz., then the width of line will be 1 Hz. So, laser radiation is said to be highly monochromatic. The degree of non-monochromaticity has been expressed as

$\xi = (d\lambda/\lambda) = d\nu/\nu$, where $d\lambda$ or $d\nu$ is the variation in wavelength or variation in frequency of radiation.

(iii) Directionality: Laser beam is highly directional because laser emits light only in one direction. It can travel very long distances without divergence. The directionality of a laser beam has been expressed in terms of divergence. Suppose r_1 and r_2 are the radii of laser beam at distances D_1 and D_2 from a laser, and then we have.

Then the divergence, $\Delta\theta = (r_1 - r_2)/ D_2-D_1$

The divergence for a laser beam is 0.01 mille radian where as incase of search light it is 0.5 radian.

(iv) High intensity: In a laser beam lot of energy is concentrated in a small region. This concentration of energy exists both spatially and spectrally, hence there is enormous intensity for laser beam. The power range of laser is about 10^{-13}w for gas laser and is about 10^9 w for pulsed solid state laser and the diameter of the laser beam is about 1 mm. then the number of photons coming out from a laser per second per unit area is given by

$N_1 = P/ h\nu\pi r^2 \approx 1022 to 1034$ photons/m^{-2}-sec

By assuming $h\nu = 10^{-19}$ Joule, Power $P = 10^{-3}$ to 10^9 watt $\qquad r = 0.5 \times 10^{-3}$ meter

Based on Planck's black body radiation, the number of photons emitted per second per unit area by a body with temperature T is given by

$N_{th} = (2h\pi C/ \lambda^4) (1/e^{(h\nu/k T)}-1) d\lambda \approx 10^{16}$ photons/m^2.sec

By assuming T=1000k, $\lambda = 6000A^0$

This comparison shows that laser is a highly intensive beam.

Einstein's Coefficients

LetN_1 be the number of atoms per unit volume with energy E_1 and N_2 the number of atoms per unit volume with energy E_2. Let 'n' be the number of photons per unit volume at frequency ν such that $h\nu = E_2 - E_1$. Then the energy density of interacting photons $\rho (\nu)$ is given by

$\rho(\nu) = n h \nu$ $\qquad\qquad\qquad\qquad \rightarrow(1)$

When these photons interact with atoms, both upward (absorption) and downward (emission) transitions occur. At equilibrium these transition rates must be equal.

Upward Transition

Stimulated absorption rate depends on the number of atoms available in the lower energy state for absorption of these photons as well as the energy density of interacting radiation.

I.e. stimulated absorption rate α N_1

$$\alpha \, \rho\,(v)$$
$$= B_{12}N_1 \, \rho\,(v) \qquad\qquad \rightarrow (2)$$

Where the constant of proportionality B_{12} is the Einstein coefficient of stimulated absorption

Downward transition

Once the atoms are excited by stimulated absorption, they stay in the excited state for a short duration of time called the lifetime of the excited state. After their life time they move to their lower energy level spontaneous by emitting photons. This spontaneous emission rate depends on the number of atoms in the excited energy state.

i.e., spontaneous emission rate α N_2

$$= N_2 \, A_{21} \qquad\qquad \rightarrow (3)$$

Where the constant of proportionality A_{21} is the Einstein coefficient of spontaneous emission.

Stimulated emission

Before excited atoms de excites to their lower energy states by spontaneous emission they may interact with photons resulting in stimulated emission of photons. Therefore stimulated emission rate depends on the number of atoms available in the excited state as well as energy density of interacting photons

I.e. stimulated emission rate α N_2

$$\alpha \, \rho(v)$$
$$= N_2 \rho\,(v) \, B_{21} \qquad\qquad \rightarrow (4)$$

Where the constant of proportionality B_{21} is the Einstein coefficient of stimulated emission.

During stimulated emission, the interacting photon called the stimulating photon and the photon due to stimulated emission are in phase with each other. Please note that during stimulated absorption, the photon density decreases where as during stimulated emission it increases.

For a system in equilibrium, the upward and down word transition rates must be equal and hence we have

$$N_1 \, \rho\,(v) \, B_{12} = N_2 \rho(v) B_{21} + N_2 A_{21} \qquad\qquad \rightarrow (5)$$

Hence $\qquad\qquad \rho(v) = \dfrac{N_2 A_{21}}{N_1 B_{12} - N_1 B_{21}} \rightarrow (6)$

$$\rho\,(v) = \frac{A_{21}/B_{21}}{\left(B_{12}/B_{21}\right)\left(N_1/N_2\right) - 1}$$

The population of various energy levels in thermal equilibrium is given by Boltzmann distribution law.
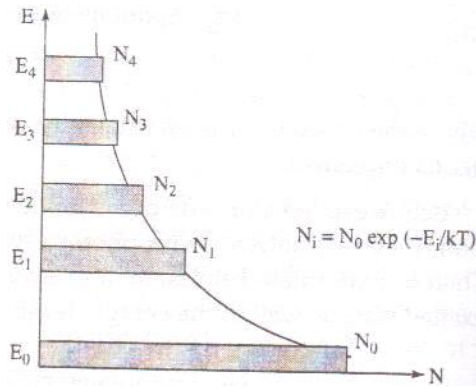
Fig.Boltzmann distribution for several energy levels

$$N_i = g_i N_o \exp(-E_i/KT)$$

Where N_i is the population density of the energy level E_i, N_0 is the population density of the ground state at temperature T_1, g_i is the degeneracy of the ith level and k is the Boltzmann constant $(=1.38 \times 10^{-23}$ joule/k$)$. The concept of degeneracy occurs since more than one level has the same energy.

$$\text{Hence } N_1 = g_1 N_o \exp(-E_1/kT)$$

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} \exp\left[\frac{(E_2 - E_1)}{kT}\right]$$

$$= \frac{g_1}{g_2} \exp\left[\frac{h\nu}{kT}\right] \qquad \rightarrow (7)$$

Substituting eq (6) in eq (4) $\rho(\nu) = \dfrac{[A_{21}/B_{21}]}{[\frac{B_{12}}{B_{21}} \frac{g_1}{g_2} \exp\left[\frac{h\nu}{kT}\right]}$ $\qquad \rightarrow (8)$

From Planck's law of blackbody radiation, the radiation density is given by

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp\left[\frac{h\nu}{kT}\right] - 1} \qquad \rightarrow (9)$$

Comparing equations (8) and (9), we get

$$\frac{B_{12}}{B_{21}} \frac{g_1}{g_2} = 1$$

$$g_1 B_{12} = g_2 B_{21}$$

$$\rightarrow (10)$$

And $\quad \dfrac{A_{21}}{B_{21}} = \dfrac{8\pi h\nu^3}{c^3}$ $\qquad \rightarrow (11)$

Equation's (10) and (11) are referred to as the Einstein relations.

The ratio of spontaneous emission rate to the stimulated emission rate is given by

$$R = \frac{N_2 A_{21}}{N_2 \rho(\nu) B_{21}} = \frac{A_{21}}{\rho(\nu) B_{21}}$$

From equation (9)

$$R = \frac{A_{21}}{\rho(\nu) B_{21}} = \left[exp^{\left[\frac{h\nu}{kT}\right]} - 1\right] \qquad \rightarrow (12)$$

In practice, the absorption and emission phase occur simultaneously. Let us calculate the rates of spontaneous emission and stimulating emission for a tungsten filament lamp operating at a temperature of 2000k. Taking the average frequency to be 5×10^{14}Hz, this ratio is

$$R = e^{\frac{(6.6 \times 10^{-34}) \times (5 \times 10^{14})}{(1.38 \times 10^{-23} \times 2000)}} - 1 \qquad \rightarrow (13)$$

This confirms that under conditions of thermal equilibrium, even for sources operating at higher temperatures and lower frequencies, spontaneous emission predominates.

From equation (12), we understand that to make R smaller $\rho(v)$ the energy density of interacting radiation has to be made larger. Let us consider the relation of stimulated emission rate to stimulated absorption rate

Thus at thermal equilibrium stimulated absorption predominates over stimulated emission. Instead if we create a situation that $N_2 > N_1$. Stimulated emission will predominate over stimulated absorption. If stimulated emission predominates the photon density increases and light amplifies the photon density increases and light amplification by stimulated emission of radiation (LASER) occurs. Therefore, in order to achieve more stimulated emission, population of the excited state (N_2) should be made larger than the population of the lower state (N_1) and this condition is called population inversion. Hence if we wish to amplify a beam of light by stimulated emission, then we must create population inversion and increase the energy density of interacting radiation.

Population inversion

Usually in a system the number of atoms (N_1) present in the ground state (E_1) is larger than the number of atoms (N_2) present in the higher energy state. The process of making $N_2 > N_1$ called population inversion. Conditions for population inversion are:

a) The system should posses at least a pair of energy levels ($E_2 > E_1$), separated by an energy of equal to the energy of a photon (hv).

b) There should be a continuous supply of energy to the system such that the atoms must be raised continuously to the excited state.

Population inversion can be achieved by a number of ways. Some of them are (i) optical pumping (ii) electrical discharge (iii) inelastic collision of atoms (iv) chemical reaction and (v) direct conversion

Helium-Neon gas laser

Helium-Neon gas laser is a continuous four level gas laser. It consists of a long, narrow cylindrical tube made up of fused quartz. The diameter of the tube will vary from 2 to 8 mm and length will vary from 10 to 100 cm. The tube is filled with helium and neon gases in the ratio of 10:1. The partial pressure of helium gas is 1mm of Hg and neon gas is 0.1mm of Hg so that the pressure of the mixture of gases inside the tube is nearly 1 mm of Hg.

Laser action is due to the neon atoms. Helium is used for selective pumping of neon atoms to upper energy levels. Two electrodes are fixed near the ends of the tube to pass electric discharge through the gas. Two optically plane mirrors are fixed at the two ends of the tube at Brewster angle normal to its axis. One of the mirrors is fully silvered so that nearly 100% reflection takes place and the other is partially silvered so that 1%of the light incident on it will be transmitted. Optical resources column is formed between these mirrors.
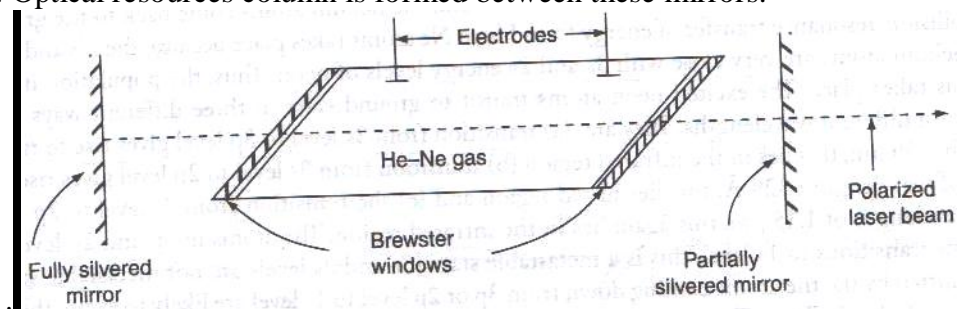
Working

When a discharge is passed through the gaseous mixture, electrons are accelerated down the tube. These accelerated electrons collide with the helium atoms and excite them to higher energy levels. The different energy levels of Helium atoms and Neon atoms is shown in fig. the helium atoms are excited to the levels F_2 and F_3 these levels happen to be meta stable energy states.

Energy levels and hence Helium atoms exited levels spend sufficiently large amount of time before getting de excited. As shown in the fig (a), some of the excited states of neon can correspond approximately to the same energy of excited levels F_2 and F_3. Thus, when Helium atoms in level F_2 and F_3 collide with Neon atoms in the ground level E_1, an energy exchange takes place. This results in the excitation of Neon atoms to the levels E_4 and E_6 and de excitation of Helium atoms to the ground level (F_1). Because of long life times of the atoms in levels F_2 and F_3, this process of energy transfer has a high probability. Thus the discharge through the gas mixture continuously populates the neon atoms in the excited energy levels E_4 and E_6. This helps to create a state of population inversion between the levels E_4 (E_6) to the lower energy level (E_3 and E_5). The various transitions $E_6 \rightarrow E_5$, $E4 \rightarrow E_3$, $E_6 \rightarrow E_3$ leads to the emission of wave lengths 3.39mm, 1.15 um and 6328 A^0. Specific frequency selection may be obtained by employing mirrors

The excited Neon atoms drop down from the level E_3 to the E_2 by spontaneously emitting a photon around wavelength 6000A^0. The pressures of the two gases in the mixture are so chosen that there is an effective transfer of energy from the Helium to the Neon atoms. Since the level E_2 is a meta stable state, there is a finite probability of the excitation of Neon, atoms from E_2 to E_3 leading to population inversion, when a narrow tube is used, the neon atoms in the level E_2 collide with the walls of the tube and get excited to the level E_1. The transition from E_5 to E_3 may be non radioactive. The typical power outputs of He-Ne laser lie between 1 and 50 mw of continuous wave for inputs of 5-10W.
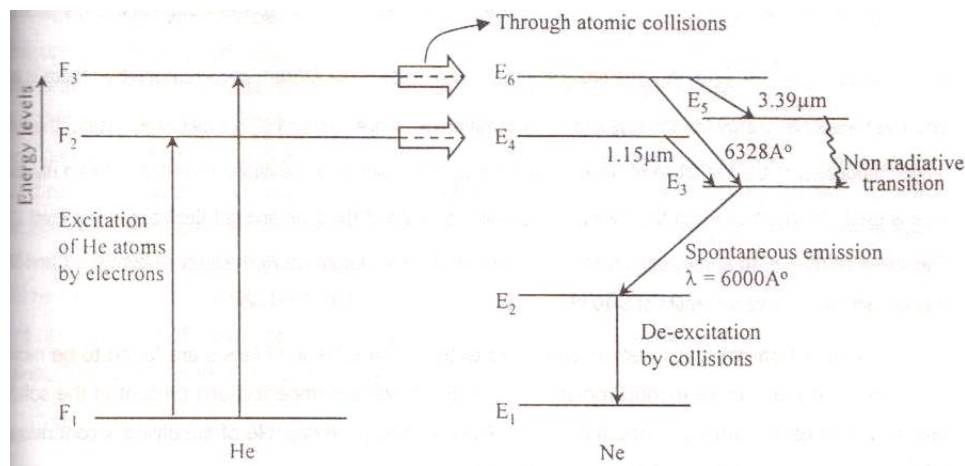

Fig. Energy level diagram of He-Ne atoms.

Ruby Laser

Ruby Laser is a solid state pulsed, three level lasers. It consists of a cylindrical shaped ruby crystal rod of length varying from 2 to 20cms and diameter varying 0.1 to 2cms. This end faces of the rod are highly flat and parallel. One of the faces is highly silvered and the other face

is partially silvered so that it transmits 10 to 25% of incident light and reflects the rest so as to make the rod-resonant cavity. Basically, ruby crystal is aluminum oxide [Al_2O_3] doped with 0.05 to 0.5% of chromium atom. These chromium atoms serve as activators. Due to presence of 0.05% of chromium, the ruby crystal appears in pink color. The ruby crystal is placed along the axis of a helical xenon or krypton flash lamp of high intensity.
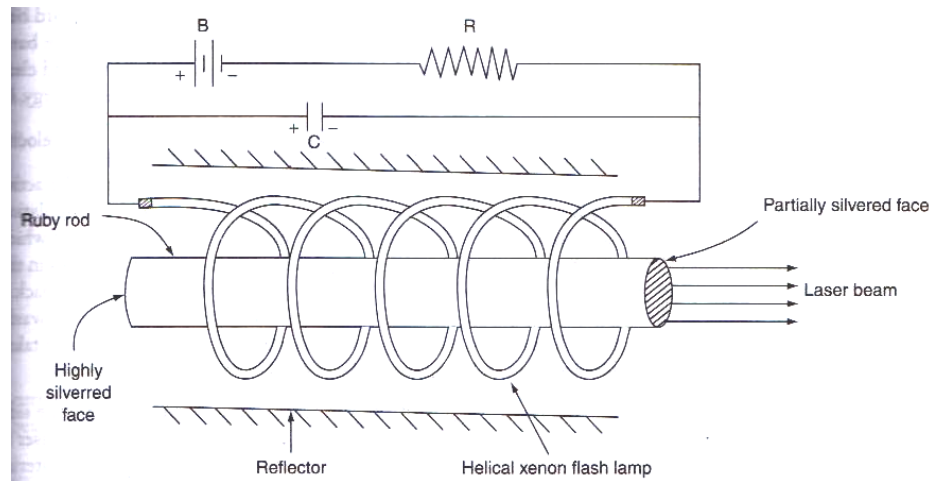

Fig. Ruby laser
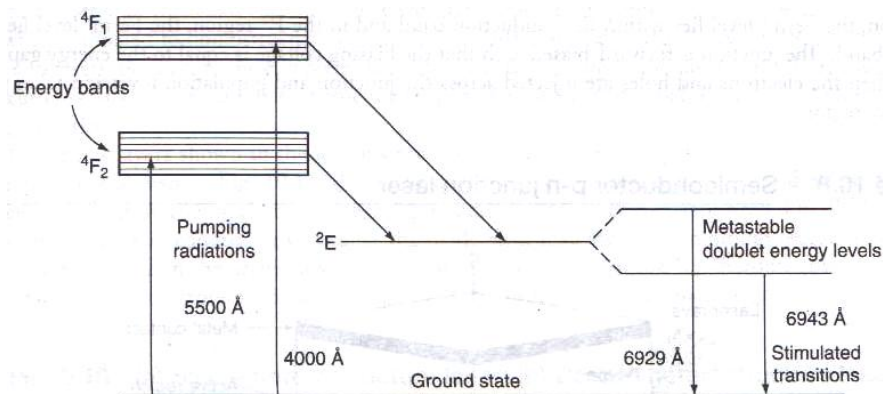
Fig. Energy level diagram of chromium ions in a ruby crystal

Construction:

Ruby ($Al_2O_3 + Cr_2O_3$) is a crystal of Aluminum oxide in which some of Al^{+3} ions are replaced by Cr^{+3} ions. When the doping concentration of Cr^{+3} is about 0.05%, the color of the rod becomes pink. The active medium in ruby rod is Cr^{+3}ions. In ruby laser a rod of 4cm long and 5mm diameter is used and the ends of the rod are highly polished. Both ends are silvered such that one end is fully reflecting and the other end is partially reflecting.

The ruby rod is surrounded by helical xenon flash lamp tube which provides the optical pumping to raise the Chromium ions to upper energy level (rather energy band). The xenon flash lamp tube which emits intense pulses lasts only few milliseconds and the tube consumes several thousands of joules of energy. Only a part of this energy is used in pumping Chromium ions while the rest goes as heat to the apparatus which should be cooled

with cooling arrangements as shown in fig. The energy level diagram of ruby laser is shown in fig.

Working:

Ruby crystal is made up of aluminum oxide as host lattice with small percentage of Chromium ions replacing aluminum ions in the crystal chromium acts as do pant. A do pant actually produces lasing action while the host material sustains this action. The pumping source for ruby material is xenon flash lamp which will be operated by some external power supply. Chromium ions will respond to this flash light having wavelength of $5600A^0$. When the Cr^{+3}ions are excited to energy level E_3 from E_1 the population in E_3 increases. Chromium ions stay here for a very short time of the order of 10-8 seconds then they drop to the level E_2 which is mat stable state of life time 10^{-3}s. Here the level E_3 is rather a band, which helps the pumping to be more effective. The transitions from E_3 to E_2 are non-radioactive in nature. During this process heat is given to crystal lattice. Hence cooling the rod is an essential feature in this method. The life time in mete stable state is 10^5times greater than the lifetime in E_3. As the life of the state E_2 is much longer, the number of ions in this state goes on increasing while ions. In this state goes on increasing while in the ground state (E_1) goes on decreasing. By this process population inversion is achieved between the exited Meta stable state E_2 and the ground state E_1. When an excited ion passes spontaneously from the metastable state E_2 to the ground state E_1, it emits a photon of wave length $6943A^0$. This photon travels through the rod and if it is moving parallel to the axis of the crystal, is reflected back and forth by the silvered ends until it stimulates an excited ion in E_2 and causes it to emit fresh photon in phase with the earlier photon. This stimulated transition triggers the laser transition. This process is repeated again and again because the photons repeatedly move along the crystal being reflected from its ends. The photons thus get multiplied. When the photon beam becomes sufficiently intense, such that part of it emerges through the partially silvered end of the crystal.

Drawbacks of ruby laser:
1. The laser requires high pumping power to achieve population inversion.
2. It is a pulsed laser.

Fig. the output pulses with time.
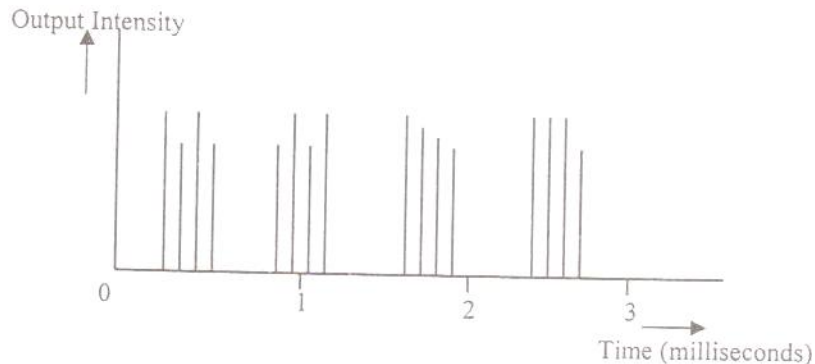
Semi conductor laser

The semiconductor laser is also called diode laser. Among the different semiconductors there are direct band gap semiconductors and indirect band gap semiconductors. In the case of direct band gap semiconductors, there is a large possibility for direct recombination of holes and electrons which emit photons. But in indirect band gap semi conductors like

silicon and germanium, direct recombination of holes and electrons is less possible and hence there is no effective emission of photons. A well know example of a direct band gap semiconductor is GaAs. Let E g be the energy gap of a material, then it emits a photon of wavelength (λ) is given the relation

$$\lambda = \frac{Ch}{E_g} \qquad \rightarrow (1)$$

Where C is the velocity of light and h is the Planck's constant
The equation (1) is reduced to

$$\lambda = \frac{1.24}{E_g} \, \mu m \qquad \rightarrow (2)$$

Where E_g is expressed in eV

As E_g increases, it emits shorter wavelengths. Diode lasers are always operated in forward bias. If p and n type materials are prepared from the same material then the p-n junction, is called as Homo junction semiconductor laser source. If p and n type materials are prepared from different materials then they are called as Hetero junction semi conductor laser source.

Construction of GaAs Semiconductor diode laser

The basic mechanism responsible for light emission from a semi conductor is the recombination of electrons and holes at p-n junction when a current is passed through the diode. The active medium is a p-n junction diode made from crystalline Gallium Arsenide (fig). The p-region and n-region in the diode are obtained by heavily doping with suitable do pants. At the junction the sides through which emitted light is coming out are well polished and parallel to each other. Since the refractive index of GaAs is high, the reflectance at the material air interface is sufficiently large so that the external mirrors are not necessary to produce multiple reflections. When a current is passed through a p-n junction p region being positively biased, holes are injected from p-region into n-region and n-region being negatively biased electrons are injected from n-region into p-region and is shown in fig.
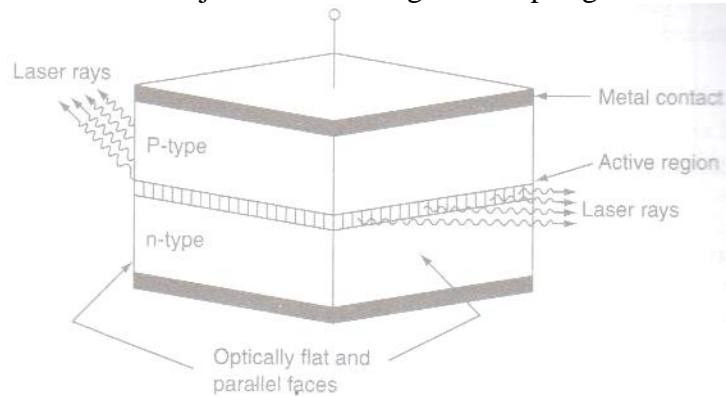
Fig. GaAs semiconductor diode laser

The connections in this p-n junction circuit are called forward bias. The electrons and holes recombine and release of light energy takes place in or near the junction. In the case of GaAs homo junction which has an energy gap of 1.44ev gives a laser beam of wavelength around 8600A^0 the electron-hole recombination takes place in the active region of the device. When the junction is forward biased, a large amount of the order of 104 amp/cm^2 is passed through the narrow junction. Thus the electrons (holes) are injected from n side to p side (p side to n side) respectively.

Fig (a)

Band diagram of heavily doped p-n junction in equilibrium.
(b) Band diagram of heavily doped p-n junction with forward bias

The electrons are minority charge carriers in p-side and holes are the minority charge carriers in n-side. The continuous injection of charge car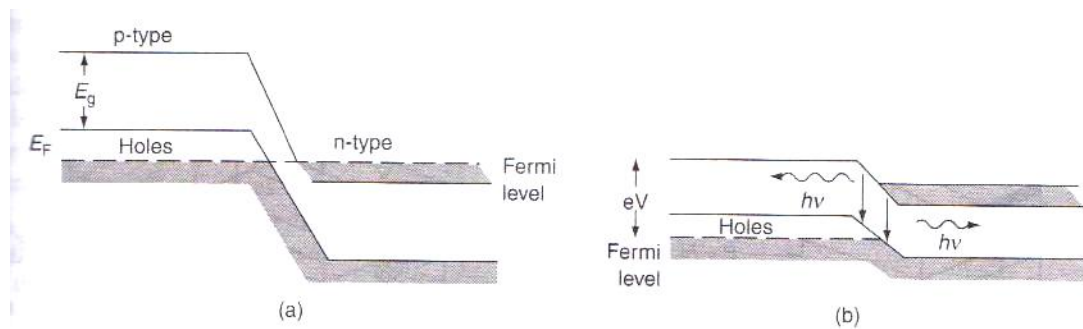riers creates the population inversion of minority carriers is n and p side respectively. The excess minority charge carries diffuse away from the region recombining with majority carriers of the n and p type material, resulting in the release of photons. Further, the emitted photons increase the recombination of injected electrons from the n-region and holes in p-region by inducing more recombinations thus the stimulated emission takes place more effectively. The efficiency of a laser emission increases when the device is cooled.

The drawbacks of homojunction lasers
1. Only output is obtained
2. The threshold current density is very large($400 amps/mm^2$)
3. Electromagnetic confinement is poor.
4. Output has more beam divergence, poor coherence.

To overcome these deficiencies, scientists have developed heterojunction laser sources like GaAs/GaAlAs structures. The advantages of heterojunction laser structures are:
1. Low threshold current density($5-10 amp/mm^2$)
2. The output is continuous
3. Carrier confinement is more effective there by less beam divergence.
4. High output power.
5. Narrow beam, high coherence, high mono chromaticity is achieved.
6. Long lifetime of the device
7. Very stable. Hence hetero junction laser diode used extensively in optical fiber communication

Applications of Lasers
Lasers find applications in various fields. They are described below.

a) **In Communications :**
   Lasers are used in optical fiber communications. In optical fiber communications, lasers are used as light source to transmit audio, video signals and data to long distances without attention and distortion.

b) The narrow angular spread of laser beam can be used for communication between earth and moon or to satellites.

c) As laser radiation is not absorbed by water, so laser beam can be used in under water (inside sea) communication networks.

2. Industrial Applications
a) Lasers are used in metal cutting, welding, surface treatment and hole drilling. Using   lasers cutting can be obtained to any desired shape and the curved surface is very smooth.
b) Welding has been carried by using laser beam.
c) Dissimilar metals can be welded and micro welding is done with great case.
d) Lasers beam is used in selective heat treatment for tempering the desired parts in automobile industry
e) Lasers are widely used in electronic industry in trimming the components of ICs

3. Medical Applications
1. Lasers are used in medicine to improve precision work like surgery. Brain surgery is an example of precision surgery Birthmarks, warts and discoloring of the skin can easily be removed with an unfocussed laser. The operations are quick and heal quickly and, best of all, they are less painful than ordinary surgery performed with a scalpel.
2. Cosmetic surgery (removing tattoos, scars, stretch marks, sun spots, wrinkles, birthmarks and hairs) see lasers hair removal.
3.Laser types used in dermatology include ruby(694nm),alexandrite(755nm),pulsed diode array(810nm), Nd:YAG(1064nm), HO:YAG(2090nm), and Er:YAG(2940nm)
4. Eye surgery and refracting surgery.
5. Soft tissue surgery: Co_2 Er :YAG laser.
6. Laser scalpel (general surgery, gynecological, urology, laparoscopic).
7.   Dental procedures.
8. Photo bio modulation (i.e. laser therapy)
9. "No-touch" removal of tumors, especially of the brain and spinal cord.
10. In dentistry for caries removal, endodontic/periodontic, procedures, tooth whitening, and oral surgery.

4. Military Applications
The various military applications are:
a) **Death rays**: By focusing high energetic laser beam for few seconds to aircraft, missile, etc can be destroyed. So, these rays are called death rays or war weapons.
b) **Laser gun:** The vital part of energy body can be evaporated at short range by focusing highly convergent beam from a laser gun.
c) **LIDAR (Light detecting and ranging)**: In place of RADAR, we can use LIDAR to estimate the size and shape of distant objects or war weapons. The differences between RADAR and LIDAR are that, in case of RADAR, Radio waves are used where as incase of LIDAR light is used.

5. In Computers: By using lasers a large amount of information or data can be stored in CD-ROM or their storage capacity can be increased. Lasers are also used in computer printers.

6. In Thermonuclear fusion: To initiate nuclear fusion reaction, very high temperature and pressure is required. This can be created by concentrating large amount of laser energy in a small volume. In the fusion of deuterium and tritium, irradiation with a high energy laser beam pulse of 1 nano second duration develops a temperature of 10^{17} ^{0}c, this temperature is sufficient to initiate nuclear fusion reaction.

7. In Scientific Research: In scientific, lasers are used in many ways including
  a) A wide variety of interferometrie techniques.
  b) Raman spectroscopy.
  c) Laser induced breakdown spectroscopy.
  d) Atmospheric remote sensing.
  e) Investigating non linear optics phenomena
  f) Holographic techniques employing lasers also contribute to a number of measurement techniques.
  g) Laser (LADAR) technology has application in geology, seismology, remote sensing and atmospheric physics.
  h) Lasers have been used abroad spacecraft such as in the cassini-huygens mission.
  i) In astronomy lasers have been used to create artificial laser guide stars, used as reference objects for adaptive optics telescope.

UNIT-IV
FIBER OPTICS

Introduction
1. An optical fiber (or fiber) is a glass or plastic fiber that carries light along its length.
2. Fiber optics is the overlap of applied science and engineering concerned with the design and application of optical fibers.
3. Optical fibers are widely used in fiber-optic communications, which permits transmission over long distances and at higher band widths (data rates) than other forms of communications.
4. Specially designed fibers are used for a variety of other applications, including sensors and fiber lasers. Fiber optics, though used extensively in the modern world, is a fairly simple and old technology.

Principle of Optical Fiber
Optical fiber is a cylinder of transparent dielectric medium and designed to guide visible and infrared light over long distances. Optical fibers work on the principle of **total internal reflection.**

Optical fiber is very thin and flexible medium having a cylindrical shape consisting of three sections
  1) The core material
  2) The cladding material
  3) The outer jacket

The structure of an optical is shown in figure. The fiber has a core surrounded by a cladding material whose reflective index is slightly less than that of the core material to satisfy the condition for total internal reflection. To protect the fiber material and also to give mechanical support there is a protective cover called outer jacket. In order to avoid damages there will be some cushion between cladding protective cover.
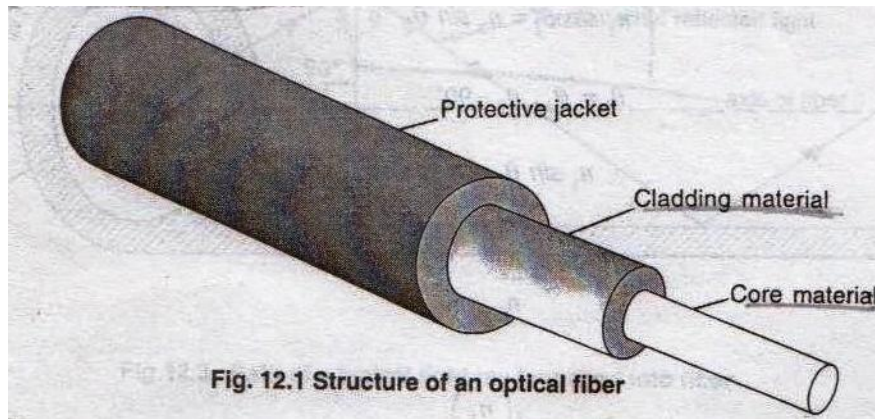
Fig. 12.1 Structure of an optical fiber

Fig..Structure of an optical fiber

When a ray of light passes from an optically denser medium into an optically rarer medium the refracted ray bends away from the normal. When the angle of incidence is increased angle of refraction also increases and a stage is reached when the refracted ray just grazes the surface of separation of core and cladding. At this position the angle of refraction is 90 degrees. This angle of incidence in the denser medium is called the **critical angle (θc)** of the denser medium with respect to the rarer medium and is shown in the fig. If the angle of incidence is further increased then the totally reflected. This is called total **internal reflection**. Let the reflective indices of core and cladding materials be n1 and n2 respectively.
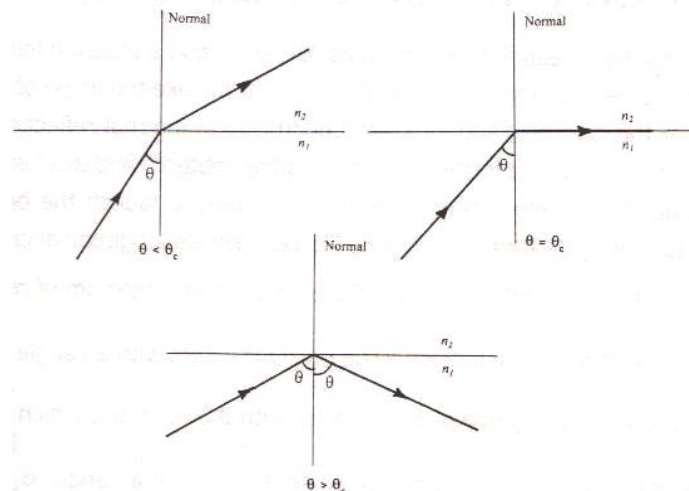

Fig. Total internal reflection.

When a light ray, travelling from an optically denser medium into an optically rarer medium is incident at angle greater than the critical angle for the two media. The ray is totally reflected back into the medium by obeying the loss of reflection. This phenomenon is known as totally internal reflection.

According to law of refraction,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Here $\theta_1 = \theta c$, $\theta_2 = 90$

$$n_1 \sin \theta c = n_2 \sin 90$$

$$\sin\theta c = \frac{n_2}{n_1}$$

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$$\theta c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad\qquad \rightarrow (1)$$

Equation (1) is the expression for condition for total internal reflection. In case of total internal reflection, there is absolutely no absorption of light energy at the reflecting surface. Since the entire incident light energy is returned along the reflected light it is called total internal reflection. As there is no loss of light energy during reflection, hence optical fibers are designed to guide light wave over very long distances.

Acceptance Angle and Acceptance Cone

Acceptance angle: It is the angle at which we have to launch the beam at its end to enable the entire light to propagate through the core. Fig.8.12 shows longitudinal cross section of the launch of a fiber with a ray entering it. The light is entered from a medium of refractive index n_0 (for air $n_0=1$) into the core of refractive index n_1. The ray (OA) enters with an angle of incidence to the fiber end face i.e. the incident ray makes angle with the fiber axis which is nothing but the normal to the end face of the core. Let a right ray OA enters the fiber at an angle to the axis of the fiber. The end at which light enter the fiber is called the **launching pad.**
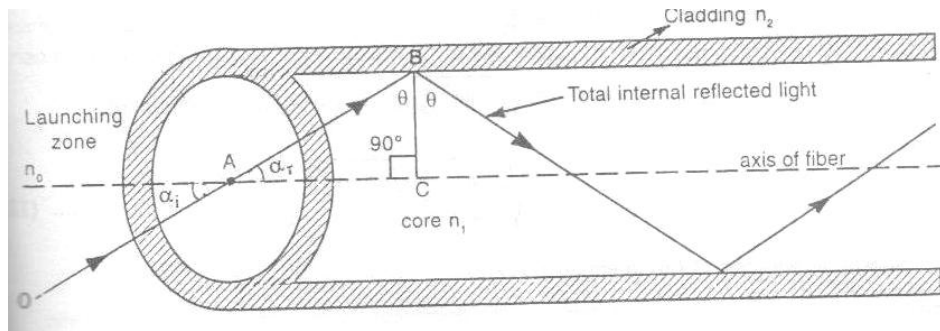

Fig. Path of atypical light ray launched into fiber.

Let the refractive index of the core be n_1 and the refractive index of cladding be n_2. Here $n_1>n_2$. The light ray reflects at an angle and strikes the core cladding interface at angle θ. If the angle θ is greater than its critical angle θc, the light ray undergoes total internal reflection at the interface.

According to Snell's law
$$n_0\sin\alpha_i=n_1\sin\alpha_r \qquad\qquad \rightarrow (2)$$
From the right angled triangle ABC
$$\alpha_r+\theta=90^0$$

$$\alpha_r=90^0-\theta \qquad\qquad \rightarrow (3)$$
Substituting (3) in (2), we get
$$n_0\sin\alpha_i =n_1\sin(90^0-\theta) = n_1\cos\theta$$

$$\sin\alpha_i=\left(\frac{n_1}{n_0}\right)\cos\theta \qquad\qquad \rightarrow (4)$$
When $\theta= \theta c$, $\alpha_i= \alpha_m$=maximum α value
$$\sin\alpha_m=\left(\frac{n_1}{n_0}\right)\cos\theta_c \qquad\qquad \rightarrow (5)$$
From equation (1) $\sin\theta c = \frac{n_2}{n_1}$ Type equation here.

$$\cos \theta_c = \sqrt{1 - sin^2\theta c} = \sqrt{1 - (\frac{n_2}{n_1})^2} = \frac{\sqrt{n_1{}^2 - n_2{}^2}}{n_1} \qquad \rightarrow (6)$$

Substitute equation (6) in equation (5)

$$\sin\alpha_m = (\frac{n_1}{n_0}) \frac{\sqrt{n_1{}^2 - n_2{}^2}}{n_1} = \frac{\sqrt{n_1{}^2 - n_2{}^2}}{n_0} \qquad \rightarrow (7)$$

If the medium surrounding fiber is air, then $n_0 = 1$

$$\sin\alpha_m = \sqrt{n_1{}^2 - n_2{}^2} \qquad \rightarrow (8)$$

This maximum angle is called the **acceptance angle or the acceptance cone half angle** of the fiber.

The **acceptance angle** may be defined as the maximum angle that a light ray can have with the axis of the fiber and propagate through the fiber. Rotating the acceptance angle about the fiber axis (fig.) describes the acceptance cone of the fiber. Light launched at the fiber end within this acceptance cone alone will be accepted and propagated to the other end of the fiber by total internal reflection. Larger acceptance angles make launching easier. Light gathering capacity of the fiber is expressed in terms of maximum acceptance angle and is termed as "**Numerical Aperture".**
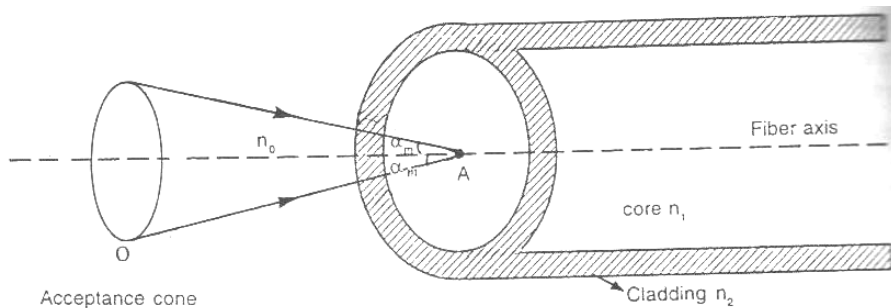

Fig. Acceptance cone

Numerical Aperture

Numerical Aperture of a fiber is measure of its light gathering power. The numerical aperture (NA) is defined as the sign of the maximum acceptance angle.

Numerical aperture (NA) = $\sin\alpha_m = \sqrt{n_1{}^2 - n_2{}^2}$ \qquad $\rightarrow (9)$

$$= \sqrt{(n_1 - n_2)(n_1 + n_2)}$$
$$= \sqrt{((n_1 + n_2) n_1 \Delta)} \qquad \rightarrow (10)$$

Where $\Delta = \frac{(n_1 - n_2)}{n_1}$ called as fractional differences in refractive indices n and n_2 are the refractive indices of core and cladding material respectively.

As $n_1 \approx n_2$, we can take $n_1 + n_2 = 2n_1$

Then numerical aperture= $(2n_1{}^2\Delta)^{1/2} = n_1 (2\Delta)^{1/2} \rightarrow (11)$

Numerical aperture is a measure of amount of light that can be accepted by a fiber. From equation (9) it is seen that numerical aperture depends only on the refractive indices of core and

cladding materials and it is independent on the fiber dimensions. Its value ranges from 0.1 to 0.5. A large NA means that the fiber will accept large amount of light from the source.

Proof:

We shall show that the light emitted by a small defuse source situated on the fiber axis near to one end face, only a fraction NA^2 can be collected by the fiber and propagated along the fiber. Consider a small defuse light source such as the isotropic radiator in which the power radiated per unit solid angle in a direction θ to the normal to the surface
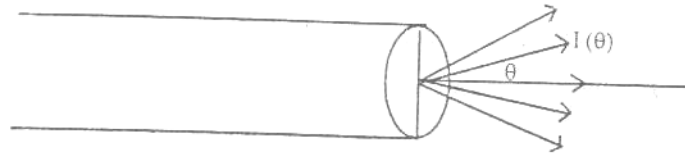
is given by

Fig. Diffuse light source

$$I(\theta) = I_0 Cos\,\theta \rightarrow (12)$$

The total power φ_0 emitted such source is obtained by integrating $I(\theta)$ over all forward directions

$$\varphi_0 = \int_0^{\Pi/2} I_0\ Cos\theta d\,\theta \qquad\qquad \rightarrow (13)$$

Where $d\Omega$=small element of solid angle=$2\pi Sin\theta\ d\theta \qquad \rightarrow (14)$

$$\varphi_0 = \int_0^{\Pi/2} I_0\ Cos\theta 2\pi Sin\theta\ d\,\theta$$
$$= \int_0^{\Pi/2} \pi I_0 Sin2\theta d\theta\ =\pi I_0 \qquad\qquad \rightarrow (15)$$

But the power from such source that can be collected by an adjacent fiber whose core diameter is greater than the diameter of the source is given by φ, where

$$\Phi = \int_0^{\alpha m} I_0\ Cos\theta 2\Pi Sin\theta\ d\theta$$
$$= \Pi I0 sin2\alpha m$$
$$(\Phi/\varphi_0) = (sin\alpha_m)^2 = (NA)^2 = 2n_1^2\Delta \qquad\qquad \rightarrow (16)$$

Where $\Delta = \frac{(n_1-n_2)}{n_1}$

Equation (16) represents the light gathering power of the fiber. In order to collect as much light as possible it is necessary to make n_1 and Δ large.

In some cases it is customary to use Δn which can be defined as $\Delta n = n_1 - n_2$

But $\Delta = \frac{(n_1-n_2)}{n_1} = \frac{\Delta n}{n_1}$

$$(\Phi/\varphi_0) = (sin\alpha_m)^2 = (NA)^2 = 2n_1\Delta n \rightarrow (17)$$
$$NA = (2n_1\Delta n) \rightarrow (18)$$

Step index fibers and graded index fiber -transmission of signals in them:

Based on the variation of refractive index of core, optical fibers are divided into: (1) step index and (2) graded index fibers. Again based on the mode of propagation, all these fibers are divided into: (1) single mode and (2) multimode fibers. In all optical fibers, the refractive index of cladding material is uniform. Now, we will see the construction, refractive index of core and cladding with radial distance of fiber, ray propagation and applications of above optical fibers.

   i.   **Step index fiber:** The refractive index is uniform throughout the core of this fiber. As we go radially in this fiber, the refractive index undergoes a step change at the core-cladding interface. Based on the mode of propagation of light rays, step index fibers are

of 2 types: a) single mode step index fiber & b) multimode step index fibers. Mode means, the number of paths available for light propagation of fiber. We describes the different types of fiber below

a) **Single mode step index fiber**: The core diameter of this fiber is about 8 to 10μm and outer diameter of cladding is 60 to 70 μm. There is only one path for ray propagation. So, it is called **single mode fiber**. The cross sectional view, refractive index profile and ray propagation are shown in fig. (i). In this fiber, the transmission of light is by successive total internal reflections i.e. it is a reflective type fiber. Nearly 80% of the fibers manufactured today in the world are single mode fibers. So, they are extensively used.
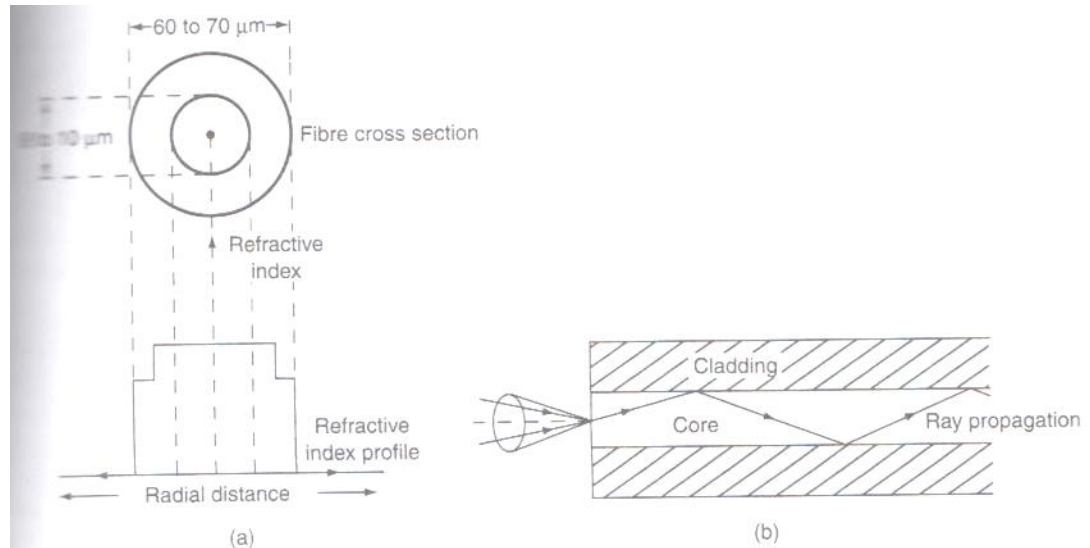

Fig (i).Single mode step index fiber ;(a) Cross sectional view and refractive index profile ;(b) Ray propagation

Multimode step index fiber: The construction of multimode step index fiber is similar to single mode step index fiber except that its core and cladding diameters are much larger to have many paths for light propagation. The core diameter of this fiber varies from 50 to 200 μm and the outer diameter of cladding varies from 100 to 250 μm. The cross-sectional view, refractive index profile and ray propagations are shown in fig 2. Light propagation in this fiber is by multiple total internal reflections i.e it is a reflective type fiber.

b) **Transmission of signal in step index fiber:** Generally the signal is transmitted through the fiber in digital form i.e. in the form of 1's and 0's. The propagation of pulses through the multimode fiber is shown in fig (i)(b). The pulse which travels along path 1(straight) will reach first at the other end of fiber. Next the pulse that travels along with path 2(zig-zag) reaches the other end. Hence, the pulsed signal received at the other end is broadened. This is known as intermodal dispersion. This imposes limitation on the separation between pulses and reduces the transmission rate and capacity. To overcome this problem, graded index fibers are used.

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2) **Graded index fiber**: In this fiber, the refractive index decreases continuously from center radially to the surface of the core. The refractive index is maximum at the center and minimum at the surface of core. This fiber can be single mode or multimode fiber. The cross sectional view, refractive index profile and ray propagation of multimode graded index fiber are shown in fig .(ii)(a). The diameter of core varies from 50 to 200μm and outer diameter of cladding varies from 100 to 250 μm.

The refractive index profile is circularly symmetric. As refractive index changes continuously radially in core, light rays suffer continuous refraction in core. The propagation of light ray is not due to total internal reflection but by refraction as shown in fig. (ii)(b). in graded index fiber, light rays travel at different speed in different paths of the fiber. Near the surface of the core, the refractive index is lower, so rays near the outer surface travel faster than the rays travel at the center. Because of this, all the rays arrive at the receiving end of the fiber approximately at the same time. This fiber is costly. .

Transmission of signal graded index fiber: In multimode graded index fiber, large number of paths is available for light ray propagation. To discuss about inter modal dispersion, we consider ray path 1 along the axis of fiber.
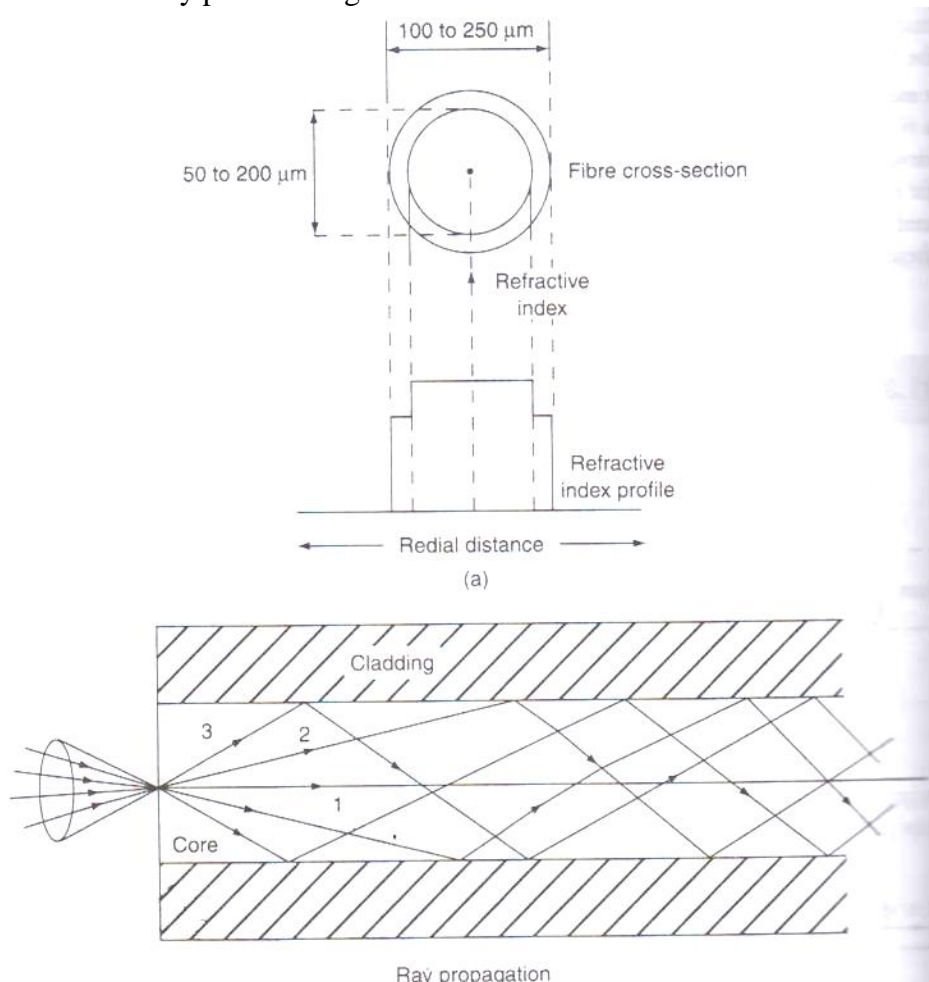

Fig. (ii).Multimode step index fibre(a)Cross sectional view and refractive index profle(b)Ray propagation

As shown in fig. (ii)(b) and another ray path 2. Along with the axis of fiber, the refractive index of core is maximum, so the speed of ray along path 1 is less. Path 2 is sinusoidal and it is longer, along this path refractive index varies. The ray mostly travels in low refractive region, so the ray

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2 moves slightly faster. Hence, the pulses of signals that travel along path 1 and path 2 reach other end of fiber simultaneously. Thus, the problem of intermodal dispersion can be reduced to a large extent using graded index fibers

Differences between step index fiber and graded index fibers:

Step index fiber	Graded index fiber
1.The refractive index of core Is uniform and step or abrupt Change in refractive index Takes place at the interface of core and cladding in step index fibers	1.The refractive index of core is non uniform, the refractive index of core decreases Para-Felicity from the axis of the fiber to its surface.
2. The light rays propagate in zigzag manner inside the core. The rays travel in the fiber as meridional rays they cross the fiber axis for every reflection.	2. Light rays propagate in the form of skew rays or helical rays. They will not cross the fiber axis

Difference between single mode fibers and multi mode fibers:

Single mode fiber	Multimode fiber
1. In single mode fiber there is only one path for ray propagation	1. In multimode fiber, large number of paths is available for light ray propagation.
2. A single mode step index fiber has less core diameter (< 10 μm) and the difference between the reflective indices of core and cladding is very small.	2. Multi mode fibers, large number of paths are available for light ray propagation.
3. In single mode fibers, there is no dispersion.	3. There is signal distortion and dispersion takes place in multimode fibers.
4. The band width is about 50 MHz for multimode step index fiber where as it is more than 1000 MHz km in case of single mode step index fiber.	4. The band width of the fiber lies in between 200 MHz km to 600 MHz km even though theoretically it has an infinite bandwidth.
5. NA of multimode step index fiber is more where as in single mode step index fibers, it is very less.	5. NA of graded index fibers is less.
6. Launching of light into single mode fibers is difficult.	6. Launching of light into multimode fibers is easy.
7. Fabrication cost is very high.	7. Fabrication cost is less

Advantage of optical fibers in communication:

Let us see the advantages of optical fiber communication over conventional communication system.

1. **Enormous bandwidths:**

The information carrying capacity of a transmission system is directly proportional to the carrier frequency of the transmitted signals. The optimal carrier frequency is in the range of 10^{14} Hz while the radio frequency is about 10^6 Hz and microwave frequency is about 10^{10} Hz. Thus the optical fibers have enormous transmission bandwidths and high data rate.

2. **Low transmission loss:**

Due to the usage of ultra low loss fibers and the erbium doped silica fibers as optical amplifiers, one can achieve almost loss less transmission. Hence for long distance communication fibers of 0.002db/km are used. Thus the repeater spacing is more than 100km.

3. **Immunity to cross talk:**

Since optical fibers are dielectric wave guides, they are free from any electromagnetic interference (EMI) and radio frequency interference (RFI). Since optical interference among different fibers is not possible, cross talk is negligible even many fibers are cabled together.

4. **Electrical isolation:**
   Optical fibers are made from silica which is an electrical insulator. Therefore they do not pick up any electromagnetic wave or any high current lightening. It is also suitable in explosive environment.

5. **The size and weight:**
   The size of the fiber ranges from 10μm to 50μm which is very very small. The space occupied by the fiber cable is negligibly small compared to conventional electrical cables. Optical fibers are light in weight. These advantages make them to use in air craft's and satellites more effectively.

6. **Signal security**:
   The transmitted signal through the fiber does not radiate. Unlike in copper cables, a transmitted signal cannot be drawn from a fiber without tampering it. Thus, the optical fiber communication provides 100% signals security.

7. **Ruggedness and flexibility**:
   The fiber cable can be easily bend or twisted without damaging it. Further the fiber cables are superior than the copper cables in terms of handling, installation, storage, transportation, maintenance, strength and durability.

8. **Low cost and availability**:
   Since the fibers are made of silica which is available in abundance. Hence, there is no shortage of material and optical fibers offer the potential for low cost communication.

9. **Reliability:**
   The optical fibers are made from silicon glass which does not undergo any chemical reaction or corrosion. Its quality is not affected by external radiation. Further due to its negligible attenuation and dispersion, optical fiber communication has high reliability. All the above factors also tend high to reduce the expenditure on its maintenance.

Fiber optic communication system :

Fiber opptics essentially deals of with the communication(including voice signals,video signals or digital data) by transmission of light through optical fibers. Optical fiber communication system essentially consists of three parts:(a)transmitter (b) optical fiber and (c) receiver.The transmitter includes modulator, encoder, light source, drive circuits and couplers. The receiver includes amplifier and decoder,binary electrical signal and light decoder.
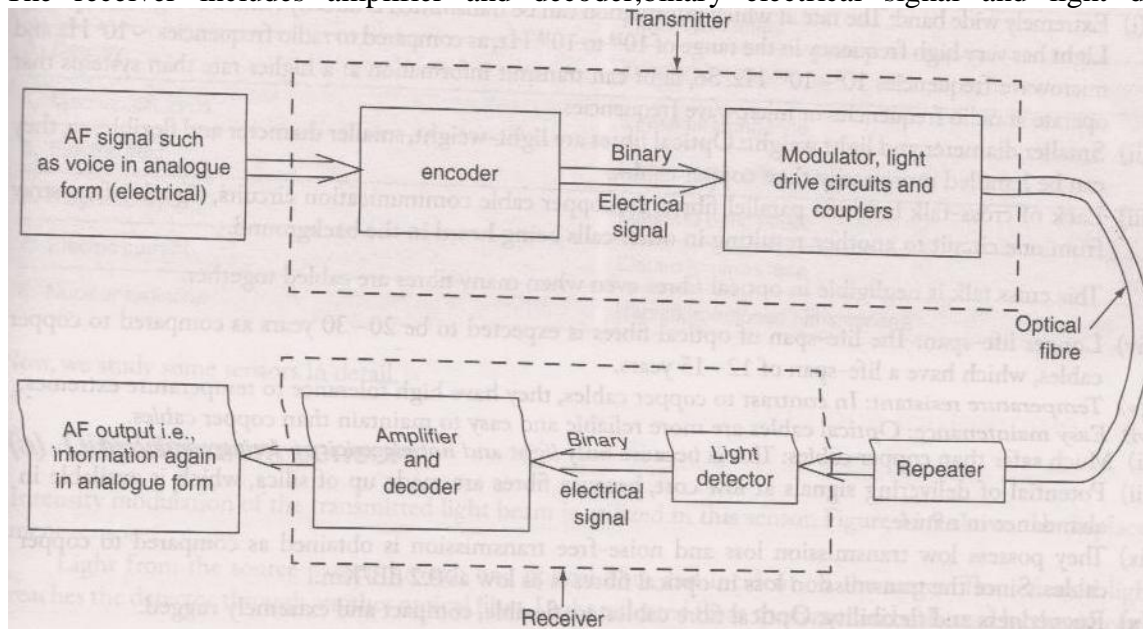

Fig. Block diagram represents optical fibre communication system

UNIT-V
INTERFERENCE AND DIFFRACTION

Interference-Introduction

Wave Theory of light attempts to understand the various optical phenomena exhibited by light waves. Interference constituted the first proof of the wave nature of light. Thomas Young first experimentally demonstrated interference in light waves. The superposition principle forms the conceptual basis for the explanation of interference. To produce interference, the light waves should be coherent, i.e., the light waves should have constant phase difference and same frequencies.

Interference

When two or more light waves superimpose in the medium then according to superposition principle, the resultant displacement at any point is equal to the algebraic sum of the displacements due to individual waves. The variation of resultant displacement influences amplitude variation, which causes intensity variations. This modification in the distribution of intensity in the region of superposition is known as interference. When the resultant amplitude is the sum of the amplitudes due to two light waves, the interference is *constructive interference*. If the resultant amplitude is equal to the difference of two amplitudes, the interference becomes *destructive interference*. The intensity variations are studied as interference fringes or patterns.

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Conditions for Interference

1. The two light sources emitting light waves should be coherent.
2. The two sources must emit continuous light waves of same wavelengths and frequency.
3. The separation between the two sources should be small.
4. The distance between the two sources and the screen should be large.
5. To view interference fringes, the background should be dark.
6. The amplitudes of the light waves should be equal or nearly equal.
7. The sources should be narrow, i.e., they must be small.
8. The sources should be monochromatic.

Interference in Thin Film by Reflection

When light is incident on a plane parallel thin film, some portion gets reflected from the upper surface and the remaining portion is transmitted into the film. Again, some portion of the transmitted light is reflected back into the film by the lower surface and emerges through the upper surface. These reflected light beams superimpose with each other, producing interference and forming interference patterns.
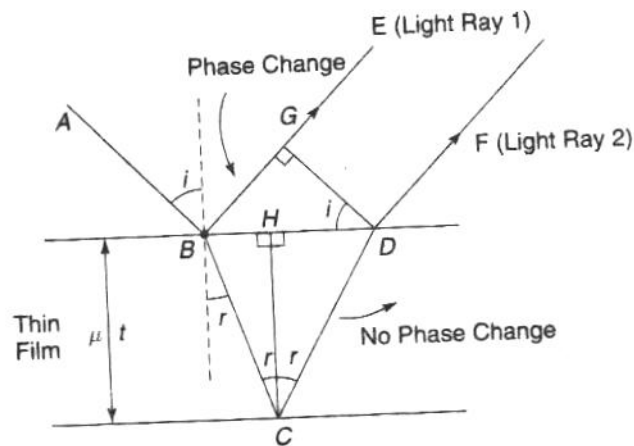

Fig Interference in thin film

Consider a transparent plane parallel thin film of thickness 't 'with refractive index μ. Let a monochromatic light ray AB be incident at an angle of incidence of 'i' on the upper surface of the film. BE and BC are the reflected and transmitted light rays. Let the angle of refraction is 'r'. The ray BC will be reflected into the film and emerge through the film in the form of light ray DF. These two light rays superimpose and depending upon the path difference between them, they produce interference patterns.

To know the path difference, draw the normal DG to BE. From the points D and G onwards, the light rays travel equal distances. By the time the light ray travels from B to G, the transmitted light ray has to travel from B to C and C to D.

The path difference between light rays (1) and (2) is,

$$\text{Path difference} = \mu(BC+CD) \text{ in film} - BG \text{ in air} \qquad \rightarrow(1)$$

Consider the ΔBCH, $\cos r = \dfrac{HC}{BC}$

$$BC = \frac{HC}{\cos r} = \frac{t}{\cos r}$$

Similarly, from ΔDCH, $CD = \dfrac{t}{\cos r}$

$$BC = CD = \frac{t}{\cos r} \qquad \rightarrow (2)$$

To calculate BG, first BD which is equal to (BH+HD) has to be obtained.

From ΔBHC
$$\tan r = \frac{BH}{CH} = \frac{BH}{t}$$
$$BH = t \tan r$$

Similarly,
$$HD = t \tan r$$
$$BD = BH + HD = 2t \tan r \; (\because BH = HD) \qquad \rightarrow (3)$$

From Δ BGD,
$$\sin i = \frac{BG}{BD}$$
$$BG = BD \sin i = 2t \tan r \sin i$$

From Snell's law, $\sin i = \mu \sin r$
$$BG = 2\mu t \tan r \sin r \qquad \rightarrow (4)$$

Substituting the above values in Eq.(1)

$$\text{Path difference} = \frac{2\mu t}{\cos r} - 2\mu t \tan r \sin r$$
$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$
$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r} \cos^2 r$$
$$= 2\mu t \cos r \qquad \rightarrow (5)$$

At the point B, reflection occurs from the upper surface of the thin film (denser medium). Light ray (1) undergoes an additional phase change of π or an additional path difference of $\frac{\lambda}{2}$

$$\text{Total path difference} = 2\mu t \cos r + \frac{\lambda}{2}$$

When the path difference is equal to integral multiple of λ then the rays (1) and (2) meet in phase and undergo constructive interference.

The condition for bright fringe is

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$
$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \qquad \text{where n= 0,1,2,3…} \qquad \rightarrow (6)$$

When the path difference is equal to half integral multiple of λ then the rays (1) and (2) meet in out of phase and undergo destructive interference.

The condition for dark fringe is

$$2\mu t \cos r + \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$
$$2\mu t \cos r = n\lambda \qquad \text{where n=0, 1, 2, 3…} \qquad \rightarrow (7)$$

Depending on the above conditions, the interference pattern consists of bright and dark fringes.

Newton's Rings

Newton's rings are one of the best examples for the interference in a nonuniform thin film. When a Plano-convex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and glass plate are seen. These circular rings were discovered by Newton and are called Newton's rings.
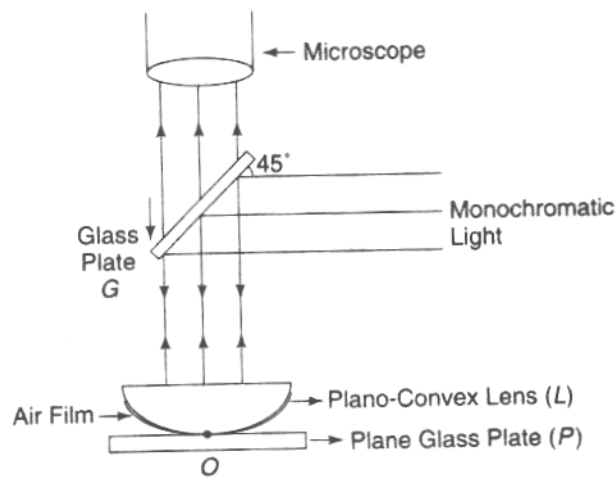
Experimental Arrangement

Fig Newton's rings (Experimental set up)

The experimental arrangement is as shown in fig. The Plano-convex lens (L) of large radius of curvature is placed with its convex surface on a plane glass plate P). The lens makes the contact with the plate at 'O'. The monochromatic light falls on a glass plate G held at an angle of 45° with the vertical. The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P. A part of the light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of the plate. These reflected rays interfere and give rise to an interference pattern in the form of circular rings. These rings are seen near the upper surface of the air film through the microscope.

Explanation of Newton's Rings
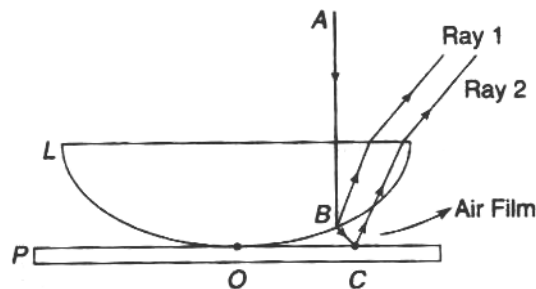
Fi.g Formation of Newton's rings

Newton's rings are formed due to interference between the light rays reflected from the top and bottom surfaces of air film between the plate and the lens. The formation of Newton's rings can be explained with the help of Fig. A part of the incident monochromatic light AB is reflected at B (glass-air boundary) in the form of the ray (1) with any additional phase (or path) change. The other part of light is refracted along BC. Then at C (air-glass boundary), it is again reflected in the form of the ray (2) with additional phase change of π or path change of $\frac{\lambda}{2}$.

As the rings are observed in the reflected light, the path difference between them is $2\mu t \cos r + \frac{\lambda}{2}$.

For air film $\mu =1$ and for normal incidence r=0, path difference is $2t + \frac{\lambda}{2}$.

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At the point of contact t=0, path difference is $\frac{\lambda}{2}$, i.e., the reflected light at the point of contact suffers phase change of π. Then the incident and reflected lights are out of phase and interfere destructively. Hence the central spot is dark.

The condition for bright ring is $2t + \frac{\lambda}{2} = n\lambda$

$$2t = (2n-1)\frac{\lambda}{2} \qquad \text{where n=1, 2, 3...}$$

The condition for dark ring is

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$
$$2t = n\lambda \text{ where } n = 0,1,2,3...$$

For monochromatic light, the bright and dark rings depend on thickness of the air film. For a Newton's rings system, the focus of points having same thickness lie on a circle having its centre at the point of contact. Thus, we get bright and dark circular rings with the point of contact as the centre.
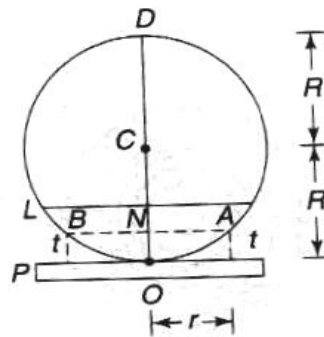
Theory of Newton's Rings

Fig. Theory of Newton's rings

To find the diameters of dark and bright rings, let 'L' be a lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface (Fig.) with center at 'C'. Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the film thickness 't'.

From the property of a circle, NA x NB = NO x ND

Substituting the values, r x r = t x (2R-t)

$$r^2 = 2Rt-t^2$$

As 't' is small, t^2 will be negligible

$$r^2 = 2Rt$$
$$t = \frac{r^2}{2R}$$

For bright ring, the condition is

$$2t = (2n-1)\frac{\lambda}{2}$$
$$2\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$
$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by $\frac{D}{2}$, the diameter of nth bright ring will be

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D = \sqrt{2n-1}\sqrt{2\lambda R}$$
$$D \propto \sqrt{2n-1}$$
$$D\sqrt{odd\ natural\ number}$$

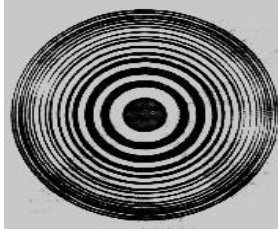Thus, the diameters of the bright rings are proportional to the square root of odd natural numbers.

Fig: Newton's ring pattern

For dark ring, the condition is

$$2t = n\,\lambda$$
$$2\frac{r^2}{2R} = n\,\lambda$$
$$r^2 = n\,\lambda R$$
$$D^2 = 4\,n\,\lambda R$$
$$D = 2\sqrt{n\,\lambda R}$$
$$D \propto \sqrt{n}$$

$$D \propto \sqrt{natural\ number}$$

Thus, the diameters of dark rings are proportional to the square root of natural numbers.
With the increase in the order (n), the rings get closer and the fringe width decreases and are shown in Fig 7.5.

Determination of Wavelength of a Light Source

Let R be the radius of curvature of a Plano-convex lens, λ be the wavelength of light used. Let D_m and D_n are the diameters of mth and n[th] dark rings respectively. Then
$$D_m^2 = 4m\lambda R$$

$$\text{And } D_n^2 = 4(n)\,\lambda R$$

$$D_n^2 - D_m^2 = 4(m\text{-}n)\ \lambda R$$

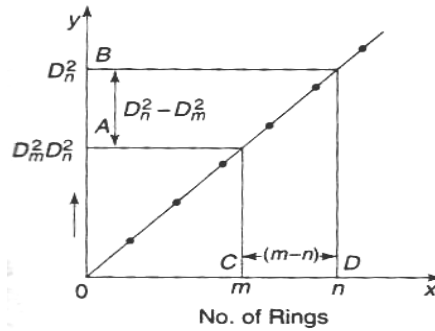$$\lambda = \frac{D_n^2 - D_m^2}{4(m-n)R}$$

Fig Plot of D^2 with respect to number of rings

Newton's rings are formed with suitable experimental setup. With the help of travelling microscope, the readings for different orders of dark rings were noted from one edge of the rings to the other edge. The diameters of different orders of the rings can be known. A plot between D^2 and the number of rings gives a straight line as shown in the fig.
From the graph,

$$\frac{D_n^2 - D_m^2}{(m-n)} = \frac{AB}{CD}$$

The radius R of the Plano-convex lens can be obtained with the help of a Spherometer..
Substituting these values in the formula, λ can be calculated.

Determination of Refractive Index of a Liquid

The experiment is performed when there is an air film between glass plate and the Plano-convex lens. The diameters of mth and nth dark rings are determined with the help of travelling microscope. We have

$$D_n^2 - D_m^2 = 4(m-n)\ \lambda R \qquad\qquad \rightarrow(8)$$

The system is placed into the container which consists of the liquid whose refractive index (μ) is to be determined. Now, the air film is replaced by the liquid film. Again, the diameters of the same mth and nth dark rings are to be obtained. Then we have

$$D_n'^2 - D_m'^2 = \frac{4(m-n)\ \lambda R}{\mu} \qquad\qquad \rightarrow(9)$$

$$\mu = \frac{D_n^2 - D_m^2}{D_n'^2 - D_m'^2}$$

$$\rightarrow(10)$$

Using the above formula, 'μ' can be calculated.

Introduction to Diffraction

The wave nature of light is further confirmed by the optical phenomenon of diffraction. The word 'diffraction' is derived from the Latin word *diffractus* which means to break to pieces. It is common experience that waves bend around obstacles placed in their path. When light waves encounter an obstacle, they bend round the edges of the obstacle. This bending is predominant when the size of the obstacle is comparable to the wavelength of light. The bending of light waves around the edge of an obstacle is diffraction. It was first observed by Gremaldy.
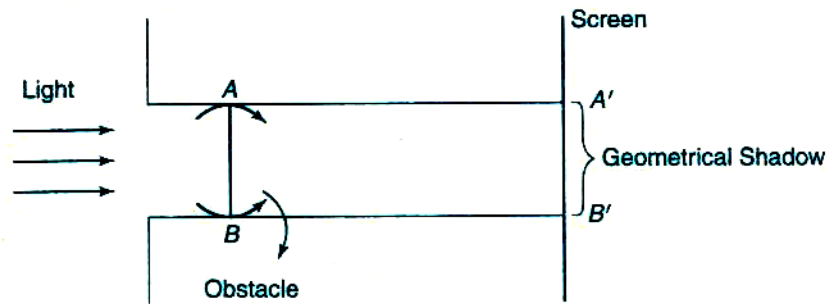
DIFFRACTION

Fig Diffraction

As shown in fig., when light falls on an obstacle, the corresponding geometrical shadow on the screen should be completely dark. In practice, the geometrical shadow consists of bright and dark fringes. These fringes are due to the superimposition of bended light waves around the corners of an obstacle. The amount of bending depends upon the size of the obstacle and wavelength of light.

When the light falls on an obstacle whose size is comparable with the wavelength of light, then the light bends around the edges or corners of the obstacle and enters into the geometrical shadow. This bending of light is known as diffraction. The bright and dark fringes in the shadow form a diffraction pattern.

The diffraction phenomena are broadly classified into two types:

Fresnel diffraction

To study diffraction, there should be a light source, obstacle and screen. In this class of diffraction, the source and screen are placed at finite distances from the obstacle. To study this diffraction, lenses are not necessary as the source and screen are at a finite distance. This diffraction can be studied in the direction of propagation of light. The incident wave fronts are either spherical or cylindrical.

Fraunhofer diffraction

In this class of diffraction, the source and screen are placed at infinite distances from the obstacle. Due to the above fact, lenses are needed to study the diffraction. This diffraction can be studied in any direction. In this case, the incident wave front is plane.

Fraunhofer Diffraction at Single Slit

Consider a slit AB of width 'e'. Let a plane wave front WW' of monochromatic light of wavelength λ propagating normally towards the slit is incident on it. The diffracted light through the slit is focused by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygens-Fresnel, every point on the wave front in the plane of the slit is a source of secondary wavelets, which spread out to the right in all directions. Those wavelets travelling normal to the slit, i.e., along the direction OP_0 are brought to focus at P_0 by the lens. Thus, P_0 is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focused at a point P_1 on the screen. Depending on path difference, the point P_1 may

have maximum or minimum intensities. To find the intensity at P_1, draw the normal AC from A to the light ray at B.
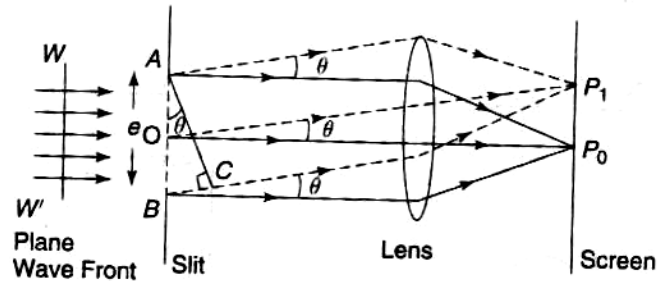

Fig Fraunhofer diffraction - Single slit

The path difference between the wavelets from A and B in the direction θ is given by

$$\text{Path diff} = BC = AB \sin\theta = e \sin\theta$$

$$\text{Corresponding phase diff} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} e \sin\theta$$

Let the width of the slit be divided into n equal parts and the amplitude of the wave from each part is 'a'. The phase difference between any two successive waves from these parts would be

$$\frac{1}{n}[\text{Total phase}] = \frac{1}{n}\left[\frac{2\pi}{\lambda} e \sin\theta\right] = d \text{ (say)}$$

Using the method of vector addition of amplitudes, the resultant amplitude R is given by

$$R = \frac{a \sin nd/2}{\sin d/2}$$

$$= \frac{a \sin(\pi e \sin\theta/\lambda)}{\sin(\pi e \sin\theta/n\lambda)}$$

$$= a \frac{\sin\alpha}{\sin\alpha/n} \text{ where } \alpha = \pi e \sin\theta/\lambda$$

$$= a \frac{\sin\alpha}{\alpha/n} \; (\because \alpha/n \text{ is very small})$$

$$= n\frac{a \sin\alpha}{\alpha} \; (\because na = A)$$

$$= A\frac{\sin\alpha}{\alpha}$$

$$\text{Intensity} = I = R^2 = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \rightarrow (1)$$

Principal Maximum

The resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\left|\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\right|$$

$$= A\left|1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right|$$

I will be maximum, when value of R is maximum. For maximum value of R, the negative terms must vanish, i.e., $\alpha = 0$

$$\frac{\pi e \sin\theta}{\lambda} = 0$$

$$\sin\theta = 0$$

$$\theta = 0 \qquad \rightarrow (2)$$

$$\text{Then } R=A$$
$$I_{max}=R^2=A^2 \qquad\qquad \rightarrow(3)$$

The condition $\theta=0$ means that the maximum intensity is formed at P_0 and is known as *principal maximum.*

Minimum Intensity Positions

'I' will be maximum, when $\sin\alpha=0$

$$\alpha=\pm\pi,\ \pm2\pi,\ \pm3\pi$$
$$\alpha=\pm m\pi$$
$$\frac{\pi e \sin\theta}{\lambda}=\pm m\pi$$
$$e\sin\theta=\pm m\lambda \qquad \text{where m} = 1,2,3\ldots \qquad \rightarrow(4)$$

Thus, the points of minimum intensity are obtained on either side of the principal maximum. For m=0, $\sin\theta=0$, which corresponds to principal maximum.

Secondary Maxima

In between these minima, we get secondary maxima. The positions can be obtained by differentiating the expression of I w.r.t α and equating to zero. We get

$$\frac{dI}{d\alpha}=\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right]=0$$
$$A^2.\frac{2\sin\alpha}{\alpha}.\frac{\alpha\cos\alpha -\sin\alpha}{\alpha^2}=0$$

Either $\sin\alpha=0$ or $\alpha\cos\alpha - \sin\alpha=0$

$\sin\alpha=0$ gives positions of minima.

Hence the positions of secondary maxima are given by

$$\alpha\cos\alpha - \sin\alpha=0$$
$$\alpha=\tan\alpha \qquad\qquad \rightarrow(5)$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $y=\alpha$ and $y=\tan\alpha$ on the same graph. The points of intersection of the two curves give the values of α which satisfy the above equation. The plots of $y=\alpha$ and $y=\tan\alpha$ are shown in fig 2.3.
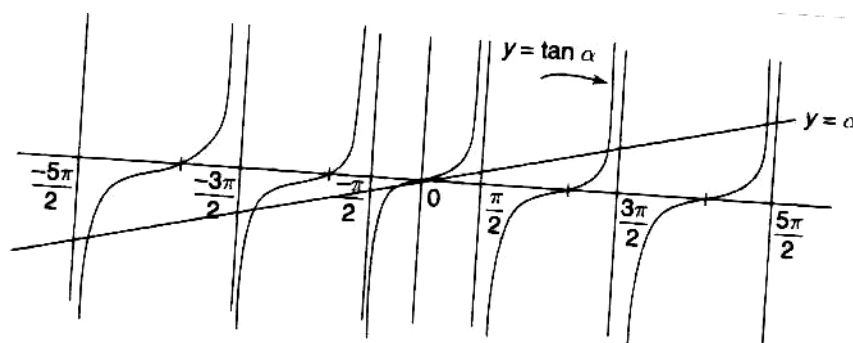

Fig Plots of y=α and y= tanα

The points of intersections are

$$\boldsymbol{\alpha} = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2} \dots$$

Substituting the above values in equations (1), the intensities in various maxima are obtained.

$$\boldsymbol{\alpha} = 0, \ I_0 = A^2 \qquad\qquad\qquad \text{(principal maximum)}$$

$$\boldsymbol{\alpha} = \frac{3\pi}{2}, \ I_1 = A^2 \left[\frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} \right]^2 \approx \frac{A^2}{22} \qquad \text{(1st secondary maximum)}$$

$$\boldsymbol{\alpha} = \frac{5\pi}{2}, \ I_2 = A^2 \left[\frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}} \right]^2 \approx \frac{A^2}{62} \qquad \text{(2nd secondary maximum)}$$

and so on.

From the above expressions, it is evident that most of the incident light is concentrated in the principal maximum.
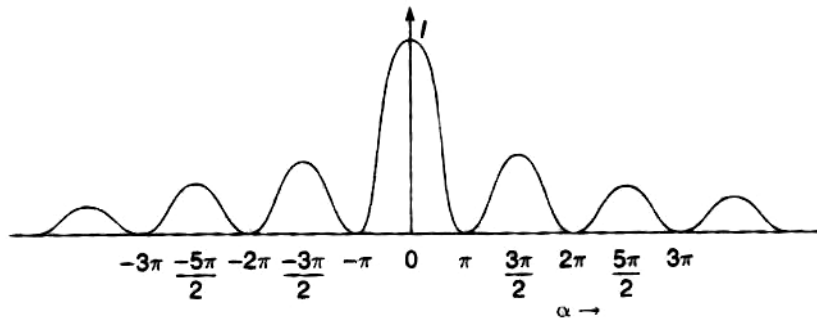
Intensity Distribution

Fig Intensity distribution

The variation of I with respect to $\propto$ is shown in fig 7.10. The diffraction pattern consists of a central principal maximum for $\boldsymbol{\alpha}$=0. There are secondary maxima of decreasing intensity on either sides of it at positions $\boldsymbol{\alpha} = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$. Between secondary maxima, there are minima at positions $\boldsymbol{\alpha} = \pm\pi, \pm 2\pi, \pm 3\pi, \dots$.

Fraunhofer Diffraction at Double Slit

Let S_1 and S_2 be double slits of equal widths e and separated by a distance d. The distance between the middle points of the two slits is (e +d). A monochromatic light of wave length λ is incident normally on the two slits. The light diffracted from these slits is focused by a lens on the screen placed in the focal plane of the lens. The diffraction at two slits is the combination of diffraction as well as interference, i.e., the pattern on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superimposed. When a plane wave front is incident normally on two slits, the secondary wavelets from the slits travel uniformly in all directions. The wavelets travelling in the direction of incident light come to a focus at P_0 while the wavelets travelling in a direction making an angle θ, come to focus at P_1.
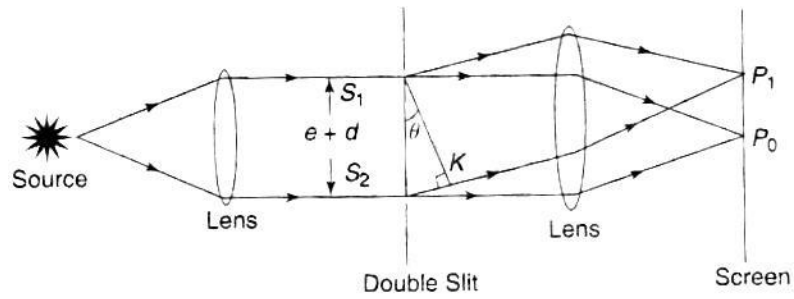
Fraunhofer diffraction-double slit

From the study of diffraction due to single slit, the resultant amplitude $= \dfrac{A \sin \alpha}{\alpha}$

Where,

$$\alpha = \dfrac{\pi e \, \sin\theta}{\lambda}.$$

Since a double slit is used, from each slit a wavelet of amplitude $\dfrac{A \sin \alpha}{\alpha}$ in a direction θ is obtained. These two wavelets interfere and meet at a point P_1 on the screen. To calculate the path difference between the wavelets, draw a normal S_1K to the wavelet through S_2.

$$\text{Path difference} = S_2K$$
$$= (e + d) \sin \theta$$
$$\text{Phase difference } \delta = \dfrac{2\pi}{\lambda} (e + d) \sin \theta \qquad \rightarrow(6)$$
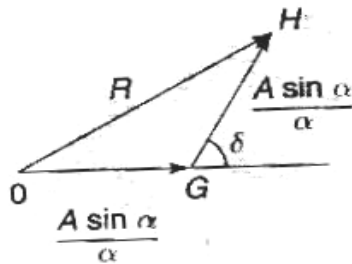
Vector addition method

To find the resultant amplitude at P_1 vector addition method is used. (Fig. 7.12) in which the two sides of a triangle are represented by the amplitudes through S_1 and S_2. The third side gives the resultant amplitude.

From the figure,

$$(OH)^2 = (OG)^2 + (GH)^2 + 2 \, (OG) \, (GH) \cos\delta$$
$$R^2 = \left[\dfrac{A \sin \alpha}{\alpha}\right]^2 + \left[\dfrac{A \sin \alpha}{\alpha}\right]^2 + 2 \left[\dfrac{A\sin \alpha}{\alpha}\right] \left[\dfrac{A\sin \alpha}{\alpha}\right] \cos\delta$$
$$= A^2 \dfrac{\sin^2\alpha}{\alpha} [2 + 2\cos\delta]$$
$$= 2\left[\dfrac{A\sin\alpha}{\alpha}\right]^2 (1 + \cos\delta)$$
$$= 2\left[\dfrac{A\sin\alpha}{\alpha}\right]^2 (1 + 2\cos^2 \ \delta/2 - 1)$$

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$$= 4\left[\frac{A\sin\alpha}{\alpha}\right]^2 \cos^2 \delta/2$$

$$R^2 = 4A^2\left[\frac{\sin\alpha}{\alpha}\right]^2 \cos^2\left[\frac{\pi(e+d)\sin\theta}{\lambda}\right]$$

$$\text{Let } \beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$

$$R^2 = 4A^2\left[\frac{\sin\alpha}{\alpha}\right]^2 \cos^2\beta$$

The resultant intensity $\qquad I = R^2 = 4A^2\left[\frac{\sin\alpha}{\alpha}\right]^2 \cos^2\beta \qquad\qquad \rightarrow (7)$

From the above expression, it is clear that the resultant intensity is the product of two factors, i.e,

1. $A^2\left[\frac{\sin\alpha}{\alpha}\right]^2$ which represents the diffraction pattern due to a single slit
2. $\cos^2\beta$, which gives the interference pattern due to wavelets from double slits. The resultant intensity is due to both diffraction and interference effects.

Diffraction Effect

The diffraction term $A^2\left[\frac{\sin\alpha}{\alpha}\right]^2$ gives the principal maximum as the centre of the screen with alternate minima and secondary maxima of decreasing intensity. We get principal maximum for $\theta = 0$. We get minima for $\sin\alpha = 0$

$$\alpha = \pm m\pi, \text{ where m = 1, 2, 3…}$$

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$e\sin\theta = \pm m\lambda$$

The positions of secondary maxima occurs for

$$\alpha = \frac{\pm 3\pi}{2}, \frac{\pm 5\pi}{2}, \frac{\pm 7\pi}{2} \ldots..$$

Interference Effect

The interference term $\cos^2\beta$ gives the equidistant bright and dark fringes.
The maxima will occur for $\cos^2\beta = 1$

$$\beta = \pm n\pi, \quad \text{where n = 0, 1, 2, 3…}$$
$$\beta = 0, \pm\pi, \pm 2\pi, \pm 3\pi \ldots\ldots\ldots$$

$$\frac{\pi(e+d)\sin\theta}{\lambda} = \pm n\pi$$
$$(e+d)\sin\theta = \pm n\lambda$$

The minima will occur for $\cos^2\beta = 0$

$$\beta = \pm(2n+1)\frac{\pi}{2} \qquad where\ n = 0, 1, 2, 3…$$

$$(e+d) \sin\theta = \pm(2n + 1)\frac{\lambda}{2}$$

Intensity Distribution

Figure 7.13 (a) and (b) represents the intensity variations due to diffractions as well as interference effect individually. When both effects are combined then the resultant intensity variation is obtained. From the figure it is clear that the resultant minima are not equal to zero, still they have some minimum intensity due to interference effect.
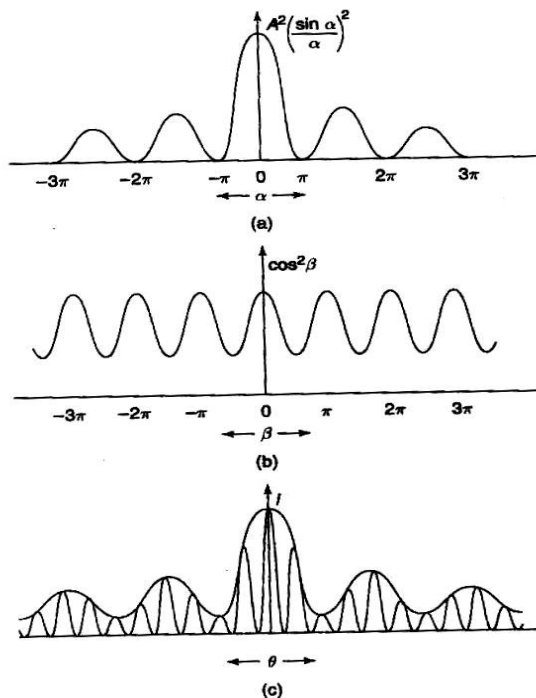
(a) Diffraction effect (b) Interference effect (c) Resultant intensity

Diffraction Grating-Normal incidence–(Diffraction at N parallel slits)
Construction

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed very closely side by side at regular intervals. The diameter of the wires was of the order of 0.05mm and their spacing varied from0.0533 mm to 0.687 mm. Now gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit. This is known as *Plane transmission grating*. On the other hand, if the lines are drawn on a silvered surface (plane or concave) then the light is reflected from the positions of mirrors in between any two lines and it forms a *plane or concave reflection grating*. When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of light is produced.

Theory

Fig.7.14 represents the section of a plane transmission grating placed perpendicular to the plane of the paper. Let 'e' be the width of each slit and'd' be the width of each opaque part. Then (e+d) is known as grating element. XY is the screen placed perpendicular to the plane of a

paper. Suppose a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. By Huygen's principle, each of the slit sends secondary wavelets in all directions. The secondary wavelets travelling in the same direction of incident light will come to a focus at a point P_0 of the screen as the screen is placed at the focal plane of the convex lens. The point P_0 will be the central maximum. Now, consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light. These waves reach the point P_1 on passing through the convex lens in different phases. As a result, dark and bright bands on both sides of the central maximum are obtained.
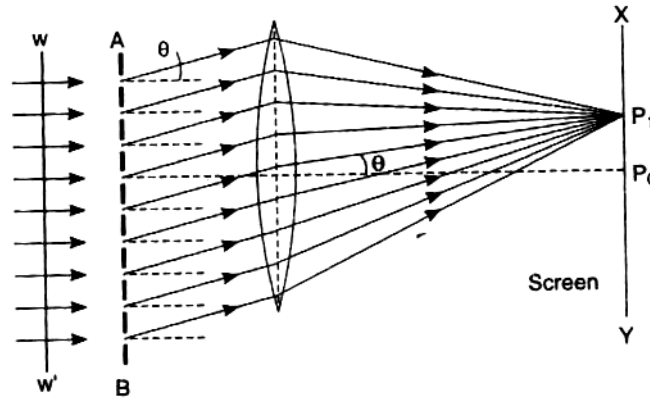

Fig7.14 Section of a Plane transmission grating

The intensity at point P_1 may be considered by applying the theory of Fraunhofer diffraction at a single slit. The wavelets proceeding from all points in a slit along the direction Ө are equivalent to a single wave of amplitude $(A \sin\alpha/\alpha)$ starting from the middle point of the slit, where

$$\alpha = (\pi \ e \ \sin\text{Ө}/\lambda).$$

If there are N slits, then there will be N diffracted waves, one each from the middle points of the slits. The path difference between two consecutive slits is $(e+d) \sin\text{Ө}$. Therefore, there is a corresponding phase difference of $(2\pi/\lambda). (e+d) \sin\text{Ө}$ between the two consecutive waves. The phase difference is constant and it is 2β.

Hence, the problem of determining the intensity in the direction Ө reduces to finding the resultant amplitude of N vibrations each of amplitude $(A \sin\alpha/\alpha)$ and having a common phase difference

$$\frac{2\pi}{\lambda} \ (e+d) \ \sin\text{Ө} = 2\beta \qquad\qquad \rightarrow (1)$$

Now, by the method of vector addition of amplitudes, the direction of Ө will be

$$R^* = \frac{A \sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin \beta}$$

$$\text{And } I = R^2 = (\frac{A\sin\alpha}{\alpha})^2 \ (\frac{\sin N\beta}{\sin\beta})^2 = I_o \ (\frac{\sin\alpha}{\alpha})^2 \ (\frac{\sin^2 N\beta}{\sin^2\beta}) \qquad\qquad \rightarrow (2)$$

The factor $(\frac{A\sin\alpha}{\alpha})^2$ gives the distribution of intensity due to single slit while the factor $(\sin^2 N\beta/\sin^2\beta)$ gives the distribution of intensity as a combined effect of all the slits.

Intensity distribution in N-Slits
Principle maxima

The intensity would be maximum when $\sin\beta = 0$.

$$\text{or } \beta = \pm n\pi \text{ where, } n=0, 1, 2, 3,\ldots$$

but at the same time $\sin N\beta = 0$, so that the factor $(\sin N\beta/\sin\beta)$ becomes indeterminate. It may be evaluated by applying the usual method of differentiating the numerator and the denominator, i.e., by applying the Hospital's rule. Thus,

$$\lim_{\beta\to\pm n\pi} \frac{\sin N\beta}{\sin\beta}$$

$$=\lim_{\beta\to\pm n\pi} \left(\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}\right)$$

$$\lim_{\beta\to\pm n\pi} \left(\frac{N\cos N\beta}{\cos\beta}\right) = \pm N$$

$$\text{Hence, } \lim_{\beta\to\pm n\pi} \left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$

The resultant intensity is $\left(\frac{A\sin\alpha}{\alpha}\right) N^2$. The maxima are most intense and are called as principal maxima.

The maxima are obtained for

$$\beta = \pm n\pi$$
$$\frac{\pi}{\lambda}(e+d)\sin\Theta = \pm n\pi$$
$$\text{or } (e+d)\sin\theta = \pm n\lambda \text{ Where, } n= 0, 1, 2, 3\ldots.$$

$n = 0$ corresponds to zero order maximum. For n=1, 2, 3...etc., the first, second, third, etc., principal maxima are obtained respectively. The $\pm$ sign shows that there are two principal maxima of the same order lying on the either side of zero order maximum.

Minima

A series of minima occur, when

$$\text{Sin } N\beta = 0 \qquad\qquad \text{but } \sin\beta \neq 0$$

For minima,

$$\text{Sin } N\beta = 0$$
$$N\beta = \pm m\pi$$
$$N\frac{\pi}{\lambda}(e+d)\sin\Theta = \pm m\pi$$
$$N(e+d)\sin\Theta = \pm m\lambda$$

Where m has all integral values except 0, N, 2N …nN, because for these values $\sin\beta$ becomes zero and the principal maxima is obtained. Thus, m = 1, 2, 3… (N-1). Hence, they are adjacent principal maxima.

Secondary maxima

As there are (N-1) minima between two adjacent principal maxima, there must be (N-2) other maxima between two principal maxima. To find out the position of these secondary maxima, equation (2) must be differentiated with respect to β and then equate it to zero. Thus,

$$\frac{dI}{d\beta} = \left(\frac{A\sin\alpha}{\alpha}\right)^2 . 2\left(\frac{\sin N\beta}{\sin\beta}\right)$$

$$\left[\frac{N\cos N\beta \sin\beta - \sin N\beta \cos\beta}{\sin^2\beta}\right] = 0 \text{ or}$$

$$N \cos N\beta \sin\beta - \sin N\beta \cos\beta = 0$$
$$N \tan\beta = \tan N\beta$$

The roots of this equation other than those for which $\beta = \pm n\pi$ (which correspond to principal maxima) give the positions of secondary maxima. To find out the value of $(\sin^2 N\beta/\sin^2\beta)$ from equation $N \tan\beta = \tan N\beta$, a triangle shown below, in the figure 7.15 is used.
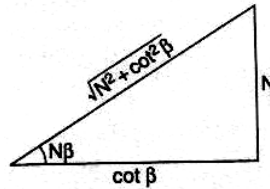

Fig 7.15

$$\text{Sin } N\beta = \frac{N}{\sqrt{(N^2 + \cot^2\beta)}}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2 + \cot^2\beta) \times \sin^2\beta}$$

$$= \frac{N^2}{N^2 \sin^2\beta + \cos^2\beta}$$

$$= \frac{N^2}{1 + (N^2 - 1)\sin^2\beta}$$

Therefore, $\dfrac{Intensity\ of\ secondary\ maxima}{Intensity\ of\ principa\ l\ maxima} = \dfrac{1}{1 + (N^2 - 1)\sin^2\beta}$

As N increases, the intensity of secondary maxima relative to principal maxima decreases and becomes negligible when N becomes large.

Figures 7.16 (a) and (b) show the graphs of variation of intensity due to the factors $sin^2 \propto/\propto^2$ and $sin^2 N\beta/sin^2 \beta$ respectively. The resultant is shown in figure 7.16 (c).
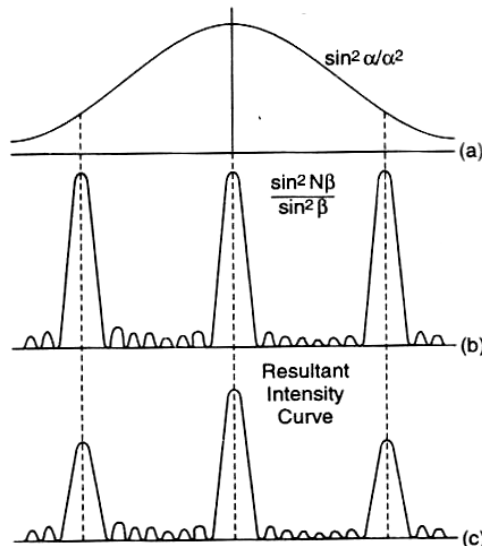
(a) & (b) Graphs showing the variation of intensity (c) The Resultant

Diffraction Grating

An arrangement which consists of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular intervals. Now gratings are constructed by ruling equidistant parallel lines on a transparent

material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit (Fig.7.17).

Commercial gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate. Solution of cellulose acetate is poured on the ruled surface and allowed to dry to form a thin film, detachable from the surface. These impressions of a grating are preserved by mounting the film between two glass sheets.
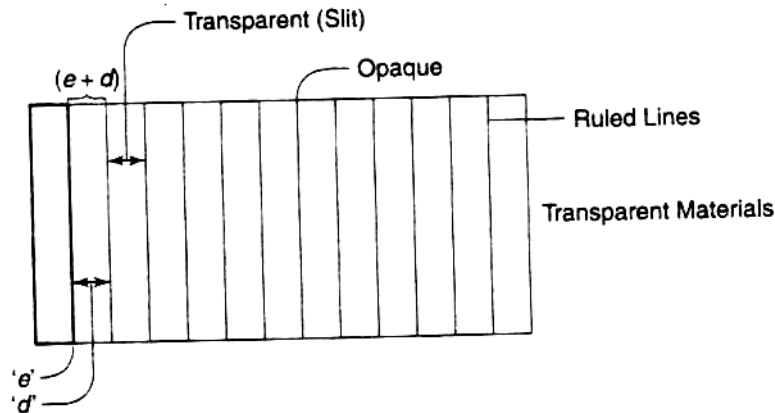

Fig7.17 Diffraction Grating

Let 'e' be the width of the line and 'd' be the width of the slit. Then (e+d) is known as *grating element.* If 'N' is the number of lines per inch on the grating, then

$$N (e+d) =1"=2.54 \text{ cm}$$
$$e+d= \frac{2.54}{N} \text{ cm}$$

There will be nearly 30,000 lines per inch of a grating. Due to the above fact, the width of the slit is very narrow and is comparable to the wavelength of light. When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as *diffraction spectrum.*

Grating Spectrum

The condition to form the principal maxima in a grating is given by

(e+d) sin θ = nλ

Where (e+d) is the grating element and the above equation is known as grating equation.

From the grating equation, the following is clear.

1. For a particular wavelength λ, the angle of diffraction θ is different for principal maxima of different orders.
2. As the number of lines in the grating is large, maxima appear as sharp, bright parallel lines and are termed as spectral lines.
3. For white light and for a particular order of n, the light of different wavelengths will be diffracted in different directions.
4. At the center, θ=0 gives the maxima of all wavelengths which coincides to form the central image of the same colour as that of the light source. This forms zero order.(Fig.2.9)

5. The principal maxima of all wavelengths forms the first, second,… order spectra for n=1,2,…
6. The longer the wavelength, greater is the angle of diffraction. Thus, the spectrum consists of violet being in the innermost position and red being in the outermost positions.
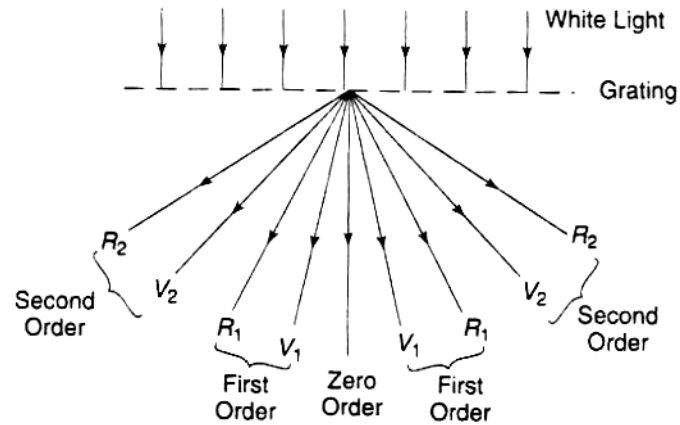

Fig7.18 Grating Spectrum

7. Most of the intensity goes to zero order and the rest is distributed among other orders.
8. Spectra of different orders are situated symmetrically on both sides of zero order.
9. The maximum number of orders available with the grating is $n_{max} = \frac{(e+d)}{\lambda}$