INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

TUTORIAL QUESTION BANK

Course Title	**PROBABILITY THEORY AND STOCHASTIC PROCESSES**				
Course Code	AECB08				
Programme	B.Tech				
Semester	III	ECE			
Course Type	Core				
Regulation	IARE - R18				
Course Structure	**Theory**			**Practical**	
	Lectures	**Tutorials**	**Credits**	**Laboratory**	**Credits**
	3	1	4	-	-
Chief Coordinator	Dr. M V Krishna Rao, Professor				
Course Faculty	Dr. M V Krishna Rao, Professor Mrs. G Ajitha, Assistant Professor Mr. N Nagaraju, Assistant Professor				

COURSE OBJECTIVES

The course should enable the students to:	
I	Understand the random experiments, sample space and event probabilities.
II	Study the random variables, density and distribution functions, moments and transformation of random variables.
III	Understand the concept of random process and sample functions (signals).
IV	Explore the temporal and spectral characteristics of random processes.

COURSE OUTCOMES (COs):

CO 1	Describe the concept of probability, conditional probability, and Baye's theorem and analyze the concepts of discrete, continuous random variables.
CO 2	Understand the Operations on single & multiple random variables– expectations.
CO 3	Explore the concepts of random processes – temporal characteristics.
CO 4	Explore the concepts of random processes – spectral characteristics

CO 5	Explore the spectral characteristics of random processes, and filtered random processes

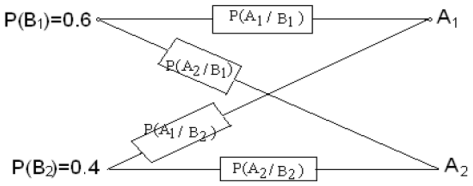
COURSE LEARNING OUTCOMES (CLOs):

AECB08.01	Describe the basic concepts of the random experiments, event probabilities, joint and conditional probabilities- bayes theorem
AECB08.02	Learn and understand the concept of random variables, continuous and discrete variables, the probability density functions (pdfs), probability distribution functions (pdfs), different random variables and their properties
AECB08.03	Learn and understand the functions of a random variable, standard and central moments, and their physical significance
AECB08.04	Monotonic and non-monotonic transformations of single random variables (continuous and discrete)
AECB08.05	Learn and understand of vector random variables, joint distribution function and its properties, marginal distribution functions, joint density function and its properties, marginal density functions,
AECB08.06	Learn and understand of conditional distribution and density – point conditioning, conditional distribution and density – interval conditioning, statistical independence,
AECB08.07	Sum of two and more random variables, central limit theorem: equal and unequal distribution
AECB08.08	Learn and understanding of functions of vector random variables, joint standard and central moments, joint characteristic functions
AECB08.09	Learn and understanding of jointly gaussian random variables; and transformations of multiple random variables
AECB08.10	Learn and understanding of random process, sample functions and time domain characteristics: stationary, independence and ergodicity
AECB08.11	Contrasting of correlation and covariance functions, gaussian and poisson random processes
AECB08.12	Distinguish between auto- and cross- power density spectra, properties, relationship between correlation functions and power density spectra
AECB08.13	Understanding of linear time invariant (lti) systems driven by random process, input-output spectral relations, white and colored noises

TUTORIAL QUESTION BANK

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
MODULE -I PROBABILITY AND RANDOM VARIABLES				
PART – A (SHORT ANSWER QUESTIONS)				
1	Define the probability of occurrence of an event.	Remember	CO 1	AECB08.01
2	Explain probability with axioms.	Understand	CO 1	AECB08.01
3	Define sample space and classify the types of sample.	Remember	CO 1	AECB08.01
4	Discuss how probability can be considered as relative frequency.	Understand	CO 1	AECB08.01
5	Define conditional, joint and total probability.	Remember	CO 1	AECB08.01
6	Define Mutually Exclusive Events.	Understand	CO 1	AECB08.01
7	Explain independent events.	Remember	CO 1	AECB08.01
8	Explain concept of random variable.	Understand	CO 1	AECB08.01
9	Describe the classifications of Random variable	Remember	CO 1	AECB08.01
10	Define Cumulative distribution function (CDF) and Probability density function (pdf).	Understand	CO 1	AECB08.01
11	List any four properties of density function.	Understand	CO 1	AECB08.02
12	List any four properties of distribution function.	Understand	CO 1	AECB08.02
13	Define Bernoulli distribution function.	Understand	CO 1	AECB08.02
14	Define Binomial Distribution function.	Understand	CO 1	AECB08.02
15	Define Poisson Random Variable.	Understand	CO 1	AECB08.02
16	Define Uniform Random Variable.	Understand	CO 1	AECB08.02
17	Define gaussian random variable	Understand	CO 1	AECB08.02
18	Define exponential random variable.	Understand	CO 1	AECB08.02
19	Define Rayleigh random variable.	Understand	CO 1	AECB08.02
20	Explain mean and mean square values.	Remember	CO 1	AECB08.03
PART – B (LONG ANSWER QUESTIONS)				
1	State and prove bayes theorem.	Understand	CO 1	AECB08.02
2	State and prove total probability theorem.	Remember	CO 1	AECB08.02
3	Derive expressions for mean and variance for uniform random variable.	Understand	CO 1	AECB08.02
4	Explain conditional probability theorem with properties.	Understand	CO 1	AECB08.01
5	Explain the properties the probability density function.	Understand	CO 1	AECB08.01
6	Derive the expression for mean and variance for Poisson random variable.	Remember	CO 1	AECB08.03
7	Derive expressions for mean and variance for binomial random variable.	Remember	CO 1	AECB08.03
8	Explain the method of defining a conditioning event.	Understand	CO 1	AECB08.01
9	Derive expressions for mean and variance for exponential random variable.	Remember	CO 1	AECB08.03
10	Derive the expressions for mean and variance for gaussian random variable.	Remember	CO 1	AECB08.03

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
11	What is the probability that exactly 5 of the items tested are defective if A batch of 50 items contains 10 defective items? Suppose 10 items are selected at random and tested.	Understand	CO 1	AECB08.01
12	Assume that the height of clouds σ, above the ground at some location is a gaussian random variable X with mean value 2km and σ =0.25km. Find the probability of clouds higher than 2.5km.	Understand	CO 1	AECB08.02
13	A production line manufactures 1KΩ resistors that must satisfy 10% tolerance. If a resistor is described by the gaussian random variable X, for which μ=1000Ω, σ =40Ω, what fraction of the resistors is expected to be rejected?	Understand	CO 1	AECB08.01
14	The power reflected from an aircraft received by radar is described by an exponential distribution. The pdf is given by $f_X(x) = 0.1e^{-x/10}$, x>0. The average power is 10W. What is the probability that the received power is greater than the average power?	Understand	CO 1	AECB08.02
15	Explain the Standard Moments and Central Moments.	Remember	CO 1	AECB08.02
16	Given that two events $A_1$and A_2 are statistically independent 1. show that $A_1$is independent of A_2^C 2. show that A_1^C is independent of A_2 3. show that A_1^C is independent of A_2^C	Understand	CO 1	AECB08.01
PART – C (Problem Solving and Critical Thinking Questions)				
1	Assume that the number of automobiles arriving at a gasoline station is passion distributed and occur at an average rate of 50/hour. The station has only one gasoline pump. If all cars are assumed to require one minute to obtain fuel, what is the probability that a waiting line will occur at the pump?	Understand	CO 1	AECB08.02
2	Find the value of C, for A random variable X has a probability density function $f_X(x)= C(1-x^4)$; -1<x<1 = 0 ; elsewhere.	Remember	CO 1	AECB08.01
3	Find the value of K and also find $P\{2 \le X \le 5\}$ Let X be a Continuous random variable with density function f(x)=x/9+k for 0<x<6; =0; otherwise	Understand	CO 1	AECB08.01
4	A shipment of components consists of 3 identical boxes. one box contains 2000 components of which 25%are defective , the second box has 5000 components of which 20%are defective, third box contains 2000 components of which 55% are defective. A box is selected at random and a component is removed at random from the box what is the probability that this component is defective? What is the probability by that it came from the second box?	Understand	CO 1	AECB08.01

<table>
<tr><th>S.No</th><th>QUESTION</th><th>Blooms taxonomy level</th><th>Course Outcomes</th><th>Course Learning Outcomes</th></tr>
<tr><td>5</td><td>The lifetime of IC chips manufactured by a semiconductor manufacturer is approximately normally distributed with a mean =5x10^6 hours and standard deviation of 5x10^6 hours. A mainframe manufacturer requires that at least 95% of a batch should have a lifetime greater than 4x10^6 hours. Will the deal be made?</td><td>Understand</td><td>CO 1</td><td>AECB08.01</td></tr>
<tr><td>6</td><td>Find the probability that at least one diode is defective. If a box contains 75 good diodes and 25 defective diodes and 12 diodes are selected at random?</td><td>Remember</td><td>CO 1</td><td>AECB08.02</td></tr>
<tr><td>7</td><td>A certain large city averages three murders per week and their occurrences follows a Poisson distribution
1. What is the probability that there will be five or more murders in a given week?
2. On the average, how many weeks a year can this city expect to have no murders?
3. How many weeks per year (average) can the city</td><td>Remember</td><td>CO 1</td><td>AECB08.02</td></tr>
<tr><td>8</td><td>Calculate the probabilities of system error and correct system transmission of symbols for an elementary binary communication system shown in below figure consisting of a transmitter that sends one of two possible symbols (a 1 or a 0) over a channel to a receiver. The channel occasionally causes errors to occur so that a ‟1‟ show up at the receiver as a ‟0? And vice versa. Assume the symbols „1‟ and „0‟ are selected for a transmission as 0.6 and 0.4 respectively.

</td><td>Understand</td><td>CO 1</td><td>AECB08.01</td></tr>
<tr><td>9</td><td>Find the individual, joint and conditional probabilities. For a given problem as shown below.
In a box there are 100 resistors having resistance and tolerance values given in table. Let a resistor be selected from the box and assume that each resistor has the same likelihood of being chosen. Event A: Draw a 47Ω resistor, Event B: Draw a resistor with 5% tolerance, Event C: Draw a 100Ω resistor.
<table>
<tr><td rowspan="2">Resistance (Ω)</td><td colspan="2">Tolerance</td><td rowspan="2">Total</td></tr>
<tr><td>5%</td><td>10%</td></tr>
<tr><td>22</td><td>10</td><td>14</td><td>24</td></tr>
<tr><td>47</td><td>28</td><td>16</td><td>44</td></tr>
<tr><td>100</td><td>24</td><td>8</td><td>32</td></tr>
<tr><td>Total</td><td>62</td><td>38</td><td>100</td></tr>
</table></td><td>Understand</td><td>CO 1</td><td>AECB08.01</td></tr>
</table>

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
10	A man wins in a gambling game if he gets two heads in five flips of a biased coin. The probability of getting a head with the coin is 0.7. i) Find the probability the man will win. Should he play this game? ii) What is the probability of winning if he wins by getting at least four heads in five flips? Should he play this new game?	Understand	CO 1	AECB08.02
MODULE -II **OPERATIONS ON SINGLE & MULTIPLE RANDOM VARIABLES – EXPECTATIONS**				
PART – A (SHORT ANSWER QUESTIONS)				
1	Give the expression for an arbitrary transformation of a single random variable, with a brief explanation.	Remember	CO 2	AECB08.04
2	Define N-dimensional random vector.	Remember	CO 2	AECB08.05
3	Define the joint distribution function of two continuous random variables X and Y.	Remember	CO 2	AECB08.05
4	Define the joint distribution function of two discrete random variables X and Y.	Remember	CO 2	AECB08.05
5	Define the joint distribution and density functions, in mathematical form, of N-dimensional continuous random vector X	Remember	CO 2	AECB08.05
6	Define the joint distribution and density functions, in mathematical form, of N-dimensional discrete random vector X	Remember	CO 2	AECB08.05
7	List any five properties of joint distribution function of two random variables.	Remember	CO 2	AECB08.05
8	List any five properties of joint density function of two random variables.	Remember	CO 2	AECB08.05
9	Define the relation between the marginal distribution functions and their joint density function of a 2-dimensional random vector	Remember	CO 2	AECB08.05
10	Comment on distribution and density function of a sum of two random variables.	Understand	CO 2	AECB08.07
11	Explain joint probability distribution function properties.	Remember	CO 2	AECB08.05
12	State the Central Limit Theorem.	Remember	CO 2	AECB08.07
13	Define Joint Moments about the origin.	Remember	CO 2	AECB08.05
14	Define the expected value of joint random variable.	Remember	CO 2	AECB08.05
PART – B (LONG ANSWER QUESTIONS)				
1	Obtain the expression for a monotonic transformation of a single random variable.	Understand	CO 2	AECB08.04
2	Obtain the expression for a nonmonotonic transformation of a single random variable.	Understand	CO 2	AECB08.04
3	Define and explain joint distribution function and joint density function of two random variables X and Y.	Understand	CO 2	AECB08.05

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
4	Distinguish between joint distribution and marginal distribution.	Understand	CO 2	AECB08.05
5	Define conditional distribution and density function of two random variables X and *Y*, and Explain.	Understand	CO 2	AECB08.06
6	Define and explain conditional probability mass function. Give its properties.	Understand	CO 2	AECB08.06
7	Define and explain the properties of conditional density functions.	Understand	CO 2	AECB08.06
8	What is the probability distribution function of the sum of two uniform random variables. Justify.	Understand	CO 2	AECB08.06
9	State and prove the central limit theorem.	Understand	CO 2	AECB08.07
10	State the properties of a joint distribution function.	Understand	CO 2	AECB08.05
11	Define two joint central moments for two-dimensional random variables X and *Y*.	Understand	CO 2	AECB08.07
12	Describe joint characteristic function.	Understand	CO 2	AECB08.05
13	How is the expected value of a conditional event defined? Explain.	Understand	CO 2	AECB08.06
14	The characteristic function for a Gaussian random variable X, having a mean value of 0, is $\Phi_X(\omega)$ =exp(-w ^{2}/ σ ^{2}) Find first three moments of X using $\Phi_X(w)$.	Understand	CO 2	AECB08.02
15	Calculate the moment generating function of Y. If X is a random variable with a Moment generating function of $M_X(v)$, Y=aX+b	Understand	CO 2	AECB08.02
16	A random variable has a probability density function $f_X(x)=(5/4)(1-x^4)$ for 0<x<1 = 0 other wise Find a)E[X]b)E[4X+2] and E[X^2]	Understand	CO 2	AECB08.03
17	The pdf of a random variable X is given by fx(x)= x/20 ; $2 \leq x \leq 5$, find the pdf of Y=3X-5.	Understand	CO 2	AECB08.03
18	The pdf of a random variable X is fx(x)=(1/2)cos (x) ; $-\pi/2<x<\pi/2$, find the mean of the function $g(X)=4X^2$.	Understand	CO 2	AECB08.03
19	Explain the significance of the characteristic function of a random variable.	Understand	CO 2	AECB08.02
20	Explain the significance of the moment generating function of a random variable.	Understand	CO 2	AECB08.02
PART-C (Problem Solving and Critical Thinking Questions)				
1	Calculate the following i) The variance of the sum of X and Y ii) The variance of the difference of X and Y for two random variables X and Y have zero mean and variance $\sigma_X^2 = 16$ and $\sigma_Y^2 = 36$ and correlation coefficient is 0.5.	Understand	CO 2	AECB08.07

<table>
<tr><th>S.No</th><th>QUESTION</th><th>Blooms taxonomy level</th><th>Course Outcomes</th><th>Course Learning Outcomes</th></tr>
<tr><td>2</td><td>Find $f_X(x)$ and $f_Y(y)$, the marginal PDFs of X and Y .The joint probability density function of random variables X and Y is
$f_{XY}(x, y) = 6(x + y^2)/5 \quad 0 \le x \le 1, 0 \le y \le 1,$
$= 0$, otherwise.</td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
<tr><td>3</td><td>Calculate the marginal PDFs $f_X(x)$ and $f_Y(y)$
The joint Pdf of X and Y is
$f_{X,Y}(x, y) = 5xy/4 \quad -1 \le x \le 1, -2 \le y \le 1,$
=0 otherwise.</td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
<tr><td>4</td><td>Find f(y/x) and f(x/y) for The joint density function of random variables X and Y is
Fxy(x,y)=8xy;0<x<1,0<y<1
=0,otherwise</td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
<tr><td>5</td><td>Show that the linear transformation of Gaussian random variable is another Gaussian random variable. What is the mean and variance of the resulting variable?</td><td>Understand</td><td>CO 2</td><td>AECB08.04</td></tr>
<tr><td>6</td><td>Find the probability density function of the random variable Y obtained by the transformation $Y = 3X^3 - 3X^2 + 2$ of the discrete random variable X whose density function is given below.
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(x,y)$</td><td>0.2</td><td>15</td><td>0.3</td><td>15</td><td>.2</td></tr></table></td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
<tr><td>7</td><td>The probabilities of the random variables X and Yare given in Table . Find (a) value of K and (b) the joint distribution function and marginal distribution functions.
Joint probabilities of X and Y
<table><tr><td>X/Y</td><td>-1</td><td>0</td><td>1</td></tr><tr><td>0</td><td>$3/18$</td><td>$2/18$</td><td>$3/18$</td></tr><tr><td>1</td><td>$1/18$</td><td>$K/18$</td><td>$1/18$</td></tr><tr><td>2</td><td>$1/18$</td><td>$1/18$</td><td>$2/18$</td></tr></table></td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
<tr><td>8</td><td>Find the probability density function of the random variable Y, obtained by a quadratic transformation a random variable X.</td><td>Understand</td><td>CO 2</td><td>AECB08.04</td></tr>
<tr><td>9</td><td>The joint pdf is given as
$f_{X,Y}(x,y) = Ae^{-(2x+y)}$ for $x \ge 0$ and $y \ge 0$.
Find (a) the value of A and (b) the marginal density functions.</td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
<tr><td>10</td><td>Find the conditional density functions for the joint density function $f_{X,Y}(x,y) = 4xye^{-(x^2+y^2)}u(x)u(y)$</td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
<tr><td>11</td><td>The joint pdf of two variables X and Y is (x) for
$f_{X,Y}(x,y) = \frac{1}{18}e^{-\left(\frac{x}{6}+\frac{y}{3}\right)}$ for $x \ge 0$ and $y \ge 0$.
Show that X and Y are independent random variables</td><td>Understand</td><td>CO 2</td><td>AECB08.05</td></tr>
</table>

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
12	Show that the variance of a weighted sum of uncorrelated random variables equals the weighted sum of the variances of the random variables.	Understand	CO 2	AECB08.07
MODULE –III **OPERATIONS ON MULTIPLE RANDOM VARIABLES – EXPECTATIONS**				
PART – A (SHORT ANSWER QUESTIONS				
1	Define the joint moments of order 3 about the origin of two random variables.	Understand	CO 3	AECB08.06
2	Define the correlation between two random variables X and Y for both continuous and discrete cases.	Understand	CO 3	AECB08.08
3	Define the joint central moments of order 3 of two continuous random variables X and Y.	Understand	CO 3	AECB08.08
4	Define the covariance between two random variables X and Y for both continuous and discrete cases.	Understand	CO 3	AECB08.08
5	Define the normalized second central moment of two continuous random variables X and Y.	Understand	CO 3	AECB08.08
6	What are the conditions for the 2-dimentional random variable to be uncorrelated and orthogonal?	Understand	CO 3	AECB08.08
7	State the properties of correlation coefficient.	Understand	CO 3	AECB08.08
8	Define the expected value of a function of two random variables.	Understand	CO 3	AECB08.08
CIE-II				
1	Define the expected value of a joint random variable of 2-dimensions.	Understand	CO 3	AECB08.08
2	Define the joint characteristic function of two random variables for both continuous and discrete cases.	Understand	CO 3	AECB08.08
3	Define the joint moment generating function of two random variables for both continuous and discrete cases.	Understand	CO 3	AECB08.08
4	List any four properties of joint Gaussian variables.	Understand	CO 3	AECB08.08
5	What are the elements of the covariance matrix of two random variables?	Understand	CO 3	AECB08.08
6	What is Jacobian? Where is it used in probability theory?	Understand	CO 3	AECB08.08
7	How is the joint characteristic function of a 2-dimensional random variable defined?	Understand	CO 3	AECB08.08
8	How is the joint moment generating function of a 2-dimensional random variable defined?	Understand	CO 3	AECB08.08
PART – B (LONG ANSWER QUESTIONS)				
1	State and prove the properties of correlation between two random variables.	Understand	CO 3	AECB08.06
2	For two random variables X and Y, prove that $cov(X+a, Y+b) = C_{XY}$ and $cov(aX, bY) = abC_{XY}$.	Understand	CO 3	AECB08.08

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
3	Prove that any two random variables X and *Y*, the inequality $\lvert\sigma_{XY}\rvert \leq \sigma_X\sigma_Y$ is true.	Understand	CO 3	AECB08.08
4	List any four properties of the covariance between two random variables.	Understand	CO 3	AECB08.08
5	Define the expected value of a function of two random variables.	Understand	CO 3	AECB08.08
6	Show that the variance of a weighted sum of uncorrelated random variables equals the weighted sum of the variances of the individual random variables.	Understand	CO 3	AECB08.08
7	Show that the joint characteristic function of two independent random variables is equal to the product of the individual characteristic functions.	Understand	CO 3	AECB08.08
8	Show that the characteristic function of sum of two independent random variables is equal to the product of the individual characteristic functions.	Understand	CO 3	AECB08.08
9	Find variance & covariance of X&Y. If E[X]=2, E[Y]=3, E[XY]=10, $E[X^2]$=9, and $E[Y^2]$=16.	Understand	CO 3	AECB08.08
CIE-II				
1	How is the expected value of a conditional event defined? Explain.	Understand	CO 3	AECB08.08
2	For random variables *X*, *Y* and *Z*, prove that $cov(X+Y,Z) = cov(X,Z) + cov(Y,Z)$.	Understand	CO 3	AECB08.08
3	Explain how a discrete random variable is transformed to a new discrete random variable.	Understand	CO 3	AECB08.08
4	Prove that the moment generating function of sum of two independent random variables is equal to the product of the individual moment generating functions.	Understand	CO 3	AECB08.08
5	Prove that the moment generating function of two independent random variables is equal to the product of the individual moment generating functions.	Understand	CO 3	AECB08.08
6	Explain the Gaussian density function for *N* random variables.	Understand	CO 3	AECB08.08
7	Explain the linear transformations of Gaussian random variables.	Understand	CO 3	AECB08.08
8	Find the PDF of W = X+ Y. Where X and Y have joint Pdf $f_{XY}(x, y) = 1/15 \quad 0 \leq x \leq 3, 0 \leq y \leq 5$, =0 otherwise.	Understand	CO 3	AECB08.08

<table>
<tr><th>S.No</th><th>QUESTION</th><th>Blooms taxonomy level</th><th>Course Outcomes</th><th>Course Learning Outcomes</th></tr>
<tr><td colspan="5">PART – C (Problem Solving and Critical Thinking Questions)</td></tr>
<tr><td>1</td><td>The joint density function of two random variables X and Y is
$f_{X,Y}(x,y) = \begin{cases} \frac{(x+y)^2}{40} & -1 \le x \le 1 ; -3 \le y \le 3 \\ 0 & elsewhere \end{cases}$
Find (a) the variances of X and Y, and (b) the correlation coefficient.</td><td>Understand</td><td>CO 3</td><td>AECB08.07</td></tr>
<tr><td>2</td><td>If the joint density function is
$f_{X,Y}(x,y) = \begin{cases} x+y & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & otherwise \end{cases}$
Find the correlation coefficient.</td><td>Understand</td><td>CO 3</td><td>AECB08.08</td></tr>
<tr><td>3</td><td>Find the coefficient of correlation between X and Y from the data given. Assume that X and Y are uniform random variables.
<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Y</td><td>2</td><td>4</td><td>8</td><td>10</td></tr></table></td><td>Understand</td><td>CO 3</td><td>AECB08.08</td></tr>
<tr><td>4</td><td>For two random variables X and Y, the joint density function is
$f_{X,Y}(x,y) = 0.15\delta(x+1)\delta(y) + 0.1\delta(x)\delta(y) + 0.1\delta(x)\delta(y-2) + 0.4\delta(x-1)\delta(y+2) + 0.2\delta(x-1)\delta(y-1) + 0.05(x-1)\delta(y-3)$
Find (a) the correlation and (b) the correlation coefficient of X and Y. (c) Are X and Y either uncorrelated or orthogonal?</td><td>Understand</td><td>CO 3</td><td>AECB08.08</td></tr>
<tr><td>5</td><td>Two random variables X and Y have the joint characteristic function
$Ø_{X,Y}(w_1,w_2)=\exp(-2w_1^2-8w_2^2)$.
Show that X and Y are both zero mean and that they are uncorrelated.</td><td>Understand</td><td>CO 3</td><td>AECB08.08</td></tr>
<tr><td>6</td><td>Show that the variance of a weighted sum of uncorrelated random variables equals the weighted sum of the variances of the random variables.</td><td>Understand</td><td>CO 3</td><td>AECB08.08</td></tr>
<tr><td colspan="5">CIE-II</td></tr>
<tr><td>1</td><td>The joint density function of X and Y is
$f(x) = \begin{cases} \frac{1}{100}, & 0 < x < 5, 0 < y < 20 \\ 0, & elsewhere \end{cases}$
Find the expected value of the functions (a). XY (b). X^2Y and (c) $(XY)^2$</td><td>Understand</td><td>CO 3</td><td>AECB08.08</td></tr>
<tr><td>2</td><td>Let two random variables Y_1 and Y_2 be linear transformations of X_1 and X_2 given by $Y_1 = X_1 + X_2$,$Y_1 = 2X_1 + 3X_2$. If $f_{x_1\,x_1}(x_1,x_2)$ is a joint density function, then find the joint density function of Y_1 and Y_2.</td><td>Understand</td><td>CO 3</td><td>AECB08.09</td></tr>
</table>

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
3	Consider two random variables X and Y such that $Y = -4X + 20$. The mean value and the variance of X are 4 and 2 respectively. Find the correlation and comment on the result.	Understand	CO 3	AECB08.08
4	Two random variables X and Y have mean values $\overline{X} = 1$ and $\overline{Y} = 1$, variances $\sigma_x^2 = 4$ and $\sigma_y^2 = 2$ and a correlation coefficient $\rho_{XY} = 0.2$. Define two new random variables $V = -X - Y$ and $W = 2X + Y$. Find (a) correlations of V and W and (b) correlation coefficient ρ_{VW}.	Understand	CO 3	AECB08.09
5	Consider two correlated random variables X and Y with variances σ_x and σ_y respectively. These variables are to be transformed to uncorrelated random variables X_1 and Y_1 by coordinate rotation as $X_1 = X\cos\theta + Y\sin\theta$ and $Y_1 = Y\cos\theta - X\sin\theta$, where θ is the rotation angle. Show that the angle of rotation is given by $$\theta = \frac{1}{2}\tan^{-1}\left[\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}\right]$$	Understand	CO 3	AECB08.09
6	Two Gaussian random variable X_1 and X_2 have zero mean and variances ${\sigma^2}_{X_1} = 4$ and ${\sigma^2}_{X_2} = 9$. Their covariance is $C_{X_1X_2} = 3$. Find the covariance $C_{Y_1Y_2}$ of new random variables Y_1 and Y_2, if the transformation is given as $Y_1 = X_1 - 2X_2$; $Y_2 = 3X_1 + 4X_2$	Understand	CO 3	AECB08.09
MODULE -IV RANDOM PROCESSES – TEMPORAL CHARACTERISTICS				
PART – A (SHORT ANSWER QUESTIONS)				
1	Define random process.	Remember	CO 4	AECB08.10
2	Define ergodicity.	Remember	CO 4	AECB08.10
3	Explain the first order stationary process.	Remember	CO 4	AECB08.10
4	Explain second order stationary process.	Understand	CO 4	AECB08.10
5	State wide sense stationary random process.	Understand	CO 4	AECB08.10
6	Define strict sense stationary random process.	Remember	CO 4	AECB08.10
7	Explain briefly about time average and Ergodicity.	Understand	CO 4	AECB08.10
8	Define deterministic random process.	Understand	CO 4	AECB08.10
9	Define distribution Function of a random process.	Understand	CO 4	AECB08.10
11.	Define mean ergodic process.	Remember	CO 4	AECB08.10
12.	State correlation ergodic process.	Remember	CO 4	AECB08.10
13.	Define auto correlation function of a random process.	Remember	CO 4	AECB08.11
14.	Explain cross correlation function of a random.	Understand	CO 4	AECB08.11
15.	Define time mean square function.	Understand	CO 4	AECB08.11
16.	Define Autocorrelation Ergodic Process.	Remember	CO 4	AECB08.11
17.	State Cross Correlation Ergodic Process.	Remember	CO 4	AECB08.11
18.	Define Auto Covariance function.	Remember	CO 4	AECB08.11
19.	State Cross Covariance Function.	Remember	CO 4	AECB08.11
20.	Define Gaussian Random Process.	Understand	CO 4	AECB08.11

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
PART – B (LONG ANSWER QUESTIONS)				
1.	Explain classification of random processes with neat sketches.	Remember	CO 4	AECB08.10
2.	Explain and write conditions for a wide sense stationary random process.	Remember	CO 4	AECB08.10
3.	Briefly explain the distribution and density function in the context of stationary and independent random process.	Understand	CO 4	AECB08.10
4.	Show that the process X(t)= A Cos (w_0t+θ) is wide sense stationery if it is assumed that A and w_0 are constants and θ is uniformly distributed random variable over the interval (0,2π).	Understand	CO 4	AECB08.10
5.	A random process Y(t)= X(t)Cos (wt+θ), where X(t) is wide sense stationery, w is constant and θ is uniformly distributed random variable over the interval (π, -π). Find E[Y(t)].	Remember	CO 4	AECB08.11
6.	The two level semi random binary process is defined by X(t)=A or –A (n-1)T<t<nT Where the levels A and –A occur with equal probability, T is a positive constant, and n=0, ±1, ±2, …. a) sketch a typical sample function b) classify the process c) is the process deterministic?	Understand	CO 4	AECB08.10
7.	A random process consists of three sample functions X(t,s_1)=2, X(t,s_2)=2 cos(t), and X(t,s_3)=3 sin(t), each occurring with equal probability. Is the process stationary in any sense?	Understand	CO 4	AECB08.10
8.	Distinguish between stationary and non stationary random processes.	Understand	CO 4	AECB08.10
9.	A random process X(t)= a.cos (wt+θ), where w and θ are constants and a is uniformly distributed random variable over the interval (-A, A). Check X(t) for stationary.	Understand	CO 4	AECB08.10
10.	A random process X(t)= at+b, where b is constant and a is uniformly distributed random variable over the interval (-2, 2). Check X(t) for stationary.	Remember	CO 4	AECB08.10
11.	State and prove any four properties of cross correlation function.	Understand	CO 4	AECB08.11
12.	Explain and prove any four properties of auto correlation function.	Understand	CO 4	AECB08.11
13.	State and prove any four properties of cross covariance function.	Understand	CO 4	AECB08.11
14.	State and prove any four properties of auto covariance function.	Remember	CO 4	AECB08.11
15.	Explain about Poisson Random process and also find its mean and variance.	Understand	CO 4	AECB08.12
16.	Explain the concept of Gaussian random process.	Understand	CO 4	AECB08.12
17.	Derive the expression for Mean value of Output Response	Understand	CO 4	AECB08.12

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
18.	Derive the expression for Autocorrelation Function of Output Response	Remember	CO 4	AECB08.11
19.	Derive the expression for Mean square value of Output Response.	Understand	CO 4	AECB08.11
20.	Derive the expression for cross correlation Function of Output Response.	Understand	CO 4	AECB08.11
PART – C (Problem Solving and Critical Thinking Questions)				
1	Find whether X(t) is wide sense stationary or not .If A random process is given as X(t) = At, where A is a uniformly distributed random variable on (0,2).	Understand	CO 4	AECB08.10
2	Let two random processes X(t) and Y(t) be defined by X(t) = A Cos(w_0t) + B Sin(w_0t) and Y(t) = B Cos(w_0t) - A Sin(w_0t). Where A and B are random variables and w_0 is constant. Show that X(t) and Y(t) are jointly wide sense stationery, assume A and B are uncorrelated zero- mean random variables with same variance.	Understand	CO 4	AECB08.10
3	Determine whether X_1(t) and X_2(t) are jointly wide sense stationary. X(t) is a wide sense stationary random process. For each process X_i(t) defined below (a) $X_1(t) = X(t + a)$ (b) $X_2(t) = X(at)$	Understand	CO 4	AECB08.10
4	Define a random process by X(t)=A cos(πt), where A is a gaussian random variable with zero mean and variance σ^2_A a) find the density functions of X(0) and X(1) b) is X(t) is stationary in any sense	Understand	CO 4	AECB08.10
5	Show that Z(t) is WSS but not strictly stationary, where Z(t) = Acos(ω_0t) + Bsin(ω_0t) is a random process. If A and B are uncorrelated zero mean rv's having different density functions but the same variance σ^2.	Understand	CO 4	AECB08.10
6	Given the random process X(t)=A sin(wt+ θ) where A and w are constants and θ is a random variable uniformly distributed on the interval (-π ,π) define a new random process Y(t)= X^2(t).Find the auto correlation function of Y(t).	Understand	CO 4	AECB08.11
7	X(t) is a stationary random process with a mean of 3 and an auto correlation function of $9+2e^{-\|\tau\|}$. Find the variance of the random variable.	Understand	CO 4	AECB08.11
8	Find E[Z]. E[Z^2] and var(z) if the function of time Z(t) = $X_1\cos\omega_0 t$- $X_2\sin\omega_0 t$ is a random process. If X_1 and X_2 are independent Gaussian random variables, each with zero mean and variance σ^2.	Understand	CO 4	AECB08.11
9	Find mean, variance and average power for a stationary ergodic random processes has the Auto correlation function with the periodic components as $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$	Understand	CO 4	AECB08.11

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
10	A random process is defined as $X(t)=A\cos(w_c t+\theta)$ where θ is a uniform random variable over$(0,2\pi)$ .Verify the process is ergodic in the mean sense and auto correlation sense.	Understand	CO 4	AECB08.10
11	Statistically independent zero mean random process X(t) and Y(t) have auto-correlation functions $R_{XX}(\tau)=e^{-\|\tau\|}$ and $R_{YY}(\tau)=\cos(2\pi\tau)$ Find the auto-correlation function of the sum w(t)=x(t)+y(t).	Understand	CO 4	AECB08.11
12	Statistically independent zero mean random process X(t) and Y(t) have auto-correlation functions $R_{XX}(\tau)=e^{-\|\tau\|}$ and $R_{YY}(\tau)=\cos(2\pi\tau)$ Find the auto-correlation function of the difference w(t)=x(t)-y(t).	Understand	CO 4	AECB08.11
13	Given E[X(t)]=6 and Rxx(t,t+ τ)= 36+25 exp(-τ) for a random process X(t). Indicate which of the following statements are true based on what is known with certainty: X(t) i. is first order stationary. ii. has total average power of 61W. iii. is ergodic. iv. is wide sense stationary.	Understand	CO 4	AECB08.11
MODULE -V RANDOM PROCESSES – SPECTRAL CHARACTERISTICS				
PART – A (SHORT ANSWER QUESTIONS)				
1	State wiener Khinchin theorem.	Remember	CO 5	AECB08.12
2	Explain any two properties of cross-power density.	Remember	CO 5	AECB08.12
3	Define cross –spectral density and its examples.	Remember	CO 5	AECB08.12
4	Explain any two uses of spectral density.	Remember	CO 5	AECB08.12
5	Define power density spectrum.	Understand	CO 5	AECB08.12
6	State any two properties of power density spectrum.	Understand	CO 5	AECB08.12
7	Define Narrow band random processes function.	Remember	CO 5	AECB08.12
8	Prove that $S_{XX}(\omega) = S_{XX}(-\omega)$	Remember	CO 5	AECB08.12
9	Define Average cross power.	Understand	CO 5	AECB08.12
10	The random process $X(t)= A_0\cos(w_0t+\theta)$, where A_0 and w_0 are constants and θ is uniformly distributed random variable over the interval $(0,\pi)$. Find the power in X(t).	Understand	CO 5	AECB08.12
11	The random process $X(t)= u(t)\cos(w_0t+\theta)$, where w_0 is constant and θ is uniformly distributed random variable over the interval $(0,\pi)$. Find the power in X(t). If u(t) is a step function.	Remember	CO 5	AECB08.12
12	Calculate the power density spectrum for the function w^2/w^6+3w^2+3	Remember	CO 5	AECB08.12
13	The random process $X(t) = A_0\cos(\omega_0t) + B_0\sin(\omega_0t)$ where A_0 and B_0 are random variables. Find the power density spectrum.	Remember	CO 5	AECB08.12
14	Calculate the power density spectrum for the function $(w^2/w^4+1)-\delta(w)$	Understand	CO 5	AECB08.12

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
15	Calculate the power density spectrum for the function $\exp[-(w-1)^2]$	Understand	CO 5	AECB08.12
16	Calculate the power density spectrum for the function $w^4/1+ w^2+jw^6$	Remember	CO 5	AECB08.12
17	Calculate the power density spectrum for the function $\cos(3w)/1+ w^2$	Remember	CO 5	AECB08.12
18	Calculate the power density spectrum for the function $1/(1+ w^2)^2$	Remember	CO 5	AECB08.12
19	Calculate the power density spectrum for the function $\|w\|/1+ 2w+w^2$	Understand	CO 5	AECB08.12
20	Explain the power density spectrum of white noise.	Understand	CO 5	AECB08.13
PART – B (LONG ANSWER QUESTIONS)				
1	Explain the concept of power density spectrum in detail and derive the expression for it.	Understand	CO 5	AECB08.12
2	Explain the concept of cross power density spectrum in detail and derive the expression for it.	Understand	CO 5	AECB08.12
3	Explain the concept of cross power spectral density of input and output of a linear system.	Understand	CO 5	AECB08.13
4	Prove Wiener Khinchin relation.	Remember	CO 5	AECB08.12
5	State and derive the properties of power density spectrum.	Understand	CO 5	AECB08.12
6	State and derive the properties of cross power density spectrum.	Remember	CO 5	AECB08.12
7	Derive the relation between power spectrum and auto correlation function.	Remember	CO 5	AECB08.12
8	Derive the relation between cross power spectrum and cross correlation function.	Remember	CO 5	AECB08.12
9	Explain power spectrums for discrete-time random processes and sequences.	Remember	CO 5	AECB08.12
10	Explain the power density spectrum of a system response.	Understand	CO 5	AECB08.12
11	Given that $X(t)=\sum_{i=1}^{N} \alpha_i X_i(t)$, where $\{\alpha_i\}$ is a set of real constants and the process $X_i(t)$ are stationary and orthogonal, show that $S_{XX}(w) = \sum_{i=1}^{N} (\alpha_i)^2 S_{XiXi}(w)$.	Understand	CO 5	AECB08.12
12	The random process $X(t) = A_0\cos(\Omega t+\theta)$ where A_0 is real constant and Ω is a random variable with density function $f_\Omega(.)$, θ is uniformly distributed random variable over the interval $(0,2\pi)$. Find the power density spectrum of X(t).	Understand	CO 5	AECB08.12
13	An LTI system with an impulse response $h(t)$ was driven by a WSS process $X(t)$. Find the mean and MS values of the system output $Y(t)$.	Understand	CO 5	AECB08.13
14	An LTI system with an impulse response $h(t)$ was driven by a WSS process $X(t)$ having an ACF $R_{xx}(\tau)$. Show that the ACF of the system output $Y(t)$ is given by the convolution of $R_{yy}(\tau)$ with $h(\tau)$ and $h(\tau)$	Understand	CO 5	AECB08.13

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
15	Find the rms bandwidth of the power spectrum $S_{XX}(w)= \omega^2 /[1+(\omega/W)^2]^3$, where W>0 is a constant.	Remember	CO 5	AECB08.13
16	Find the power density spectrum of the random process for which $R_{XX}(\tau)=P \cos^4(\omega_0\tau)$, if P and ω_0 are constants. Determine the power in the process.	Remember	CO 5	AECB08.12
17	Find the rms bandwidth of the power spectrum $S_{XX}(w)=1/[1+(\omega/W)^2]^N$, where W>0 is a constant and N>=2 is an integer.	Remember	CO 5	AECB08.12
18	A random process has the power density spectrum $S_{XX}(w)= 6\omega^2 /(1+\omega^2)^4$, find the average power.	Understand	CO 5	AECB08.12
19	Distinguish between white and colored noises. Where these noises are observed? Explain.	Understand	CO 5	AECB08.13
20	Define noise equivalent bandwidth. Derive the expression for noise bandwidth.	Understand	CO 5	AECB08.13
21	An LTI system with an impulse response $h(t)$ was driven by white noise having a power density spectrum of $N_o/2$. Find the mean square value of the system output $Y(t)$.			
PART – C (Problem Solving and Critical Thinking Questions)				
1	Find an expression for its power spectral density S_{XX} (ω).Let the auto correlation function of a certain random process X (t) be given by $R_{XX}(\tau) = A^2/2 \cos(\omega\tau)$.	Understand	CO 5	AECB08.12
2	Describe the power spectral density function For a wide sense stationary process X(t) has autocorrelation function $R_{XX}(\tau) =A e^{-b\|\tau}$ | where b > 0. $S_{XX}(f)$ and calculate the average power $E[X^2(t)]$.	Understand	CO 5	AECB08.12
3	Find the average power in a random process defined by $X(t)=A \cos(w_ot+\Theta)$ where A and w_o are constants and Θ is a random variable uniformly distributed on the interval $(0,\pi/2)$.	Understand	CO 5	AECB08.12
4	Find the autocorrelation function The power Spectral density of X(t) is given by $S_{XX}(w)=1/(1+w^2)$ for w>0.	Understand	CO 5	AECB08.12
5	The auto correlation function of an a periodic random process is $R_{XX}(T)= e^{-\|\tau\|}$. Find the PSD and average power of the signal.	Understand	CO 5	AECB08.12
6	The cross spectral density of two random process X (t) and Y (t) is $S_{XY}(w)=1+(jw/k)$ for $-k<w<k$ and 0 elsewhere Where k > 0. Find the cross correlation function between the processes?	Understand	CO 5	AECB08.12
7	A random process has the power density spectrum $S_{XX}(w)=w^2/(w^2+1)$.Find the average power in the random process.	Understand	CO 5	AECB08.12
8	Estimate the power spectral density of a stationary random process for which auto correlation function is $R_{XX}(\tau)=6.e^{-\alpha\|\tau\|}$.	Understand	CO 5	AECB08.12

S.No	QUESTION	Blooms taxonomy level	Course Outcomes	Course Learning Outcomes
9	Find i) The Auto correlation function of a random process Y(t), if it has the power spectral density $S_{YY}(\omega)=9/(\omega^2+64)$. ii) The average power of the process.	Understand	CO 5	AECB08.12
10	Find the rms bandwidth of the power spectrum $S_{XX}(w)= \omega^2 /[1+(\omega/W)^2]^4$, where W>0 is a constant.	Understand	CO 5	AECB08.13
11	A random process has the power density spectrum $S_{XX}(w)= 6\omega^2 /1+\omega^4$, find the average power.	Understand	CO 5	AECB08.13
12	Find the cross correlation function of sin(wt) and cos(wt) and hence find its cross power spectral density.	Understand	CO 5	AECB08.12
15	An LTI system with an impulse response $h(t)$ was driven by a WSS process $X(t)$ having an ACF $R_{xx}(\tau)$. Show that the CCF of the system output $Y(t)$ with system input is given by the convolution of $R_{xx}(\tau)$ with $h(\tau)$.	Understand	CO 5	AECB08.13
16	Find the rms bandwidth of the power spectrum $S_{XX}(w)=1/[1+(\omega/W)^2]^3$, where W>0 is a constant.	Remember	CO 5	AECB08.13
17	A random process has a power spectrum $S_{XX}(w)= 4-(\omega^2/9)$; $\|\omega\|<=6$ = 0 ; elsewhere Find (a) the average power. (b) autocorrelation function of the process.	Understand	CO 5	AECB08.12

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