LECTURE NOTES

ON

THERMODYNAMICS

B. Tech IV semester (IARE-R16)

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UNIT -I

-: BASIC CONCEPTS :-

Thermodynamics :- It is the science of energy transfer and its effects on properties of system.

The main aim of thermodynamics study is to convert disorganised form of energy (heat) into organised form of energy (work) in an efficient manner.

System :- It is a region in space upon which the study is focussed or concentrated.

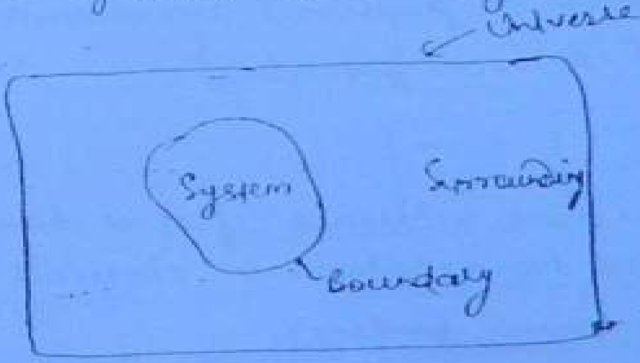
Surroundings :- Anything external to the system is known as surroundings.

Boundary :- The separation between systems and surrounding is known as boundary.

Boundary can be either "rigid or movable" and boundary can be "real or imaginary."

Note: Universe = System + Surrounding

⑤



** TYPES OF SYSTEM:

Types of System	Mass Transfer	Energy transf	Examples
Closed	✗	✓	Piston cylinder without valves
Open	✓	✓	Turbine, Comp., Pumps etc.
Isolated	✗	✗	Universe, the coffee in a well insulated flask

* Microscopic and Macroscopic approach of Thermodynamics :-

In microscopic approach, the behaviour of individual molecules is taken into consideration. This approach is also known as statistical thermodynamics and this approach is useful at low densities (higher altitudes).

$\rho = m/V \rightarrow$ No. of molecules

In macroscopic approach, the behaviour of individual molecules is not taken into consideration but the average behaviour of molecules is taken into consideration. This approach is also known as classical thermodynamics.

In our course we follow Classical Thermodynamics.

⑥

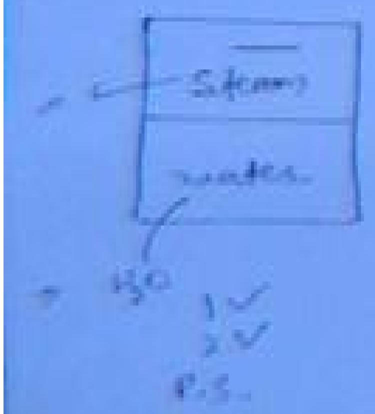
* Thermodynamic Equilibrium :-

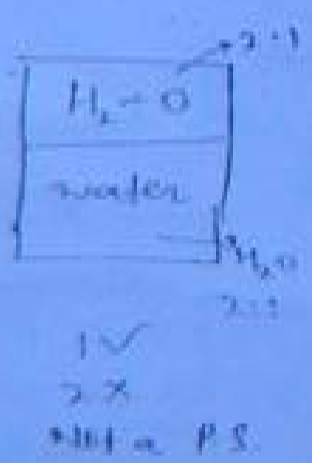
A system is said to be in thermodynamic equilibrium, if it is in thermal equilibrium (equality of temperature), mechanical equilibrium (equality of force or pressure) and chemical equilibrium (equality of chemical potential).

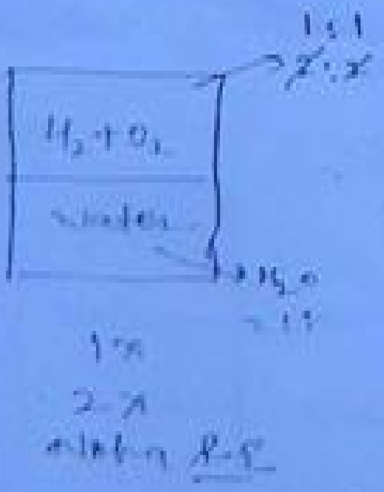
* Pure Substances:-

A substance is said to be a pure substance if it is homogeneous in chemical composition and homogeneous in chemical aggregation.

* (Aggregation means bonding).







* PROPERTIES OF A SYSTEM :- (Important).

Properties are characteristics of system. Examples Pressure, temp^{r}, volume density, mass, etc. Properties are of two types:-

① Intensive (or intrinsic)
② Extensive (or extrinsic)

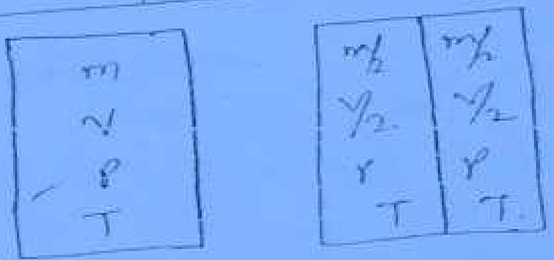
Intensive properties are independent of size or mass.

Examples - pressure, temperature, density, thermal-conductivity, viscosity, etc.

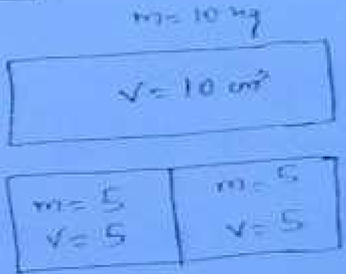
Extensive properties depend on size or mass of the system.

Examples - volumes, all forms of energy, etc.

Note :- Ratio of two extensive properties is an intensive property.



$$\overset{I}{\rho} = \frac{m \rightarrow E}{v \rightarrow E}$$



$\rho = m/v = 1.$

$\rho = m/v = 1.$

Note :- Specific properties are extensive properties per unit mass and these ~~specific~~ properties are intensive propertie

Specific Properties are intensive properties.

Example :- specific volume (v)
specific enthalpy (h)
specific internal energy (u)
specific entropy (s).

* Important points with respect to properties :-

① Properties are point functions or state functions.
② Properties are independent of past history.
③ Properties are exact differentials.

⑧

3.* State of a system :- The condition of a system is known as state of a system. The condition of a system is specified by means of its properties. As long as the properties are fixed, the condition is also fixed.

Process: A change of state is known as a process.

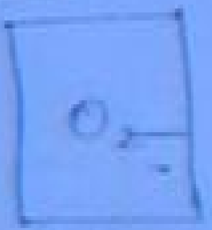
* GIBB'S PHASE RULE :-

According to Gibb's phase rule :-

$$\boxed{P+F = C+2}$$

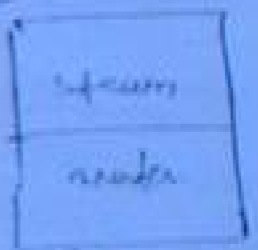
where,
P = No. of Phases
F = Degree of freedom
or
Minimum no. of independent intensive variables required.
C = No. of components.

Ex:-1



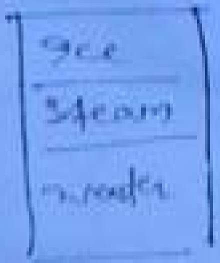
$P = 1$
$C = 1$
$P+F = C+2$
$1+F = 1+2$
$\therefore \boxed{F=2}$

Ex:-2



$P = 2$
$C = 1$
$P+F = C+2$
$\boxed{F = 1}$

Ex:-3



$P = 3$
$C = 1$
$P+F = C+2$
$\boxed{F = 0}$

⇒* THERMODYNAMIC CYCLES :-

A system is said to have undergone a cycle if the initial and final points are same. Minimum number of processes required for a cycle are 2.

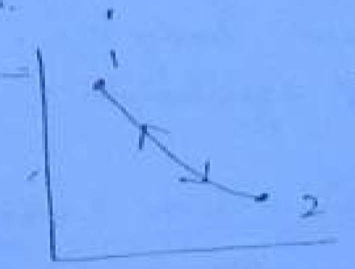
Note:- for a cycle the change in property is equal to zero. because property depends on point and for a cycle initial and final points are same.

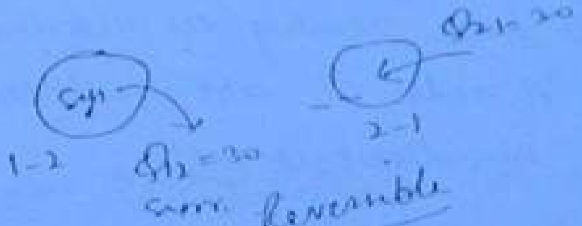
⇒* REVERSIBLE AND IRREVERSIBLE PROCESSES :-

A process is said to be a reversible process, if when reversed in direction follows the same path as that of the forward path without leaving any effect on system and surroundings.

A process which is not a reversible process is an irreversible process.

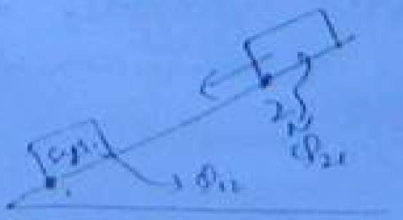
friction is one of the reasons which makes a process irreversible. Reversible process is the most efficient process.





Reversible

System	Surroundings
$Q_{12} = -30$	$Q_{12} = +30$
$Q_{21} = +30$	$Q_{21} = -30$
0	0



Irreversible

⇒ * QUASI- STATIC PROCESS :-

$\underset{\text{(almost)}}{\downarrow}$ $\underset{\text{(rest)}}{\downarrow}$

A process is said to be Quasi-static process, if it is carried out in a very slow manner with small gradient. Frictionless Quasi-static process is a reversible process.

⑩

⇒* ZEROTH LAW OF THERMODYNAMICS (Concept of Temperature)

Statement:- When a body 'A' is in thermal equilibrium with body 'B' and body 'B' is in thermal equilibrium with body 'C' separately, then 'A' and 'C' are in thermal equilibrium.

In zeroth law of thermodynamics one body acts as thermometer. Let us assume that body 'B' is thermometer. Let us emerge thermometer 'B' in body 'A'. When the thermal equilibrium is reached, body 'B' records some temperature. Now place the body 'B' (thermometer) in body 'C'. If the thermometer records same reading as recorded with body 'A', then we can say that 'A' and 'C' are in thermal equilibrium without bringing them in direct contact.

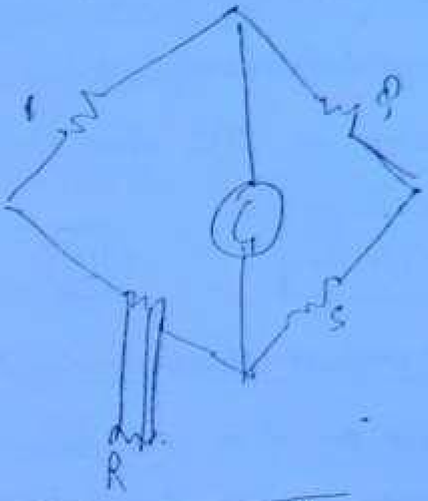
* Principle of Thermometers:-

In finding temperature, first the property which varies with temperature is found and this property is known as thermometric property and with the help of thermometric property, the unknown temp. 'T' can be found.

⇒* Types of thermometers:-

① Resistance Thermometer (Thermistor):

These thermometers are based on wheatstone bridge principle. In these thermometers, resistance plays the role of thermometric property.



$P/Q = R/S$

$R = \frac{PS}{Q}$

⑪

$$\boxed{R = R_o(1 + \alpha t + \beta t^2)}$$

② **Thermocouple :-** Thermocouples are based on Seebeck effect. According to this Seebeck Effect, when two dissimilar metals are joined to form two separate junctions and if these junctions are maintained at different temp^n emf (voltage) is generated and this ~~temp~~ emf is proportional to two temperature difference.

③ **Constant Volume Gas ~~ther~~ thermometer :-** In constant volume gas thermometer, pressure plays the role of thermometric property.

④ **Constant Pressure Gas thermometer :-** Volume plays the role of thermometric property.

Type of thermometer — Thermometric Property

1) Thermistoo →
2) Thermocouple →
3) constant Vol^m Gas thermometer →
4) Constant Pressure " " →

⊃ * Temperature Scale:-

Temperature scales are arbitrary.

In celsius scale, ice point is taken as 0°C and steam point is taken as 100°C.

$$\boxed{K = \_\,^\circ C + 273.15}$$

⑫

Method used before 1954 for temperature measurement :-

This method is based on two reference temperatures i.e. ice point (0°C) and steam point (100°C).

$$t = ap + b$$

$$t_i = a p_i + b$$

$$t_s = a p_s + b.$$

$$100 = a p_s + b$$

$$\underline{\;0 = a p_i + b\;}$$

$$100 = a(p_s - p_i)$$

$$\text{or}\quad a = \frac{100}{(p_s - p_i)}$$

Now

$$0 = a p_i + b$$

$$\text{or,}\quad b = -a p_i$$

$$= -\frac{100}{(p_s - p_i)} \times p_i$$

$$\therefore\quad t = \frac{100}{(p_s - p_i)} p - \frac{100}{(p_s - p_i)} . p_i$$

$$\therefore\quad \boxed{t = \frac{100}{(p_s - p_i)} . (p - p_i)}$$

By knowing the unknown property "p", the temperature 't' can be found.

* Method used after 1954 for temperature measurement :-

It is based on single fixed temperature i.e. triple point of water.

$$\overset{c}{P}V = \overset{c}{m}\overset{c}{R}T$$

let $V = c$.

$$P \propto T$$

$$P = cT.$$

$$c = P/T$$

At triple point

$$c = \frac{P_{tp}}{T_{tp}}$$

$$\therefore P = \frac{P_{tp}}{T_{tp}} \times T.$$

$$T = \frac{P}{P_{tp}} \cdot T_{tp}$$

$$\boxed{T = 273.16\left(\frac{P}{P_{tp}}\right)}$$

At triple point.

$T = 0.01°C$

$K = 0.01 + 273.15$

$\underline{K = 273.16\,K.}$

The triple point of water is assigned a value of 0.01°C (273.1

Note:- Ideal gas thermometers are independent of material of construction. ∵ All ideal gas behave same.

PROBLEM

13 The reading 't_A' and 't_B' of 2°C thermometer 'A' & 'B' agree at ice point (0°C) and steam point (100°C) and are related by the equation, $t_A = l + m t_B + n t_B^2$ between these temp where l, m and n are constants when both are immersed in oil 'A' reads 51°C and 'B' reads 50°C. Determine the reading on 'A', when 'B' reads 25°C. And discuss the question which thermometer is correct?

S: $t_A = l + m t_B + n t_B^2$

$t_A = 0°C = t_B$

$0 = l + m(0) + n(0)$

$\Rightarrow \boxed{l = 0}$

$t_A = m t_B + n t_B^2$

$t_A = 100°C = t_B$

$100 = m(100) + n(100)^2$

$1 = m + 100n$ —①

$t_A = 51°C,\ t_B = 50°C$

$51 = 0 + m(50) + n(50)^2$

$\Rightarrow 51 = 50m + 2500n$ —②

from ① & ②

$m = 1.04$

$n = -4\times10^{-4}$

$t_A = 0 + 1.04\,t_B - 4\times10^{-4}\,t_B^2$ —③

$\rightarrow t_B = 25°C$

$t_A = 1.04(25) - 4\times10^{-4}(25)^2$

$\boxed{t_A = 25.75°C}$

Though the end point temperatures are same, it does not necessarily mean the intermediate point temperatures are also same.

① **Assertion (A):-** If alcohol and mercury thermometers read exactly at ice point and steam point, these two thermometers will also give same reading exactly at 50°C.

Reasoning (R):- Temperature scales are arbitrary.

Ans: Assertion is wrong and Reasoning is independently correct.

② Which of the following are intensive properties?

1. Kinetic Energy 2. Thermal conductivity
3. ~~Volume~~ Pressure 4. Volume

15

Ans: (b) (2 and 3)

③ Match the following Question:-

List I (Type of thermometer)	List II (Thermometric Property)
A) Mercury in Glass	1) Pressure
B) Thermocouple	2) Resistance
C) Thermistor	3) Volume or length
4) Constant Volm Gas Thermometer	4) Emf

Ans: 3-4-2-1

④ The thermometric property x varies with tempr T according to the relation $t = ax^2 + b$, where 't' is in °C and 'x' is in cm. ~~A~~ 'a' and 'b' are constants. At ice point (0°C) and steam point (100°C), the values of 'x' are 5cm and 20cm respectively. When this thermometer is brought in contact with heated body, the value of 'x' is recorded as 15cm. Then the tempr of heated body is?

Soln:

$0 = 25a + b \Rightarrow b = -25a$ — ①

$100 = 400a + b$

$\Rightarrow 100 = 400a - 25a \Rightarrow a = 100/345 = 0.27$

$\therefore b = -25 \times 0.27 = -6.75$

$t = 0.27(15)^2 + -6.75$

$a = 0.26$
$b = -6.66$

$\Rightarrow \boxed{t = 54°C} \simeq \boxed{53.3°C}$ ...

V.V.I
P.S In a new temperature scale, °P, the boiling and freezing points of water are 100°P and 300°P respectively. Find the reading corresponding to 0°P on celcius scale.

Soln Let us assume a linear relation between °C and °P scale.

Let it be,

$$°P = a \times °C + b.$$

⑯

Now

$$300 = a \times (0) + b$$

$$\Rightarrow \boxed{b = 300}$$

Again

$$100 = a \times 100 + 300$$

$$\Rightarrow \boxed{a = -2}$$

$$\therefore °P = -2 \times °C + 300$$

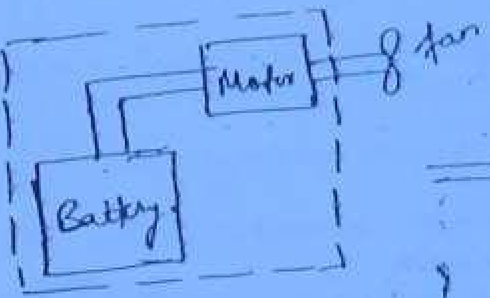
when 0°P $0 = -2 \times °C + 300.$

$$\therefore \boxed{°C = 150°}$$

CHAPTER-2 :-

ENERGY INTERACTIONS (WORK & HEAT)

⇒* Thermodynamic work :- Work is said to be done by the system if the sole effect on things external to the system can be equated to raising of weights (weights may not be actually raised but the effect can be equated to raising of weights).



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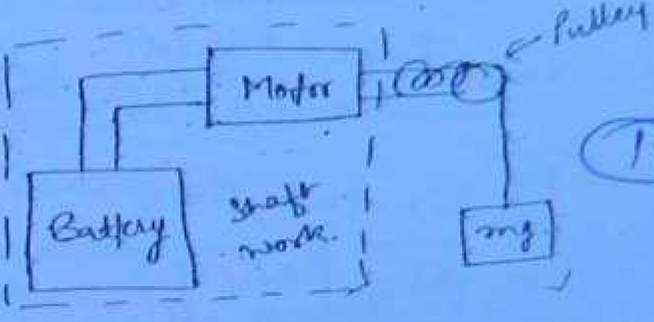

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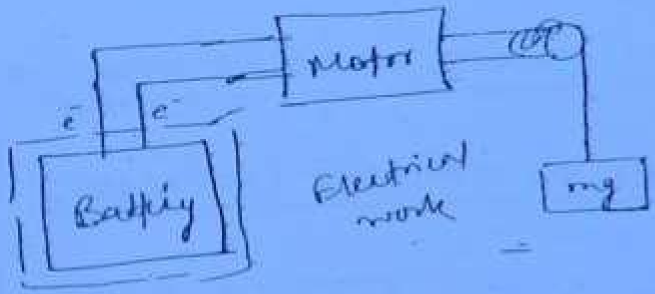
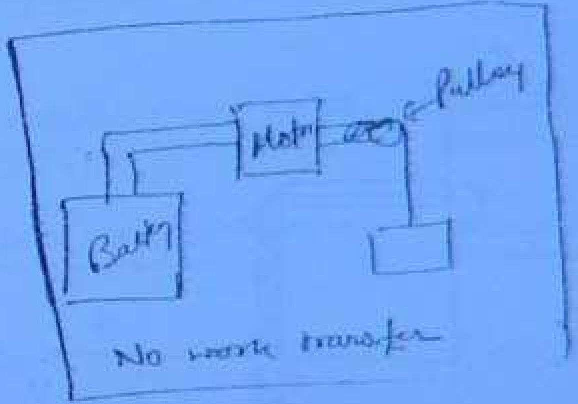
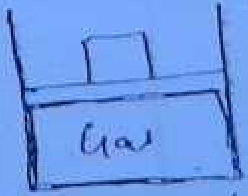


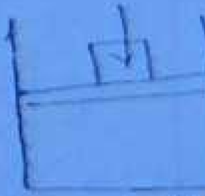


⇒* Conventions of work transfer :-

Work done by the system is taken as positive and work done on the system is taken as negative.



+ve work (raising of weights)
(Boundary weights)



-ve work (lowering of Boundary contacts. weights)

⇒ * Generalised Equation for Non-flow work or closed system work :-

$P = F/A$

$F = PA$

$$W = F \times x$$

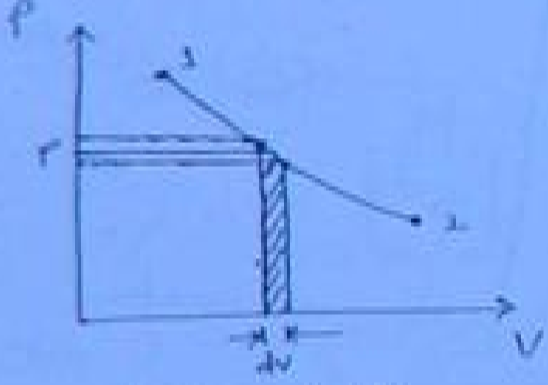
$$dW = F \times dx$$

⑱

$$= \underbrace{PA \times dx}$$

$$\therefore dW = P.dV$$

or $\boxed{W = \int P dV}$ ⟶ Non-flow work
(Closed system work).



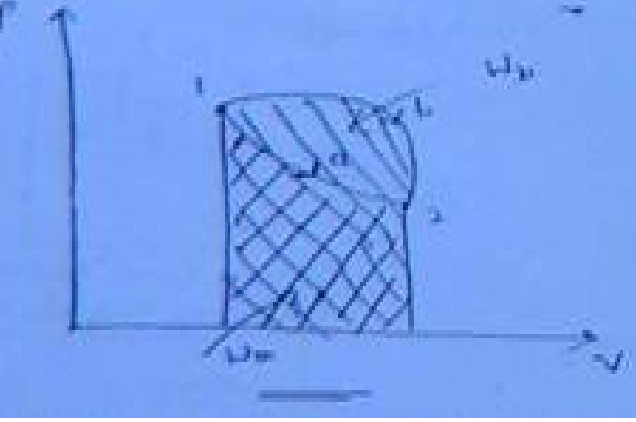
Area = P.dV

Also, work = Pdv.

$\therefore$ $\boxed{\text{Area} = \text{Work}}$

ery Important Sentence)

⇒ Area under the curve when projected on volume axis gives closed system work.



$W_a \neq W_b$

→ Though the end points for 'a' and 'b' are same, work transfer is not same because areas are different. Therefore, work transfer depends on path followed by the process and hence work transfer is a path function and it is not a property and it is inexact differentials (đW or δW).

⇒* There is no work transfer when the system is in equilibrium state. Work transfer is always associated with a process.

⇒* Conditions for Applying the Equation $W = \int P dV$:→

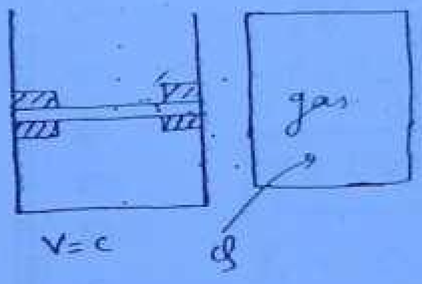
① The system must be a closed system.

② Work should cross the boundary.

③*** The process must be a reversible process.

⇒* NON-FLOW OR CLOSED SYSTEM WORK FOR VARIOUS PROCESSE

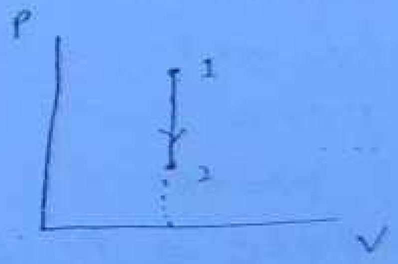
① CONSTANT VOLUME OR ISOCHORIC OR ISOMETRIC PROCESS:



$V = C \Rightarrow dV = 0$

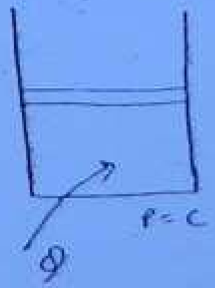
$W = \int P dV$

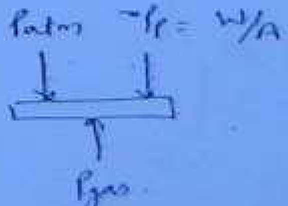
$W = 0$



Constant volume closed system work = 0.

② CONSTANT PRESSURE OR ISOBARIC OR ISOPIESTIC PROCESS:





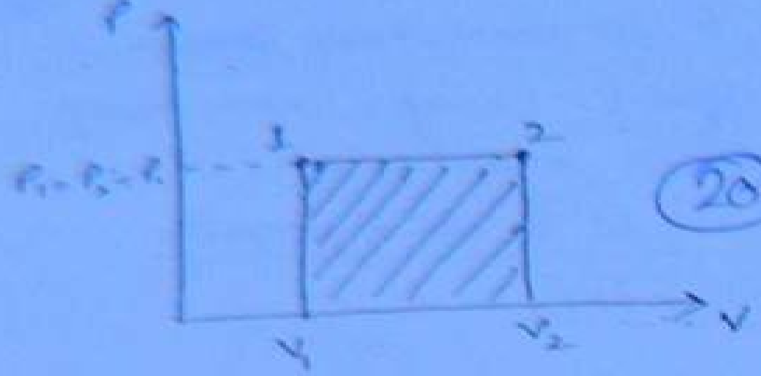
At equilibrium.

$P_{atm} + P_p = P_{gas}$

$P_{atm} + \frac{W}{A} = P_{gas}$

∵ Patm, W & A are constants.

So, P_{gas} = constant.



$$W=\int_{V_1}^{V_2} P.dV.$$

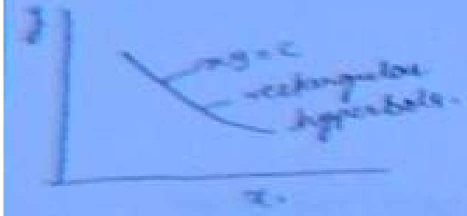
$$W = P\int_{V_1}^{V_2} dV.$$

$$\boxed{W = P(V_2-V_1)}$$

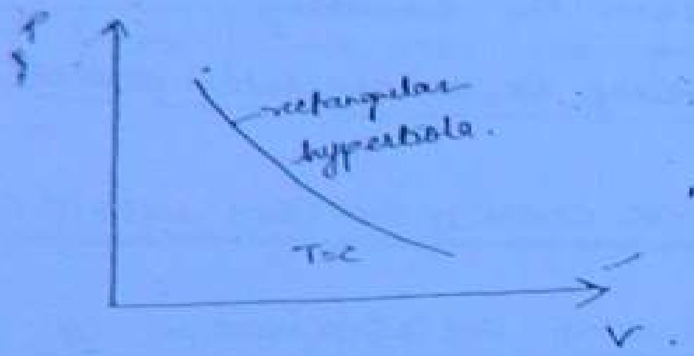
W = Area.

$$\boxed{W = P(V_2-V_1)}$$

3) CONSTANT TEMPERATURE OR ISOTHERMAL PROCESS :-



$xy=c$



T = constant.

$PV = mRT$

$PV = C.$

$\therefore PV = mRT = C.$

$PV = C \Rightarrow P = C/V.$

$P_1V_1 = P_2V_2 = C.$

Now,

$$W = \int_{V_1}^{V_2} P.dV$$

$$W = \int_{V_1}^{V_2} \frac{C}{V}.dV$$

$$\text{or, } W = C\int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = C\left[\ln V_2 - \ln V_1\right].$$

or, $$W = C\ln\left(\frac{V_2}{V_1}\right)$$

$$\therefore \boxed{W = P_1V_1\ln\left(\frac{V_2}{V_1}\right)}$$

$$\boxed{W = mRT\ln\left(\frac{V_2}{V_1}\right)}$$

Imp
An adiabatic process can never be an isolated process

㉑

((Isothermal curves for an ideal gas are rectangular hyperbola on P-V diagram.))

④ **ADIABATIC PROCESS** :- A process is said to be adiabatic process in which there is no heat transfer from the system or to the system.

For adiabatic process;

$$\boxed{PV^{\gamma} = C}$$

where, γ = adiabatic index and it is greater than 1 ($\gamma > 1$).

$$PV^{\gamma} = C$$

$$P = \frac{C}{V^{\gamma}}$$

$$\boxed{P = C\cdot V^{-\gamma}}$$

$$P_1 = C\cdot V_1^{-\gamma} \quad \text{--- ①}$$

$$P_2 = C\cdot V_2^{-\gamma} \quad \text{--- ②}$$

Now,

$$W = \int_{V_1}^{V_2} P\,dV$$

$$W = \int_{V_1}^{V_2} C\cdot V^{-\gamma}\,dV$$

$$W = C\int_{V_1}^{V_2} V^{-\gamma}\,dV$$

$$W = C\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$W = \frac{C}{-\gamma+1}\left[V_2^{-\gamma+1} - V_1^{-\gamma+1}\right] \quad (22)$$

$$W = \frac{1}{-\gamma+1}\left[\underbrace{C\cdot V_2^{-\gamma}}\cdot V_2 - \underbrace{C\cdot V_1^{-\gamma}}\cdot V_1\right]$$

$$\Rightarrow W = \frac{1}{-\gamma+1}\left[P_2\cdot V_2 - P_1\cdot V_1\right]$$

$$\Rightarrow W = \frac{\cancel{-}(P_1V_1 - P_2V_2)}{\cancel{-}\cdot(\gamma-1)}$$

$$\therefore \boxed{W = \frac{(P_1V_1 - P_2V_2)}{(\gamma-1)}}$$

⑤ POLYTROPIC PROCESS :-

A process is said to be a polytropic process, if pressure & volume follows the relation $PV^n = C$, where 'n' is known as polytropic index.

Generally, $\boxed{n > 1 \text{ and } n < \gamma.}$

In polytropic process, there is both heat transfer and work transfer but in adiabatic process, there is only work transfer.

Polytropic Work,

$$\boxed{W = \frac{(P_1V_1 - P_2V_2)}{(n-1)}}$$

Any process can be represented as $PV^K = C$. $\underline{PV^n = C}$

for constant Pressure process,

$\underline{P = C.}$

$PV^K = C.$

$K = 0,$

$PV^0 = C$

$P \cdot 1 = C$

$\boxed{P = C}$

* for thermal constant process,

$\underline{T = C}$

$PV = mRT.$

$PV = C$

$PV^1 = C$

$\boxed{K = 1}$

* for constant volume process,

$\underline{V = C}$

$PV^K = C.$

$(PV^K)^{1/K} = C^{1/K}$

$P^{1/K} \cdot V = C^{1/K}$. (constant now)

when, $K = \infty$

$P^0 \cdot V = C$

$\therefore \boxed{V = C}$

* for adiabatic process,

$PV^\gamma = C.$

$\boxed{K = \infty}$

* for polytropic process,

$PV^n = C$

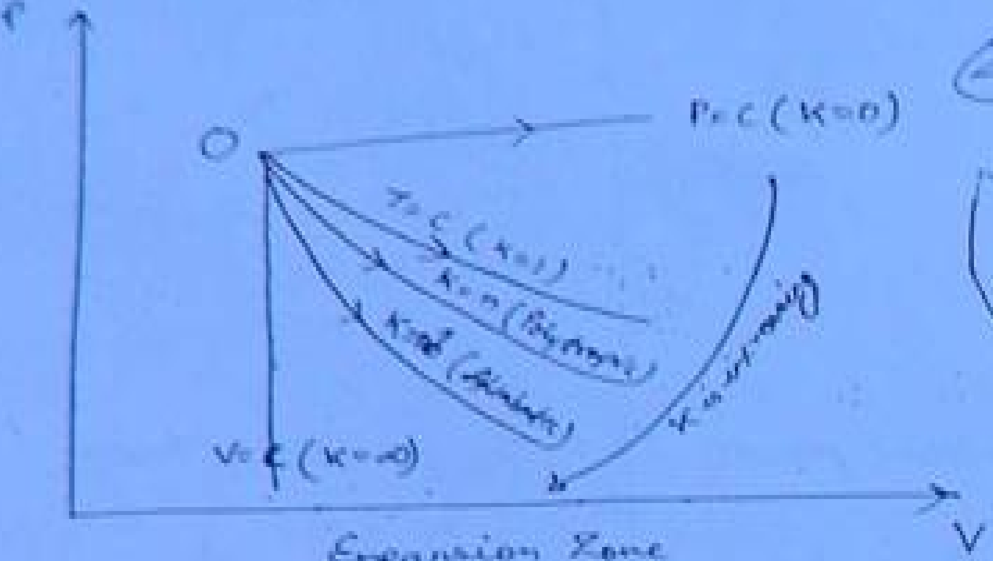
$\boxed{K = n}$

$P = C \rightarrow K = 0.$
$V = C \rightarrow K = \infty$
$T = C \rightarrow K = 1$
Adiabatic $\rightarrow K = \gamma$
Polytropic $\rightarrow K = n$

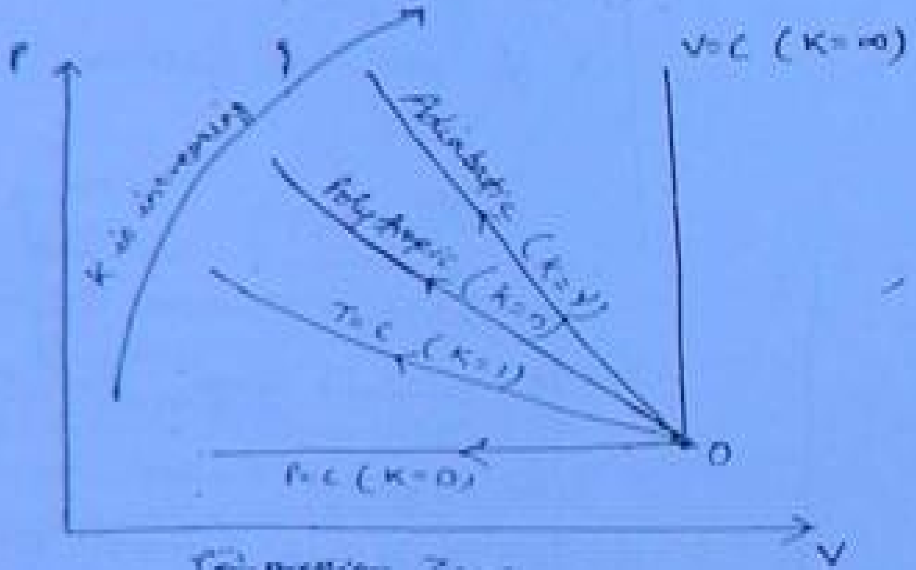
① Constant Pressure, $P = C, \rightarrow K = 0$
② Constant Temp^r, $T = C, \rightarrow K = 1$ (Isothermal)
③ Polytropic Process, $K = n$
④ Adiabatic Process $K = \gamma$.
⑤ Constant volume, $V = C \rightarrow K = \infty$

$PV^\gamma = C.$

⇒ * Representation of Various Process on P-V diagram :-



Expansion Zone



Compression Zone

⇒ * Ideal Gas Equations for Various Processes :-

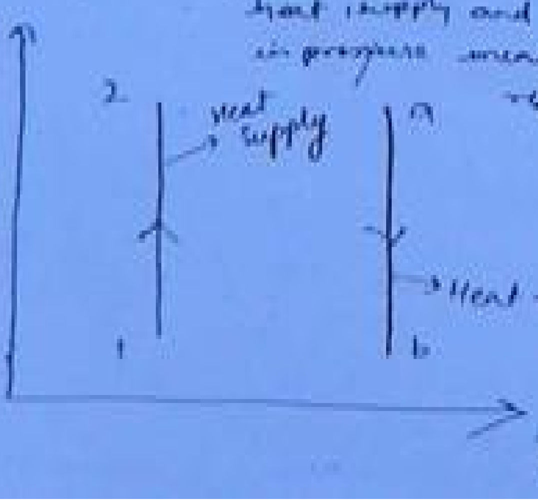
① Constant Volume Process :-

V = C.

$$P\overset{c}{\cancel{V}} = \overset{c}{\cancel{m}}\,\overset{c}{\cancel{R}}\,T$$

$P \propto T.$

$$\boxed{\frac{P_2}{P_1} = \frac{T_2}{T_1}}$$



Increase in pressure means heat supply and decrease in pressure means heat rejection.

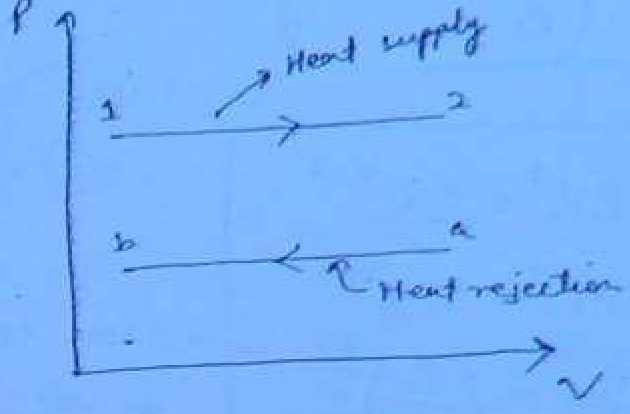
② Constant Pressure Process :-

P = constant

$\overset{c}{\cancel{P}}V = \overset{c}{\cancel{m}}\overset{c}{\cancel{R}}T$

$V \propto T$.

$$\boxed{\frac{V_2}{V_1} = \frac{T_2}{T_1}}$$



③ Isothermal Process :-

T = constant

$PV = \overset{c}{\cancel{m}}\overset{c}{\cancel{R}}\overset{c}{\cancel{T}}$

$PV = C$ ..

$$\boxed{P_1V_1 = P_2V_2}$$

④ Adiabatic Process :-

$PV^{\gamma} = C$.

$PV = mRT$

$P = \frac{mRT}{V}$.

$\frac{mRT}{V} \cdot V^{\gamma} = C$.

or, $T \cdot V^{\gamma-1} = \frac{C}{RT} = C$.

or, $T \cdot V^{\gamma-1} = \text{const}$.

$$\boxed{P_1V_1^{\gamma} = P_2V_2^{\gamma}} \quad \text{—} ① \quad \Rightarrow \left(\frac{V_1}{V_2}\right) = \left(\frac{P_2}{P_1}\right)^{1/\gamma}.$$

$$\boxed{T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}} \quad \text{—} ② \quad \Rightarrow \left(\frac{V_1}{V_2}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

Thus $\left(\frac{T_2}{T_1}\right)^{1/\gamma-1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$.

$$\boxed{\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

Thus

$$\boxed{\begin{aligned} P_1V_1^{\gamma} &= P_2V_2^{\gamma} \\ T_1V_1^{\gamma-1} &= T_2V_2^{\gamma-1} \\ \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \end{aligned}}$$

→ These equations are valid for an ideal gas undergoing adiabat process.

⑤ Polytropic Process :-

$$\boxed{\begin{aligned} P_1V_1^{n} &= P_2V_2^{n} \\ T_1V_1^{\gamma-1} &= T_2V_2^{\gamma-1} \\ \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \end{aligned}}$$

→ These equations are valid for an ideal gas undergoing ~~adia~~ polytropic process.

• SLOPE OF ISOTHERMAL CURVES ON P-V DIAGRAM :-

$T =$ Constant

$PV = mRT$

$PV = C.$

$PdV + VdP = 0$

$PdV = -VdP$

$$\frac{dP}{dV} = -\frac{P}{V}$$

Slope of isothermal curves on P-V diagram $= -\frac{P}{V}$

⇒* SLOPE OF ADIABATIC CURVES ON P-V DIAGRAM :-

$$PV^\gamma = C$$

$$P[\gamma . V^{\gamma-1} . dV] + V^\gamma [dP] = 0 .$$

$$\gamma P . \frac{\cancel{V^\gamma}}{V} . dV = -\cancel{V^\gamma} dP$$

$$\Rightarrow \quad -\frac{\gamma P}{V} = \frac{dP}{dV}$$

$$\text{or} \quad \frac{dP}{dV} = \frac{-\gamma P}{V}$$

$$\therefore \quad \boxed{\frac{dP}{dV} = \gamma\left(-P/V\right)}$$

∴ Slope of adiabatic curves = γ(Slope of Isothermal curves)

$$\frac{\text{Slope of adiabatic}}{\text{Slope of isothermal}} = \gamma$$

$$\because \quad \gamma > 1$$

$$\therefore \quad \frac{\text{Slope of adiabatic}}{\text{Slope of isothermal}} > 1.$$

Hence,

$$\boxed{\text{Slope of adiabatic curves} > \text{Slope of isothermal curves}}$$

CONVENTIONAL PROBLEM

Q. A system undergoes three processes as shown in figure.

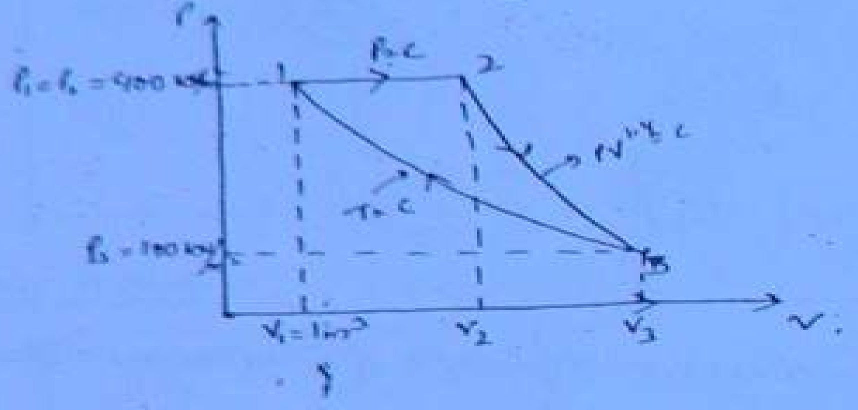
$1-2 \rightarrow$ Isobaric process

$2-3 \rightarrow$ Adiabatic process with $\gamma = 1.4$

$3-1 \rightarrow$ Isothermal process with $PV = C$.

(28)

Find the value of V_2 and also find net work transfer.



Soln.

(Process 3-1)

$T = C.$

Also $PV = C$

$P_3V_3 = P_1V_1$

$\Rightarrow 100 \times V_3 = 400 \times 1$

$\therefore V_3 = 4\ m^3$

Again (Process 2-3)

$PV^{1.4} = C$

$P_2V_2^{1.4} = P_3V_3^{1.4}$

$\Rightarrow 400 \times V_2^{1.4} = 100 \times (4)^{1.4}$

$\Rightarrow \boxed{V_2 = 1.486\ m^3}$ (Ans)

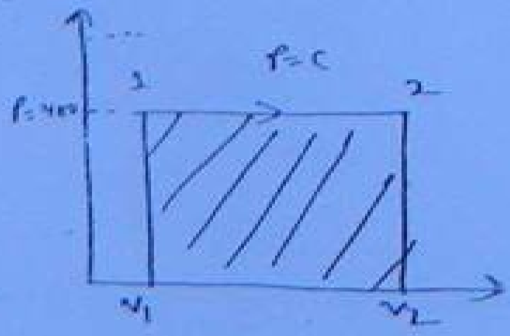
Now.

$W_{net} = W_{12} + W_{23} + W_{31}$

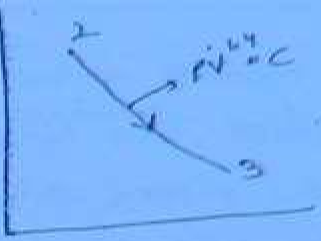
$W_{12} = P(V_2 - V_1)$

$= 400(1.486 - 1)\ \frac{kN}{m^2} \times m^3$

$\therefore W_{12} = 194.4\ kJ.$



$$W_{23} = \frac{P_2V_2 - P_3V_3}{\gamma - 1}$$



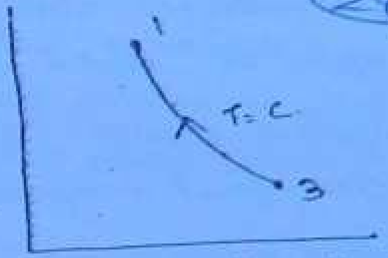
$$\Rightarrow W_{23} = \frac{400 \times 1.486 - 100 \times 4}{1.4 - 1}$$

$$\Rightarrow \boxed{W_{23} = 486 \text{ kJ}}$$

$$W_{31} = P_3V_3 \ln \frac{V_1}{V_3}$$

$$= 100 \times 4 \ln\left(\frac{1}{4}\right)$$

$$\boxed{W_{31} = -554.9 \text{ kJ}}$$



$$W = P_iV_i \ln \frac{V_f}{V_i}$$

$$W_{net} = 194.4 + 486 + (-554.5)$$

$$\therefore \boxed{W_{net} = 125.8 \text{ kJ}}$$

Note:- Net work in a cycle is equal to area of the closed region. All clockwise cycles on P-V diagram are work producing or power producing ~~por~~ cycles. All anti-clockwise cycles are power absorbing or work absorbing cycles.

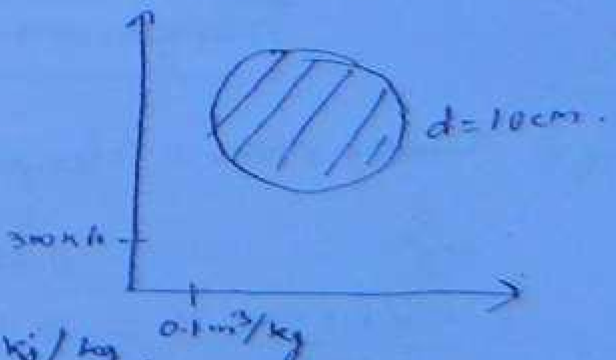
Q:- An imaginary engine recieves heat and performs work on a slowly moving piston at such a rate that the cycle of operation of 1 kg of working fluid can be represented as a circle of 10 cm diameter on P-V diagram, on which 1 cm = 300 kPa and 1 cm = 0.1 m³/kg. Then, find the net work during the cycle.

Sol^n

1 cm = 300 kPa

1 cm = 0.1 m³/kg

1 cm² = 300×0.1 = 30 kJ/kg

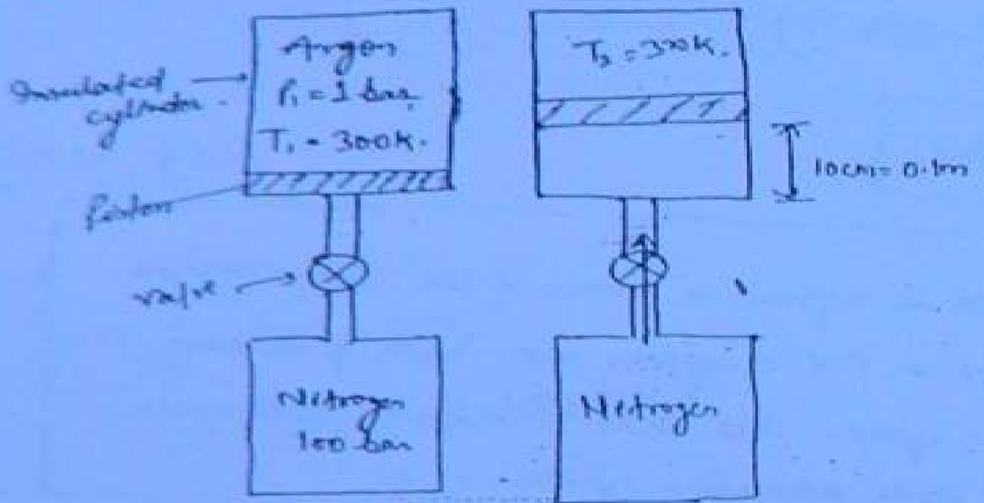


$$W_{net} = \text{Area} = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$$

$$\therefore W_{net} = (78.54 \times 30) = 2356.2 \text{ kJ/kg}.$$

Q. Important An insulated vertical cylinder contains 0.1 kg of Argon gas. With the help of a frictionless non-conducting piston as shown in figure. The mass of the piston is 5 kg and it initially rest on the bottom of the cylinder. The cylinder is connected to nitrogen tank at 100 bar to a pipeline fitted with the valve. The valve is opened and nitrogen slowly enters the cylinder. During this process, the piston is lifted to a height of 10 cm by nitrogen gas. The initial pressure and temperature of argon are 300 K and 1 bar. The final temperature of argon gas is 320K. For argon gas R = 0.208 kJ/kg K and $\gamma = 1.67$.



$$W_{argon} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{m_1R_1T_1 - m_2R_2T_2}{\gamma - 1} \quad (\because \text{ m \& R are constant})$$

$$\therefore W_{argon} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{0.1 \times 0.208 (300 - 320)}{1.67 - 1}$$

$$\therefore \boxed{W_{argon} = -0.621 \text{ kJ}}$$

$$W_{N_2} = \underset{\substack{\downarrow \\ mgh}}{W_{raising\ piston}} + \underset{\substack{\downarrow \\ 0.621 \text{ kJ}}}{W_{compressing\ Argon}}$$

$$= \frac{5 \times 9.81 \times 0.1}{1000} = 4.905 \times 10^{3} \text{ KJ.}$$

∴ Net work done by Nitrogen.

$$W_{N_2} = 4.905 \times 10^{-3}\ kJ + 0.621\ kJ$$

$$\boxed{W_{N_2} = 0.626\ kJ}\ (Ans)$$

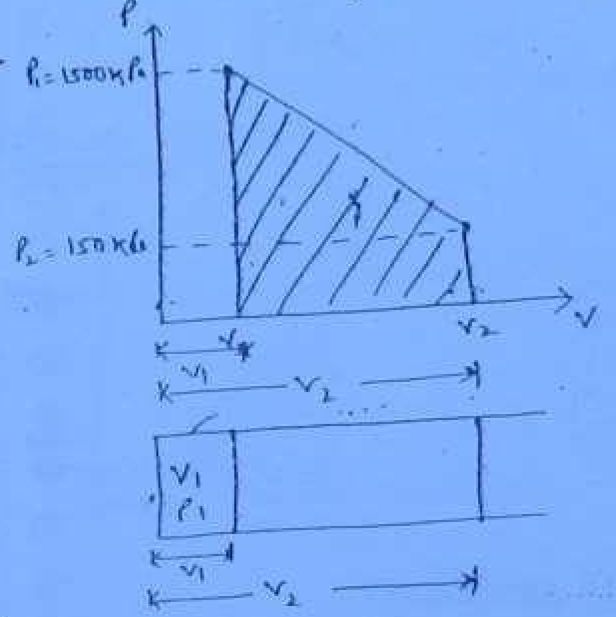
$\boxed{V_s = A \times L}$

V_s = swept vol.
L = stroke length
A = Area of Piston.

Q. Important

An engine cylinder has a Piston area of 0.12 m^2 and contains a gas at a pressure of 1.5 MPa. The gas expands according to a process which is represented by a straight line on P-V diagram. The final pressure is 0.15 MPa. Calculat the work done by the gas if the stroke length is 0.3 m.

Solⁿ.



Work = Area under curve

$$= \frac{1}{2}(1500 + 150) \times (V_2 - V_1)$$

But, $V_2 - V_1 = V_s = AL$.

$$= \frac{1}{2} \times 1650 \times (0.12 \times 0.3)$$

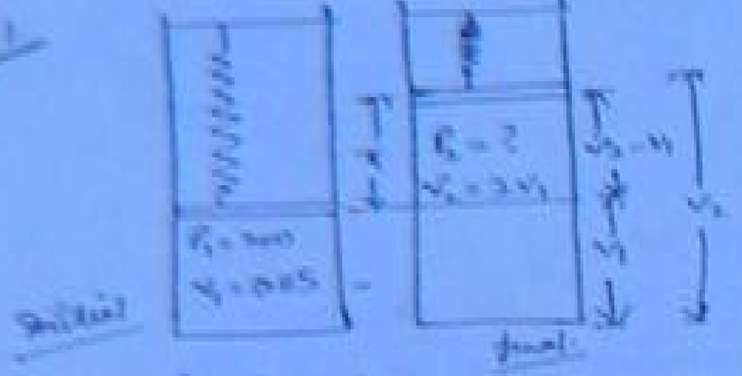
$$\therefore \boxed{W = 29.7\ kJ}\ (Ans)$$

Q. V.V.Imp

A piston cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state a linear spring which has spring constant of 150 kN/m is just touching the piston but exerting no force on it. Now Heat is transferred to the gas causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, find :-

① final pressure inside the cylinder
② Work done by the gas

Solⁿ:



(B)

$V_2 - V_1 = A \times x$

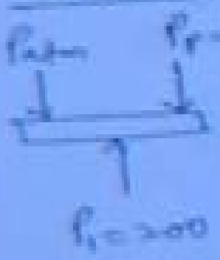
$\Rightarrow 2V_1 - V_1 = 0.25 \times x$

$\Rightarrow 0.05 = 0.25x$

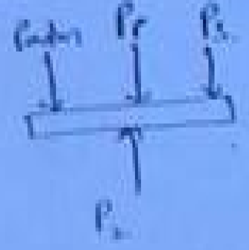
$\therefore \boxed{x = 0.2\,m}$

Now free body diagrams.

Initial



final..



At equilibrium.

$P_{atm} + W/A = 200$ —①

At equilibrium

$P_{atm} + P_P + P_s = P_2$ —②

But from eqⁿ ①, $P_{atm} + P_P = 200$

$\therefore 200 + P_s = P_2$ —③

Now $P_s = \frac{F_s}{A} = \frac{K \cdot x}{A}$

$\therefore P_s = \frac{150 \times 0.2}{0.25} = 120$ kN/m² or kPa.

$\therefore \boxed{P_2 = 200 + 120 = 320\,kPa}$ ...(Ans)

Work done by gas = work done in raising piston + work done in compressing spring.

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Spring Work

$= \frac{1}{2} Kx^2$

$= \frac{1}{2} \times 150 \times 0.2^2$

$= 3$ kJ

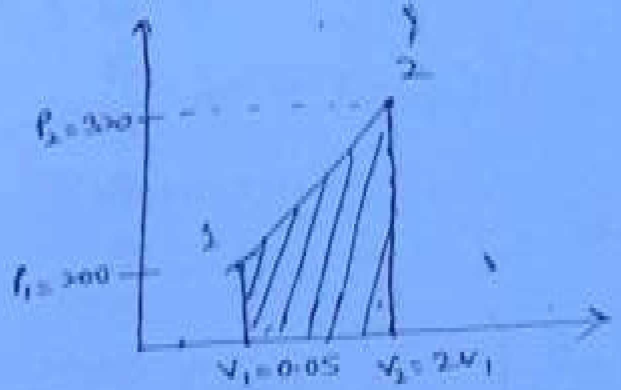
$W_{piston} = P(V_2 - V_1)$

$200(2V_1 - V_1)$

$= 200 \times (0.05) \Rightarrow W_{piston} = 10$ kJ

$$\boxed{W_{Total} = 10 + 3 = 13 \text{ kJ}}$$ (Ans)

Alternate Method:



$P_2 = 200 + P_s$.

$P_s = \frac{F_s}{A} = \frac{Kx}{A}$ — ①

Also, $V_2 - V_1 = Ax$

$\Rightarrow x = \frac{V_2 - V_1}{A} = \frac{\Delta V}{A}$.

$\therefore P_s = \frac{K}{A} \cdot \frac{\Delta V}{A}$

or $P_s = \frac{K}{A^2} \cdot \Delta V$.

Thus, $P_2 = 200 + \frac{K}{A^2} \cdot \Delta V$.

$W = \frac{1}{2}(200 + 320)(2V_1 - V_1)$

$= \frac{1}{2} \times 520 \times 0.05$

$\therefore \boxed{W = 13 \text{ kJ}}$ (Ans)

WORK BOOK.

PAF-cle
2A ㉔ ②. 30

Chapter - ①

① — ⓐ
② — ⓒ
③ — ⓑ
④ — ⓓ
⑤ — ⓐ
⑥ — ⓑ
⑦ — ⓒ
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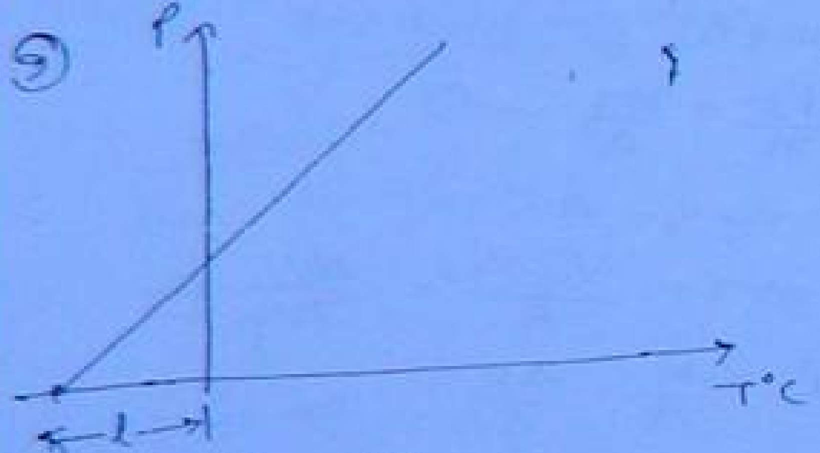
㊳

1-2-7
3-8

⑨



$Pv = mRT.$

$P = \frac{mR}{V} \cdot T$, Here , m, R & V are constants.

$\therefore P = CT.$

$\because P = 0 \cdot \quad \therefore \underline{T = 0K}.$

converting in °C.

$K = °C + 273.15$

$\therefore \; 0 = °C + 273.15$

$\Rightarrow °C = -273.15$, Hence (d) is ans.

Ans (13) (a) $\int PdV$ & (b) $\int VdP$.

Both are works and work is not considered a property.

$$\boxed{\begin{array}{l} dz = Mdx + Ndy \\ \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \end{array}}$$

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The equation $dz = Mdx + Ndy$ is exact, when

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Solving option (d):

$$\frac{dT}{T} - \frac{VdP}{T}$$

$$\frac{1}{T} \cdot dT + \left(-\frac{V}{T}\right) dP$$

$$Mdx + Ndy$$

$$M = \frac{1}{T};\ x = T;\ N = -\frac{V}{T};\ y = P.$$

Now $\left(\frac{\partial M}{\partial y}\right)_x = \left[\frac{\partial}{\partial P}\left(\frac{1}{T}\right)\right]_T$ $\therefore \left(\frac{\partial M}{\partial y}\right)_x = 0.$

$\because$ T is constant

Again $\left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial T}\left(-\frac{V}{T}\right)\right]_P$

from ideal gas equation

$$PV = mRT \quad \therefore \frac{V}{T} = \frac{mR}{P}.$$

$$\therefore \left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial T}\left(-\frac{mR}{P}\right)\right]_P$$

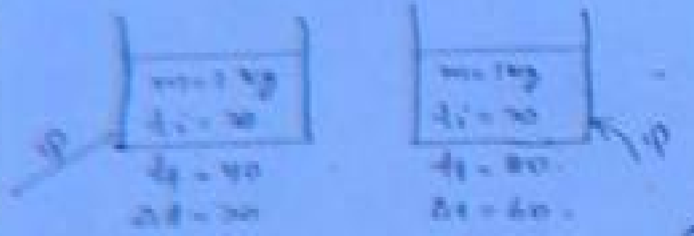
$\because$ P is constant $\therefore \left(\frac{\partial N}{\partial x}\right)_y = 0$

$$\therefore \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y = 0$$

Hence, these are exact differentials. So, it is a property.

← : HEAT : →

The energy transfer due to temperature difference is known as heat transfer.



$Q \propto \Delta t$

$Q \propto m$

$Q \propto m(\Delta t)$

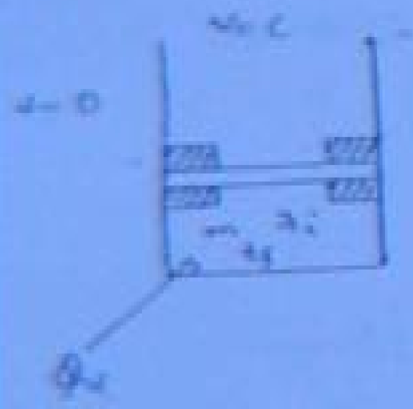
or $Q = m\underset{\text{(specific heat)}}{c}(\Delta t)$.

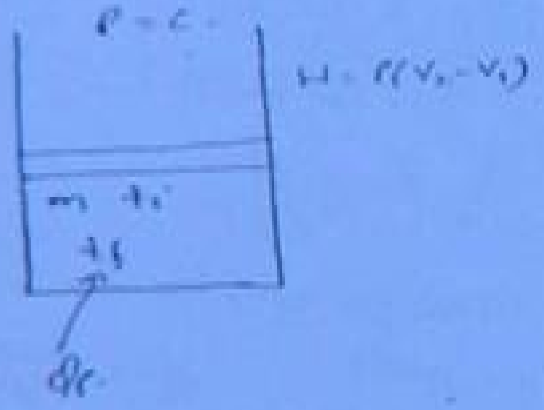
when $m=1$; $\Delta t = 1$

$$\boxed{Q = c}$$

$$\boxed{c = \frac{Q}{m\Delta t} \quad \text{Unit}\left(\frac{kJ}{kg\,K}\right)}$$

Specific heat (c) :→ It is the amount of heat required to raise the temperature of unit mass of substance through unit degree temperature difference.





$Q_p > Q_v$

$\cancel{m} c_p \cancel{\Delta t} > \cancel{m} c_v \cancel{\Delta t}$

$$\therefore \boxed{c_p > c_v}$$

$\because \gamma = \frac{c_p}{c_v}$ ~~as~~ & $\frac{c_p}{c_v} > 1$

$$\therefore \boxed{\gamma > 1}$$

Specific heat at constant pressure (C_p) is greater than specific heat at constant volume (C_v) because 'C_p' includes internal energy and external work whereas 'C_v' includes internal energy only.

FIRST LAW OF THERMODYNAMICS :→ (59)

(Law of Conservation of Energy) :-

Statement :- for a closed system undergoing a cycle net heat transfer (ΣQ) is equal to net work transfer (ΣW)

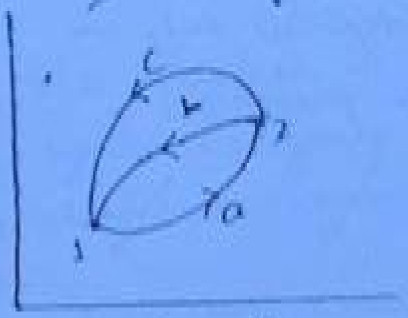
$$\boxed{\Sigma Q = \Sigma W}$$

↳ valid for a cycle...

(Results)

Consequences of first law of Thermodynamics :-

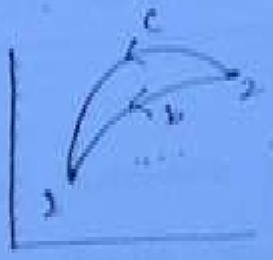
① Heat transfer is a path function.



$$\cancel{(\text{đ}Q)_{1a2}} + (\text{đ}Q)_{2b1} = \cancel{(\text{đ}W)_{1a2}} + (\text{đ}W)_{2b1}$$

$$\underline{\cancel{(\text{đ}Q)_{1a2}} + (\text{đ}Q)_{2c1} = \cancel{(\text{đ}W)_{1a2}} + (\text{đ}W)_{2c1}}$$

$$(\text{đ}Q)_{2b1} - (\text{đ}Q)_{2c1} = (\text{đ}W)_{2b1} - (\text{đ}W)_{2c1}$$



$$(\text{đ}W)_{2b1} \neq (\text{đ}W)_{2c1}$$

$$\therefore (\text{đ}W)_{2b1} - (\text{đ}W)_{2c1} \neq 0.$$

$$\therefore (\text{đ}Q)_{2b1} - (\text{đ}Q)_{2c1} \neq 0.$$

Hence, $(\text{đ}Q)_{2b1} \neq (\text{đ}Q)_{2c1}$

Though for paths 'b' and 'c', end point are same but heat transfer is not same and hence heat transfer is a path function.

* Important points with respect to heat transfer and work transfer :-

A) Both are path functions.

B) Both are not properties. $\boxed{(\text{đ} = \delta)}$

) Both are inexact differentials.

) Both are boundary phenomenon. 38

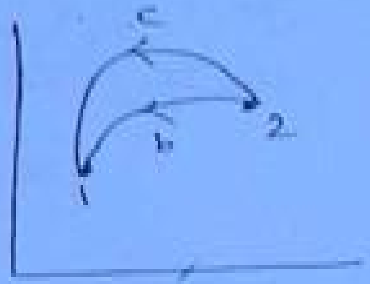
nsequences continued....

- Energy is a property.

$$\text{đ}Q_{2b1} - \text{đ}Q_{2c1} = (\text{đ}W)_{2b1} - (\text{đ}W)_{2c1}$$

$$\text{đ}Q_{2b1} - \text{đ}W_{2b1} = \text{đ}Q_{2c1} - \text{đ}W_{2c1}$$

$$\boxed{(\text{đ}Q - \text{đ}W)_{2b1} = (\text{đ}Q - \text{đ}W)_{2c1}}$$



~~quantity~~ The quantity $(\text{đ}Q - \text{đ}W)$ is same for paths 'b' and 'c' and hence it does not depend on path and it depends only on end points. Therefore, $(\delta Q - \delta W)$ must be a property. and this property is known as energy.

$$(\text{đ}Q - \text{đ}W)_{2b1} = (\text{đ}Q - \text{đ}W)_{2c1} = dE$$

$$(\delta Q - \delta W)_{2b1} = dE$$

$$\boxed{\delta Q = dE + \delta W}$$

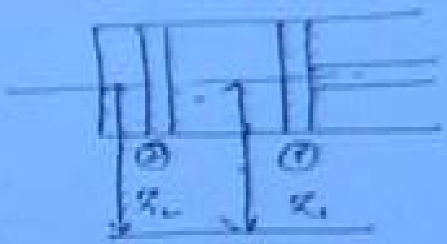
This is the first law of thermodynamics for a closed system undergoing a process (reversible or irreversible).

$\boxed{\text{đ}Q = dE + PdV}$ It is the first law of thermodynamics for a closed system undergoing reversible process because ~~δW~~ 'W = PdV' is valid for a reversible process.

Now,

$$E = \underbrace{K.E + P.E}_{\text{Macroscopic}} + U \leftarrow \text{Microscopic}$$

$$dE = d(KE) + d(PE) + dU.$$



Let the system is initially ~~and~~ first rest, so K.E = 0 and again finally it is also at rest, ∴ KE = 0 again.

The height is also same ∴ change in Potential Energy $d(PE) = 0$.

$$\therefore dE = dU.$$

Now, $dQ = dE + dW$

$$\therefore \boxed{dQ = dU + dW}$$

This equation is first law of thermodynamics equation for a system undergoing a process when kinetic and potential energy changes are neglected.

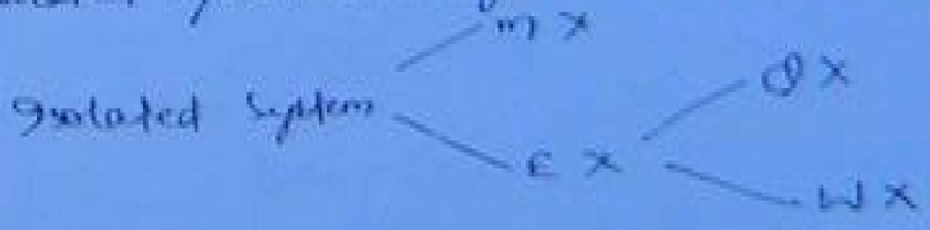
INTERNAL ENERGY (U) :-

It is the energy associated with molecules. It is an extensive property. and it is generally expressed in and it is an intensive

Specific Internal Energy = U/m

It is generally expressed in kJ/kg.

③ Energy of an isolated system is always constant.



* To show that "$(C_p - C_v) = R$" for an ideal gas :-

(Meyer's Eqⁿ):

$$H = U + PV \qquad ①$$

$$dH = dU + d(PV)$$

$$\cancel{m} c_p \cancel{dT} = \cancel{m} c_v \cancel{dT} + d(mRT) \qquad \text{(for an ideal gas)}$$

$$\Rightarrow \cancel{m} c_p \cancel{dT} = \cancel{m} c_v \cancel{dT} + \cancel{m} R \cancel{dT}$$

$$\Rightarrow c_p = c_v + R.$$

$$\text{or} \quad \boxed{C_p - C_v = R} \longrightarrow \text{Meyer's Equation}$$

Now

$$\frac{C_p}{C_v} = \gamma \Rightarrow C_p = \gamma \cdot C_v.$$

$$\therefore \gamma C_v - C_v = R$$

$$\Rightarrow \boxed{C_v = \frac{R}{\gamma - 1}} \text{ --- } ②$$

$$\because C_p = \gamma \cdot C_v$$

$$\therefore \boxed{C_p = \frac{\gamma \cdot R}{\gamma - 1}} \text{ --- } ③.$$

for Air:

$C_p = 1.005$ kJ/kg K
$C_v = 0.718$ kJ/kg K
$R = 0.287$ kJ/kg K
$\gamma = 1.4$.

③ Isothermal Process :-

$$dQ = dU + dW$$

for an ideal gas , $U = f(T)$.

Here, T = constant

$\therefore$ U = constant $\Rightarrow dU = 0$ (4a)

$$\therefore \; dQ_T = 0 + dW$$

$$\Rightarrow \boxed{dQ_T = dW}$$

When an ideal gas undergoes isothermal process, heat transfer is equal to work transfer.

④ Adiabatic Process :-

~~$dQ = dU + dW$~~ $Q = 0$.

There is no heat transfer in adiabatic process.

⑤ Polytropic Process :-

$$dQ = dU + dW$$

for an ideal gas, $dU = mc_v dT$.

& Polytropic work = $dW = \left(\frac{P_1V_1 - P_2V_2}{n-1}\right)$

$$\therefore \; dQ = mc_v dT + \frac{P_1V_1 - P_2V_2}{(n-1)}$$

$$= m.\frac{R}{\gamma-1} dT + \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{mR}{\gamma-1}(T_2 - T_1) + \frac{P_1V_1 - P_2V_2}{n-1}$$

$\because$ $mRT = PV$. $\therefore$ $mRT_2 = P_2V_2$ & $mRT_1 = P_1V_1$

$$\therefore \; dQ = \frac{P_2V_2 - P_1V_1}{\gamma-1} + \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{P_1V_1 - P_2V_2}{n-1} + \frac{P_1V_2 - P_2V_2}{n-1}$$

$$\rightarrow \quad dQ = P_1V_1 - P_2V_2 \left[\frac{1}{n-1} - \frac{1}{\gamma-1}\right].$$

$$= P_1V_1 - P_2V_2 \left[\frac{\gamma-1-(n-1)}{(n-1)(\gamma-1)}\right].$$

$$= \frac{P_1V_1 - P_2V_2}{n-1}\left[\frac{\gamma-n}{\gamma-1}\right].$$

Imp

$$\therefore \boxed{dQ = \left(\frac{\gamma-n}{\gamma-1}\right) W_{polytropic}}$$

VVI

Polytropic Specific heat (C_{poly}) :-

$dQ_v = dU = mC_v dT$ (for ideal gas)

$dQ_v = mC_v dT$ —①

$dQ_p = dH = mC_p dT$ (for ideal gas).

$dQ_p = mC_p dT$ —②

Now for polytropic process:

$$dQ_{poly} = \left(\frac{\gamma-n}{\gamma-1}\right) \times \left(\frac{P_1V_1 - P_2V_2}{n-1}\right).$$

$\because PV = mRT.$

$$\therefore dQ_{poly} = \frac{(\gamma-n)}{(\gamma-1)}\left[\frac{mRT_1 - mRT_2}{n-1}\right].$$

$$= \frac{(\gamma-n)}{(\gamma-1)} \frac{mR}{n-1}(T_1 - T_2)$$

$$= \frac{(\gamma-n)}{(\gamma-1)} \cdot \frac{mR}{(n-1)}(-dT).$$

$\because (dT = T_2 - T_1)$ Also, $\frac{R}{\gamma-1} = C_v$.

$$\therefore dQ_{poly} = \frac{(\gamma-n)}{(n-1)} mC_v(-dT).$$

or $dQ_{poly} = \frac{(n-\gamma)}{(n-1)} \cdot mc_v(dT)$.

Rearranging above equation

$$\boxed{dQ_{poly} = m\left[\frac{n-\gamma}{n-1} C_v\right] dT.}$$

$$\therefore \boxed{C_{poly} = \frac{n-\gamma}{n-1} \cdot C_v.}$$

for '$n>1$' and '$n<\gamma$'; 'C_{poly}' is negative i.e. though heat is supplied, there is a decrease in temperature. This is because in such a polytropic process, work transfer is more than heat transfer and this excess work transfer comes from internal energy of the system and as there is a decrease in internal energy & the temperature also decreases.

** To show that PV^{γ} = constant for adiabatic Process:-

$$dQ = dU + dW$$

for reversible process.

$$dQ = dU + PdV$$

Ideal gas undergoing adiabatic process.

$$dU = mc_v dT \,;\quad dQ = 0$$

$$\therefore 0 = mc_v dT + PdV$$

$$\text{or, } PdV = -mc_v dT \quad \text{—①}$$

Now, $H = U + PV$

$$\rightarrow dH = \underbrace{dU + PdV}_{dQ} + VdP$$

$$\therefore dH = dQ + VdP$$

for ideal gas :-

$dH = mc_p dT$

adiabatic $\Rightarrow dQ = 0$.

$\therefore mc_p dT = 0 + V dP$.

$\Rightarrow mc_p dT = V dP$ —— ②

Dividing eqⁿ ① by eqⁿ ② :-

$$\frac{\cancel{m}c_p \cancel{dT}}{-\cancel{m}c_v \cancel{dT}} = \frac{V dP}{P dV}$$

$$\Rightarrow -\gamma = \frac{V}{dV} \cdot \frac{dP}{P}$$

$$\Rightarrow -\gamma \cdot \frac{dV}{V} = \frac{dP}{P}$$

$$\Rightarrow \frac{dP}{P} + \frac{\gamma dV}{V} = 0$$

$$\Rightarrow \ln P + \gamma \ln V = \ln C$$

$$\Rightarrow \ln P + \ln V^{\gamma} = \ln C$$

$$\Rightarrow \ln PV^{\gamma} = \ln C$$

$$\therefore \boxed{PV^{\gamma} = C}$$

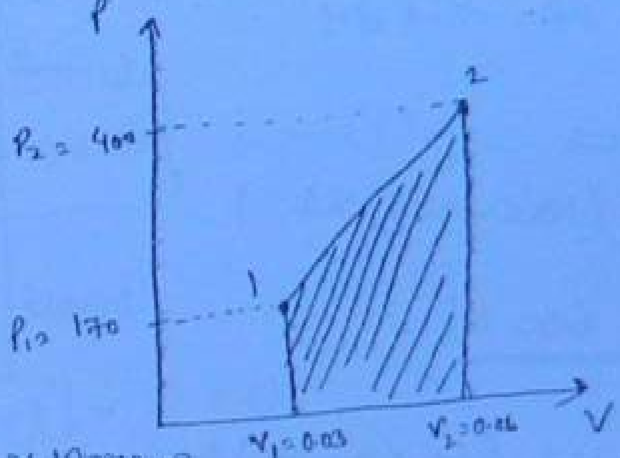
The equation "$PV^{\gamma} = C$" is valid for an ideal gas undergoing reversible adiabatic process.

-: CONVENTIONAL QUESTIONS :-

IAS
40 marks

Q: A fluid is contained in a cylinder by spring loaded friction piston so that pressure in the fluid is a linear function of volume i.e. $P = a + bV$, where 'a' and 'b' are constants. internal energy of the fluid is given by $U = 34 + 3.15\,PV$, wher 'U' is in ~~kilo~~ kj, 'P' in kPa and 'V' in m^3. If the fluid changes from initial state of $P_1 = 170$ kPa, $V_1 = 0.03\ m^3$ to a final state of $P_2 = 400$ kPa, $V_2 = 0.06\ m^3$, find the magnitude and direction of heat transfer and work transfer. [illegible]

Sol^n:



$W =$ Area under curve

$= \frac{1}{2} \times (170 + 400) \times 0.03$

$=$

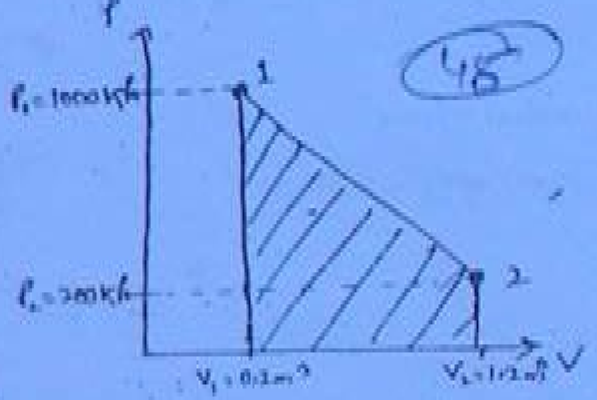
$dW = U_2 - U_1 = 3.15\,P_2V_2 - 3.15\,P_1V_1$

$= 3.15 \times 400 \times 0.06 - 3.15 \times 170 \times 0.03$

$\therefore dU =$

$dQ = dU + dW$

Q. v.3) A gas of mass 1.5 kg undergoes a reversible expansion process which follows the relationship $P = a + bV$, where 'a' and 'b' are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are $V_1 = \overset{0.2}{\cancel{0.02}}$ m³ and $V_2 = 1.2$ m³. The specific internal energy, 'u' = (1.5 Pv − 85) kJ/kg where 'P' is in kPa and small 'v' is m³/kg. Calculate net heat transfer and <u>maximum internal energy</u> of the gas.



P1

$$dQ = dU + dW$$

W = Area

$$= \frac{1}{2}(1000+200) \times 1$$

$$W = 600 \text{ kJ}.$$

Now, $u = (1.5 Pv - 85)$ kJ/kg.

$u = U/m \Rightarrow U = u \times m$

$\therefore U = m[1.5 Pv - 85]$

$= [1.5 Pvm - m.85]$

$v = \frac{V}{m} \Rightarrow V = v.m.$

$\therefore U = (1.5 PV - 1.5 \times 85)$ $\quad \because (m = 1.5 \text{ kg})$

$$\therefore \boxed{U = (1.5 PV - 127.5)}$$

$dU = U_2 - U_1$

Here
$U_2 = 1.5 P_2V_2 - 127.5$
$U_1 = 1.5 P_1V_1 \underset{+}{-} 127.5$

$dU = U_2 - U_1 = 1.5(P_2V_2 - P_1V_1)$

$\therefore\ dU = 1.5[200\times 1.2 - 1000\times 0.2]$

$= 60\ kJ.$

Now $\quad dQ = dU + dW$

$= (60+600)\,kJ$

$\therefore\ \boxed{dQ = 660\,kJ}$ (A)

Now 2nd Part:

$U = 1.5\,PV - 127.5.$ (B)

$\because\ P = a + bV$

$\therefore\ U = 1.5\,(a+bV)V - 127.5$

$\Rightarrow \boxed{U = 1.5\,[aV + bV^2] - 127.5}$ —— ①

for max^m U, $\dfrac{dU}{dV} = 0.$

$\dfrac{dU}{dV} = 1.5\,[a(1) + b(2V)] - 0 = 0$

$\Rightarrow 1.5\,[a + 2bV] = 0.$

$\therefore\ (a + 2bV) = 0$

or, $a = -2bV$ $\quad \Rightarrow \boxed{V = \dfrac{-a}{2b}.}$

$P_1 = a + bV_1 \quad \Rightarrow \quad 1000 = \cancel{a} + b\times 0.2$

$P_2 = a + bV_2 \quad \Rightarrow \quad \underline{\ \ 200 = \cancel{a} + b\times 1.2\ \ }$

$\quad\quad 800 = -b$

or $\boxed{b = -800}$

Again

$200 = a + 1.2b \quad \Rightarrow$ [illegible]

$\Rightarrow \quad a = 200 - 1.2(-800)$

$\therefore\ \boxed{a = 1160}$

Now $V = \frac{-1160}{2\times(-800)} \Rightarrow V = 0.725 m^3$

$$U = 1.5\left[1160 \times 0.725 + (-800)\,0.725^2\right] - 127.5$$

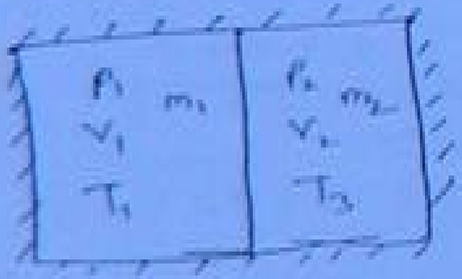
$$\therefore \boxed{U_{max} = 503.25 \text{ KJ}}$$ (Ans) (48)

Q:- 32 An insulated rigid pressure vessel is divided into two portions by a thin partition. First part of vessel is occupied by an ideal gas at a pressure 'P_1', volume V_1 and temperature 'T_1'. The other part is occupied by the same ideal gas but at a pressure 'P_2', volume 'V_2' and temperature 'T_2'. Suddenly the partition is removed and two portions mix with each other. Show that the final pressure P_3 and final temp: 'T_3' are given by

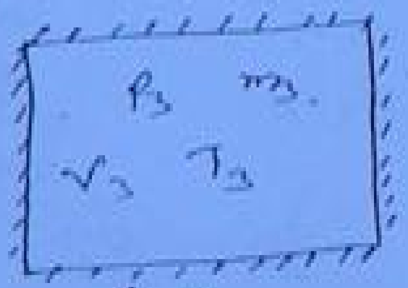
$$P_3 = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$$

$$T_3 = \frac{P_1V_1 + P_2V_2}{\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}}$$

Sol:- $PV = mRT \Rightarrow m = \frac{PV}{RT}$



Initial



final.

$$m_3 = m_1 + m_2$$

$$\frac{P_3V_3}{RT_3} = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}$$

$$\frac{P_3V_3}{T_3} = \left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)$$

$$\Rightarrow T_3 = \frac{P_3V_3}{\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)} \quad \text{——— (I)} \quad (ii)$$

Now $dQ = dU + dW$

for constant volm, closed system, $dW = 0$.

Also, ∵ the system is insulated

∴, $dQ = 0$.

$$\therefore \quad 0 = dU + 0$$

$$\Rightarrow dU = 0$$

$$\Rightarrow U_f - U_i = 0 \quad \Rightarrow U_f = U_i$$

Now $U_3 = U_1 + U_2$

$$m_3 C_v T_3 = m_1 C_v T_1 + m_2 C_v T_2$$

$$\Rightarrow \frac{m_3 R T_3}{\cancel{\gamma-1}} = \frac{m_1 R T_1}{\cancel{\gamma-1}} + \frac{m_2 R T_2}{\cancel{\gamma-1}} \quad \text{——— (II)}$$

$$\Rightarrow P_3V_3 = P_1V_1 + P_2V_2.$$

Also, $V_3 = V_1 + V_2$

$$\therefore P_3(V_1+V_2) = P_1V_1 + P_2V_2$$

$$\Rightarrow \boxed{P_3 = \frac{P_1V_1 + P_2V_2}{V_1+V_2}} \quad \text{proved}$$

$$\& \quad \boxed{T_3 = \frac{P_1V_1 + P_2V_2}{\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)}} \quad \text{proved}$$

from eqn (2)

Q: A fluid system contained in a piston-cylinder machine passes through a complete cycle of four processes. The summation of heat transfer is -340 kJ/cycle. The system completes 200 cycles per minute. Complete the following table and also find the net work transfer in kWatt

Process	Q(kj/min)	W(kj/min)	dU(kJ/min)
1-2	0	4340	-4340
2-3	42000	0	42000
3-4	-4200	69000	-73200
4-1	-105800	-141340	35540.

Soln:

For (Process - 1-2) :-

$$dQ = dU + dW$$

$$0 = dU_{12} + 4340$$

$$\Rightarrow dU_{12} = -4340.$$

For 2-3 (Process):-

$$dQ = dU + dW$$

$$42000 = dU + 0 \Rightarrow dU_{23} = 42000$$

For 3-4 (Process):-

$$dQ = dU + dW$$

$$\Rightarrow -4200 = dW_{34} - 73200$$

$$\Rightarrow dW_{34} = 69000.$$

Now,

$$\Sigma Q = -340 \text{ kj/cycle}.$$

& There are 200 cycles/minute.

$$\therefore \Sigma Q = -340 \times 200 = -68000 \text{ kj/min}.$$

Also $\Sigma Q = Q_{12} + Q_{23} + Q_{34} + Q_{41}$

$\Rightarrow -68000 = 0 + 42800 + (-4200) + Q_{41}$

$\Rightarrow \underline{Q_{41} = -105800.}$

Again $\Sigma Q = \Sigma W = -68000.$

$\Sigma W = W_{12} + W_{23} + W_{34} + W_{41}$

$\Rightarrow W_{41} = -68000 - 4340 - 0 - 69000$

$\therefore \underline{W_{41} = -141340}$

Now for Process (4-1)

$dQ = dU + dW$

$\Rightarrow dU = dQ - dW$

$= -105800 + (-141340)$

$\Rightarrow \underline{dU_{41} = 35540}$

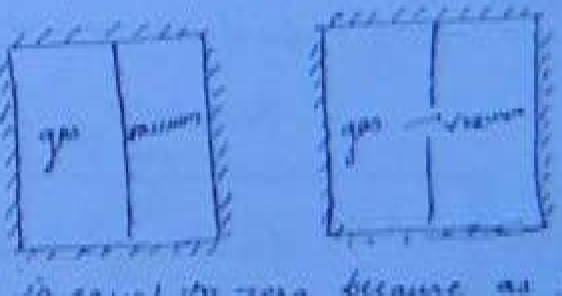
Now $W_{net} = \Sigma W = -68000 \text{ kJ/min}$

$W_{net} = -\frac{68000}{60} \text{ kJ/sec} = \text{kW}.$

$$\boxed{W_{net} = -1133.3 \text{ KW}}$$ (Ans)

THEORY

⇒* **FREE EXPANSION** :→ The expansion of a gas against vacuum is known as free expansion.



Free expansion work is equal to zero because as the gas is expanding against vacuum, there is no resistance offered from surrounding and hence work is equal to zero.

(52)

$dQ = dU + dW$

$0 = dU + 0$

$\Rightarrow dU = 0$

$U_f - U_i = 0 \Rightarrow U_f = U_i$

If an ideal gas undergoes free expansion

$U = f(T)$

$\Rightarrow \boxed{U_i = U_f \Rightarrow T_i = T_f}$

Also,

$H = U + PV$

$H = f(T) + mRT$

$\therefore H = \phi(T)$.

As, $\boxed{T_i = T_f \Rightarrow H_i = H_f}$.

Note:- When an ideal gas undergoes free expansion :-

(i) $U_i = U_f$

(ii) $T_i = T_f$

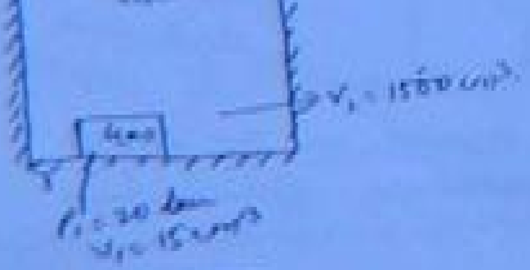
(iii) $H_i = H_f$.

PROBLEM

Q4. An ideal gas at 20 bar and 40°C is contained in a small cylinder having a volume of 15 cm³. This cylinder is placed inside a large container having a volume of 1500 cm³. The large container is perfectly insulated and evacuated. By an appropriate means, the gas is allowed to discharge and fill the large container. Find the final pressure after the entire assembly reaches the equilibrium.

$T_1 = T_2$ (for free expansion).

Sol.



$PV = mRT$

$m = \frac{PV}{RT}$

from mass conservation

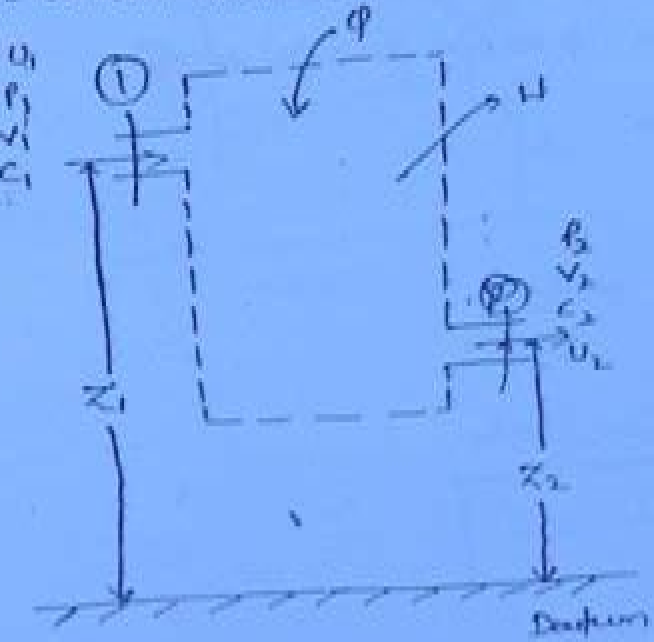
$m_1 = m_2$

STEADY FLOW ENERGY EQUATION (SFEE) :-

A flow is said to be steady flow if properties do not vary with respect to time at any given section.

For steady flow, there is no accumulation of mass and energy in the control volume i.e. mass entering is equal to mass leaving and energy entering = energy leaving the control volume.

(53)



$W = W_{\text{entry flow}} + W_{CV} + W_{\text{exit flow}}$

$W = -P_1V_1 + W_{CV} + P_2V_2$

For steady flow, $E_1 = E_2$.

$$\frac{1}{2}mC_1^2 + mgZ_1 + U_1 + Q = \frac{1}{2}mC_2^2 + mgZ_2 + U_2 + W$$

$$\Rightarrow \frac{1}{2}mC_1^2 + mgZ_1 + U_1 + Q = \frac{1}{2}mC_2^2 + mgZ_2 + U_2 + (-P_1V_1) + W_{CV} + P_2V_2$$

$$\frac{1}{2}mC_1^2 + mgZ_1 + \underbrace{U_1 + P_1V_1} + Q = \frac{1}{2}mC_2^2 + mgZ_2 + \underbrace{U_2 + P_2V_2} + W_{CV}$$

$$H_1 + \frac{1}{2}mC_1^2 + mgZ_1 + Q = \frac{1}{2}mC_2^2 + H_2 + mgZ_2 + W_{CV}$$

· Dividing the above equation by 'm' :-

$$\boxed{h_1 + \frac{C_1^2}{2} + gZ_1 + q = h_2 + \frac{C_2^2}{2} + gZ_2 + w_{CV}}$$

→ 1st law of thermodynamics for steady flow open system

Here, C_1 and C_2 are velocities at 1 and 2 respectively.

$$\frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2} \Rightarrow P_1V_1 = P_2V_2$$

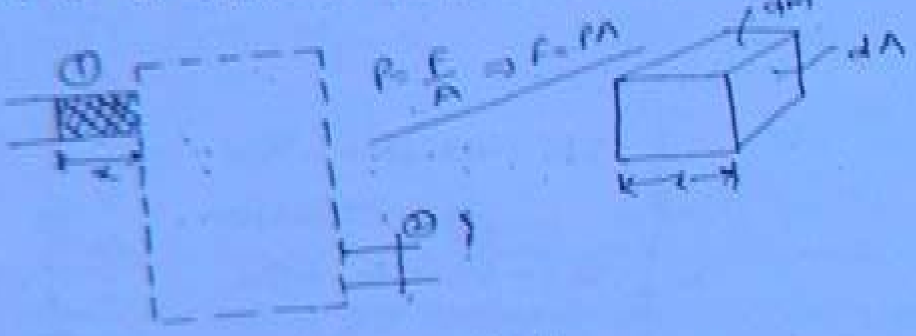
$$\Rightarrow 20 \times 15 = 1500 \times V_2$$

$$\therefore \boxed{V_2 = 0.2\ \text{ltr}}$$ (Ans)

—: FLOW WORK :—

5.4

It is the work transfer involved in causing the fluid element either to enter or leave the control volume is known as flow work.



$$\text{Work} = F \times x$$
$$= P\,\underbrace{dA \times x}$$
$$\text{Work} = P\,dV$$

$$dV = dA \times x$$
$$v = \frac{dV}{dm}$$

$$\text{Work} = P\,dV$$

$$\frac{\text{work}}{\text{mass}} = \frac{P\,dV}{dm}$$

$$\frac{\text{work}}{\text{mass}} = Pv$$

$$v = V/m$$

$$\text{Total Work} = P\,\underline{v\,m}$$

$$\therefore \underline{V = v\,m}$$

$$\boxed{\text{Total work} = PV}$$

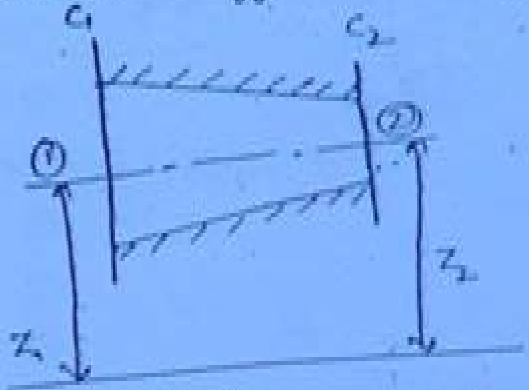
If the entry occurs at '1', then the entry flow work = $(-P_1V_1)$

If the exit occurs at '2', then the exit flow work = $(+P_2V_2)$.

This is the first law of thermodynamics equation for open system under steady flow conditions.

Special Cases :-

① **Nozzle** :- Nozzle is a device which is used for increasing kinetic energy at the expense of pressure energy.



Nozzle

$$h_1 + \frac{C_1^2}{2} + \cancel{z_1 g} + \overset{0}{\cancel{q}} = h_2 + \frac{C_2^2}{2} + \cancel{z_2 g} + \cancel{w}$$

∵ Insulated, $q = 0$.

for a nozzle, $w_{cv} = 0$.

Also, $z_1 = z_2$ ∴ P.E are same

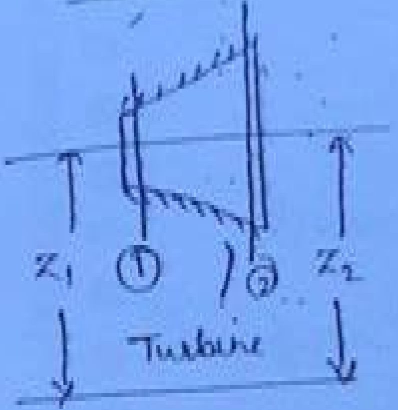
; Thus,

$$\boxed{h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}}$$

when $C_1 << C_2$, then

$$\boxed{h_1 = h_2 + \frac{C_2^2}{2}}$$

② **Turbine** :-



$$h_1 + \cancel{\frac{C_1^2}{2}} + \cancel{z_1 g} + \overset{0}{\cancel{q}} = h_2 + \cancel{\frac{C_2^2}{2}} + \cancel{z_2 g} \pm w_{cv}$$

Assumptions:

① Steady flow

② Perfectly insulated, so ∴ $q = 0$.

③ Neglecting P.E changes

④ Neglecting K.E changes.

$$h_1 = h_2 + w_{cv}$$

or, $\boxed{w_{Turbine} = h_1 - h_2}$

③ **Compressor :-**

Compressor work is always negative, Since work is done on the system.

$\boxed{W_{comp} = h_2 - h_1}$ (Assumptions are same as in case of turbine).

$W_{Turbine} = h_1 - h_2$

$W_{comp} = -W_{Turbine} = -(h_1 - h_2)$.

$\therefore \boxed{W_{comp} = h_2 - h_1}$

④ **Throttling valve :-**

Examples of throttling :-

(i) flow through a partially opened valve.

(ii) flow through a very small opening (orifice).

(iii) flow through a porous plug.

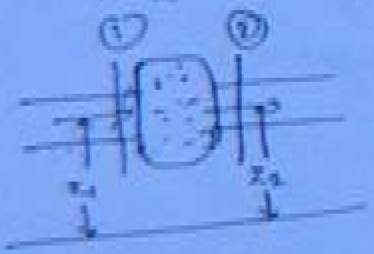
Characteristics of throttling :-

(i) No heat transfer.

(ii) No work transfer.

(iii) Highly irreversible process.

(iv) Enthalpy remains constant. (It is isenthalpic process).



$$h_1 + \frac{C_1^2}{2} + \cancel{z_1 g} + \cancel{q}^0 = h_2 + \frac{C_2^2}{2} + \cancel{z_2 g} + \cancel{W_{cv}}^0$$

$$h_1 + \cancel{\frac{C_1^2}{2}} = h_2 + \cancel{\frac{C_2^2}{2}}$$

In comparision to enthalpy values, K.E changes are negligible.

$\therefore \boxed{h_1 = h_2}$

Ideal Gas ($PV = mRT$)

Perfect Gas

↓

C_p & C_v are constants

Semi-Perfect Gas

↓

C_p & C_v vary w.r.t tem.

No. of moles $(n) = \dfrac{\text{mass } (m)}{\text{Molecular wt. } (M)}$

$$n = \frac{m}{M}$$

or $m = nM$

∴ Ideal gas equation $PV = mRT$

$$= n\underbrace{MR}T$$

$MR = \bar{R}$ (universal gas constant).

$$\boxed{PV = n\bar{R}T}$$

$\bar{R} = 8.314$ kJ/kg K.

R = characteristic gas constant.

Isothermal work $= mRT \ln \dfrac{V_2}{V_1}$

$$= \left(n\bar{R}T \ln \frac{V_2}{V_1}\right).$$

$$\boxed{R = \frac{\bar{R}}{M} = \frac{8.314}{M}}$$

Work Book Chapter-2

① — ⓐ
② — ⓒ
③ — ⓐ
④ — ⓒ
⑤ — ⓒ
⑥ — ⓐ
⑦ — ⓐ
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→ (Assignment)
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㊴ — ⓒ
㊵ — ⓐ
㊶ — ⓓ

① $T = C$, $n = 70$, $T = 354K$, $V_2 = 1m^3$, $W = -206$ kj

(57)

$$W = n\bar{R}T \ln \frac{V_2}{V_1}$$

$$\Rightarrow -206 = 70 \times 8.314 \times 354 \ln\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow \ln\left(\frac{V_2}{V_1}\right) = \frac{-206}{70 \times 8.314 \times 354}$$

$$\Rightarrow \left(\frac{V_2}{V_1}\right) = e^{(9.9989\,\text{[illegible]}5 \times 10^{-4})} = 1.001 \simeq 1.$$

$$\therefore \underline{\left(\frac{V_2}{V_1}\right) = 1} \quad \therefore \underline{V_2 = V_1}$$

② ~~[illegible]~~ $\underline{dQ_f = dU}$

$P_i = P_0$ and $T_i = T_0$

$T =$ Const (Isothermal Process).

$P_f = P_1$

$PV = \cancel{nRT}$

$PV = C$

$P_i V_i = P_f V_f$

$\underline{\frac{V_f}{V_i}} = \frac{P_i}{P_f}$

$W = Q = mRT \ln \frac{V_f}{V_i}$

$W = Q = mRT \ln \frac{P_i}{P_f}$

$Q = 1 \times R \times T_0 \ln\left(\frac{P_i}{P_f}\right)$

$Q = RT_0 \ln\left(\frac{P_0}{P_1}\right)$

$Q = -RT_0 \ln\left(\frac{P_1}{P_0}\right)$

$= -60 J/s \quad = \underline{}$

③ $W = -60W. \quad = \cancel{(60\times60) = 3600} \,]$

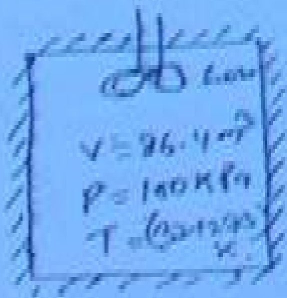
$V = 86.4 m^3.$

$T_1 = 32°C, \quad T_2 = ?$

$C_v = 0.718, \quad R = 0.287 \ kJ/kg\,K.$

$\underline{P_1 = 100 \ kPa.}$

$T_1 = (273+32)\,K = 305K.$



$$W = -60 \ J/s = -\frac{60\times60\times60\times4}{1000} \ kJ$$

$$= \underline{\underline{-864 \ kJ}}$$

Air can be treated as an ideal gas.

$\therefore \ dU = mc_v dT.$

$dQ = dU + dW.$

$\because$ Insulated. $\therefore \ dQ = 0.$

$\therefore \ 0 = mc_v dT + (-864)$

$\Rightarrow \cancel{dT} \ mc_v dT = 864$

$\Rightarrow \ dT = \frac{864}{mc_v}$ — ①

Now for ideal gas $PV = mRT$

$\Rightarrow m = \frac{PV}{RT} = \frac{100\times86.4}{0.287\times305}$

$\therefore \ \underline{m = 98.7 \ kg}$

$\therefore \ dT = \frac{864}{98.7\times0.718} = 12.19 \simeq \underline{12°C.}$

⑦ $dQ = dU + dW$

$\underline{T_1 = 15°C, \ T_2 = 45°C, \ P = 100 \ kPa.}$

for constant volume process, $V = C$

$dQ_v = dU = mc_v dT.$

$= 1\times3.5107\times30 = \underline{105.32} \ kJ$

$\frac{C_p}{C_v} = k$

$\Rightarrow C_v = \frac{C_p}{k} = \frac{1.005}{0.2868}$

$\underline{= 3.570}$

④ Area of cycle = Area enclosed by the curve:-

1st case Area of $\Delta = \frac{1}{2} bh$

$= \frac{1}{2} \times 8 \times 3 = \underline{12}$

2nd case, Area $= \frac{1}{2} \times 6 \times 4 = 12$

but for clockwise dir^n ~~area~~ work is ⊕ve
and for anticlock wise dir^n work is ⊖ve,

⑦ A-2, B-4, C-1, D-3.

$m = 1$ kg, $T_1 = 15°C$, $P = 100$ kPa, $T_2 = 45°C$

$C_p = 1.042$ kJ/kg-K. $R = 0.2968$

$dU = mC_v dT$

$= 1 \times 0.7452 \times 30$

$=$

$(C_p - C_v) = R$

$= 1.042 - 0.2968$

$= 0.7452$

$(C_p - C_v) 6 = R$

$(5.19 - C_v) = \frac{8.314}{4}$

$\Rightarrow C_v = (5.19 - 2.0785) = \underline{3.11}$

$dQ = dU + dW$

$dW = (-180 \text{ kJ})$

$dW = \frac{100 \times 60 \times 30}{1000} = \underline{-180 \text{ kJ}}$

$dU = mC_v dT$

$= 30 \times 0.7 \times dT$

$dU = -(dW)$

$\Rightarrow 30 \times 0.7 \times dT = +180$

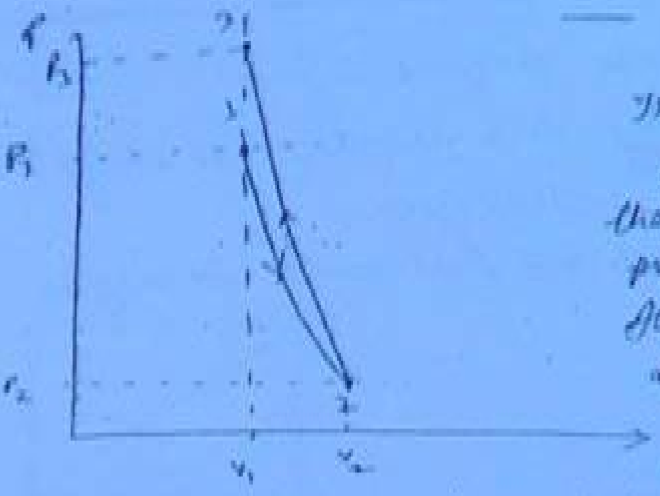
$\Rightarrow dT = \underline{+8.57}$

$T_2 = T_1 + dT$

$= 21 + 8.57$

$= \underline{29.57°C}$

(16)



The slope of adiabatic process is always greater than the slope of isothermal process.
Also expansion work is (+)ve and compression work is (−)ve
∵ Compression work here is greater

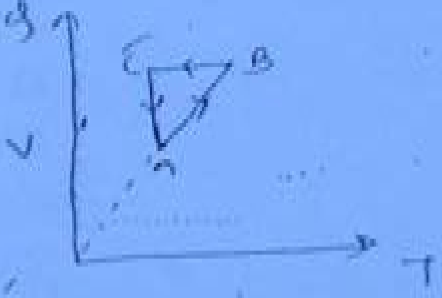
(Gd)

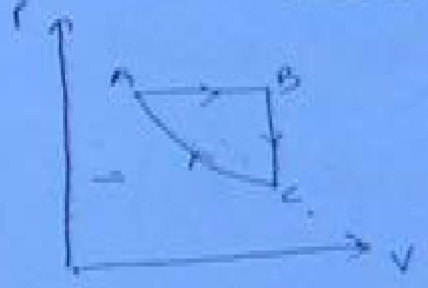
∴ W < 0. (∵ W is (−)ve)

& $P_3 > P_1$.

(17) The total work done by the gas is $(300 \times 0.01) = 3$ kj
Out of this total work, $(100 \times 0.01) = 1$ kj is used in displacing atmospheric air.
Therefore the work that could be utilised $= (3-1) = \underline{2 \text{ kj}}$

(19)





$PV = mRT$

$V = \frac{mR}{P} \cdot T$

$y = m \cdot x$

$m = \frac{mR}{P} = \text{const}$

m is const if P is constant ∴ $AB \rightarrow \underline{P = C}$.

$BC \rightarrow V = C$

$CA \rightarrow T = C$

$AB \rightarrow P = C$.

(20) For isothermal process of an ideal gas open system work is equal to closed system work because a rectangular hyperbola is symmetric about x and y axis and hence it gives same areas.

(21) Change in property for any cycle is equal to zero.

Since internal energy is a property, so change in internal energy for a cycle will be zero.

As the ~~start~~ initial and final point in a cycle are same.

3) –(?)

) W = 300 W = $\frac{300\times60\times15}{1000}$ = 5700 kJ

(Q2)

Q = 0.

(25) Assignment

$P_1 = 10$ bar. $V_1 = 1\,m^3$, $T_1 = 300\,K$.

VdW = $V_2 = 2\,m^3$.

$$\left(P + \frac{a}{V^2}\right)V = RT = C$$

$$\left(P_1 + \frac{a}{V_1^2}\right)V_1 = \left(P_2 + \frac{a}{V_2^2}\right)V_2$$

$$\Rightarrow \left(10 + \frac{a}{1}\right)\times 1 = \left(P_2 + \frac{a}{2^2}\right)2$$

$$= (10 + a) = \frac{4P_2 + a}{2}$$

$$\Rightarrow 20 + 2a = 4P_2 + a$$

$$\Rightarrow a = 4P_2 - 20.$$

$$\Rightarrow P_2 = \frac{a + 20}{4} = \frac{a}{4} + 5.$$

$$\boxed{P_2 = \frac{a}{4} + 5}$$

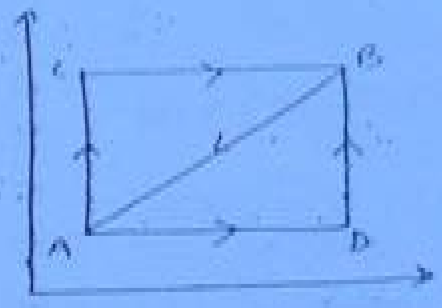
∵ a > 0 and also its value is slightly greater than 0, ∴ $\frac{a}{4}$ will be very less.

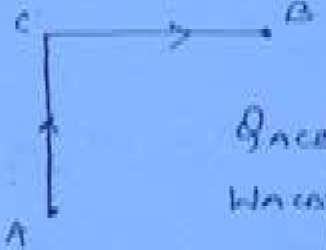
∴ P_2 will be slightly more than 5 bar.

(57) E - G - I - K - N
F - H - J - K - M

Boundary and transient both are same and can be used for both heat and work

(58)





$Q_{ACB} = 180$

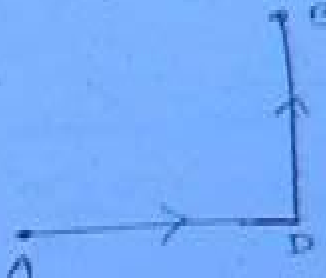
$W_{ACB} = 130$

$dQ_{ACB} = dU_{ACB} + dW_{ACB}$

$\Rightarrow 180 = (U_B - U_A) + 130$

$\therefore \boxed{(U_B - U_A) = 50}$ —①

(58)



$Q_{ADB} = ?$

$W_{ADB} = 40\,kJ$

$dQ_{ADB} = dU_{ADB} + dW_{ADB}$

$dQ_{ADB} = 50 + 40$

$\boxed{dQ_{ADB} = 90\,kJ}$ (A)

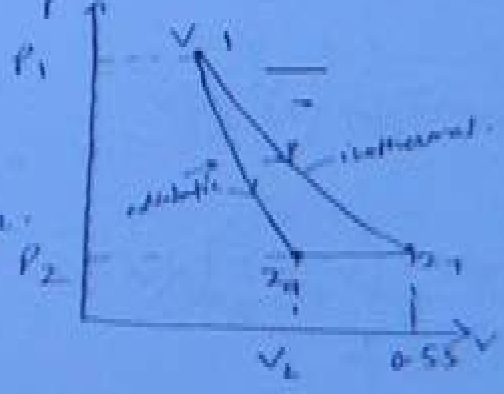
(29) ~~$P_1V_1 = P_2V_2$~~

~~$P_1V_1 = \frac{P_1}{10} \times 0.55$~~

~~$V_1 = 0.055\ m^3$~~

~~for adiabatic $P_1V_1^{\gamma} = P_2V_2^{\gamma}$~~

~~$\Rightarrow P_1 \times (0.055) = \frac{P_1}{10} \times (V_2)^{1.4}$~~



All values are greater than 0.55 but only value 0.46 is lesser. //

⑤

$T_1 = 293\,K$
$P_1 = 100\,kPa$
$V_1 = 15\,m^3$

$T_3 = ?$
$P_3 = 150$
$V_3 = 25\,m^3$

$PV = mRT$

$m = \frac{PV}{RT}$

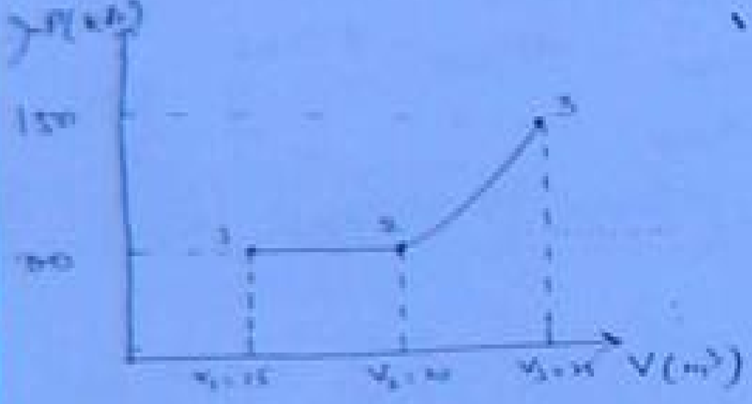
(G4)

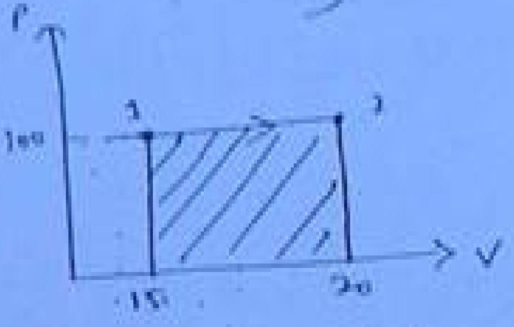
$m_1 = m_3$

$\therefore \frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2}$

$\Rightarrow T_2 = \frac{150 \times 25 \times 293}{100 \times 15}$

$= 732.5 = \underline{733\,K}$





$P = P_0 + 2(V - V_0)^2$

$W_{12} = P(V_2 - V_1)$

$= 100(20 - 15)$

$= \underline{500\,kJ}$

Again

$W = \int P\,dV$

$$W_{23} = \int_{V_2}^{V_3} \left[P_0 + 2(V - V_0)^2\right] dV$$

$$= \int_{V_2}^{V_3} \left[100 + 2(V - 20)^2\right] dV$$

$$W_{23} = \int_{20}^{25} 100\, dV + 2(V-20)^2\, dV$$

$\boxed{1\ \text{bar} = 100\ \text{kPa}}$

$$= \left[100V + \frac{2}{3}(V-20)^3 \right]_{20}^{25}$$

$$= 100 \times (25-20) + \frac{2}{3}\left[(25-20)^3 - (20-20)^3\right]$$

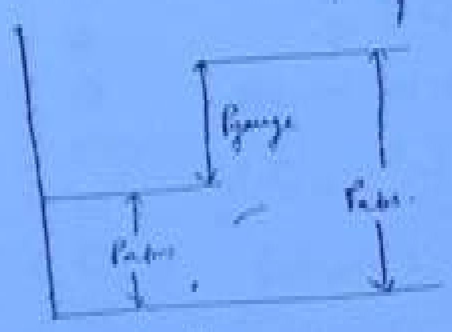
$$= 500 + \frac{2}{3}(5)^3 = 500 + 83 = 583\ \text{kJ}.$$

65

$$\therefore W_{total} = W_{12} + W_{23}$$

$$= (500 + 583)\ \text{kJ} = \underline{\underline{1083\ \text{kJ}}}$$

(38) In the equation $PV = mRT$, 'P' is in absolute scale.



$P_1 = 1$ bar (gauge)
$T_1 = 288$ K.

$P_{abs} = P_g + P_{atm}$

$P_{abs} = P_g + P_{atm}$

$P_1 = 1 + 1.013$

$P_1 = 2.013$ bar

(∵ 1 bar = 100k

$$PV = mRT$$

$$\Rightarrow m = \frac{PV}{RT} = \frac{2.013 \times 100 \times 2500}{0.287 \times 288 \times 10^{-6}}$$

$$\Rightarrow m = 6.08 \times 10^{-3}\ \text{kg}$$

for a constant volume process

$$dQ_v = dU$$

$$dQ_v = mc_v dT = mc_v(T_2 - T_1)$$

$$= 6.08 \times 10^{-3} \times 0.718 \times (278 - 288)$$

$$\Rightarrow dQ_v = -43.7 \times 10^{-3}\ \text{kJ} = -43.7\ \text{joule}$$

(Negative sign indicates rejection of heat).

$P_1 = 2.013$
$T_1 = 288$

$P_2 = ?$
$T_2 = 278$

$PV = mRT$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

(66)

$$\Rightarrow P_2 = \frac{T_2 \times P_1}{T_1} = \frac{278 \times 2.013}{288}$$

$\therefore P_2 = 1.943$ bar.

$\therefore (P_2)_{gauge} = P_2 - P_{atm}$
$= 1.943 - 1.013$
$\therefore (P_2)_{gauge} = \underline{0.93 \text{ bar}}$ (Ans).

⑤

$P_1 = ?$
$T_1 = 288$

$P_2 = 1$ bar (gauge)
$T_2 = 278$

P_2 abs $= 1 + 1.013 = \underline{2.013 \text{ bar}}$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\rightarrow P_1 = \frac{2.013 \times 288}{278} = 2.085 \text{ bar}$$

$(P_1)_{gauge} = (P_1)_{abs} - P_{atm}$
$= 2.085 - 1.013$

$\therefore (P_1)_{gauge} = \underline{1.07 \text{ bar}}$ (gauge).

(39) $W_{2-3} = P(V_3 - V_2)$

(40)

Process 1→2

$P_1V_1 = P_2V_2$

$\Rightarrow 1 \times 1.6 = P_2 \times 0.2$

$\Rightarrow P_2 = \frac{1.6}{0.2} = 8\ \text{bar}$

$W_{2-3} = 800\,(1.6 - 0.2) = \underline{1120\ \text{kJ}}$

(41) Heat transfer for constant ~~process~~ pressure process.

~~$dQ_p = dH$~~

~~$= d(U + PV)$~~

$dQ_p = dU + dW$

$dQ_{2-3} = dU_{23} + dW_{23}$ → 1120

∵ Process 1-2 is an isothermal process.

∴ $U_1 = U_2$. (∵ Change in internal energy for isothermal process is 0).

$dU = U_3 - U_2$

$U_3 - U_2 = U_3 - U_1 = 3549\ \text{kJ}$

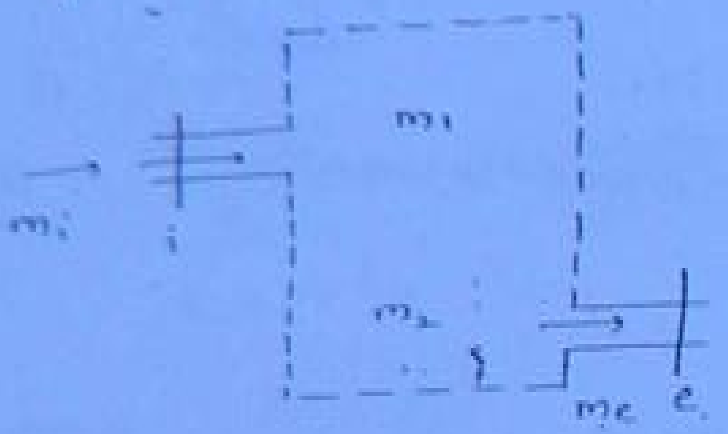
$dQ_{23} = (3549 + 1120)\ \text{kJ}$

$= \underline{4669\ \text{kJ}}$

← UNSTEADY STATE FLOW →

Let 'm_i' and 'm_e' be the masses entering and leaving the control volume.

Let 'm_1' and 'm_2' be the masses in the control volume initially and finally respectively.



$m \rightarrow kg$

$\dot{m} = \frac{m}{t} = \frac{kg}{sec}$.

<u>conservation of mass:-</u>

$$\left(\frac{dm}{dt}\right)_{cv} = \frac{dm_i}{dt} - \frac{dm_e}{dt}$$

$$\boxed{\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e} \text{ ——— } ①$$

<u>conservation of Energy:-</u>

energy per unit mass = $h_i + \frac{C_i^2}{2} + Zg + q$.

Total inlet energy, $E_i = m_i h_i + \frac{1}{2} m_i C_i^2 + m_i g z_i + Q$.

Ily, Total energy leaving,

$$E_e = m_e h_e + \frac{1}{2} m_e C_e^2 + m_e g z_e + W_{cv}$$

Now

$$\left(\frac{dE}{dt}\right)_{cv} = \frac{dE_i}{dt} - \frac{dE_e}{dt}$$

$$\left(\frac{dE}{dt}\right)_{cv} = \frac{d}{dt}\left(m_i h_i + \frac{1}{2} m_i c_i^2 + m_i g z_i + Q\right) - \frac{d}{dt}\left(m_e h_e + \frac{1}{2} m_e c_e^2 + m_e g z_e + W_{cv}\right).$$

Neglecting K.E & P.E changes, we have

$$\left(\frac{dU}{dt}\right)_{cv} = \frac{d}{dt}\left(m_i h_i + Q\right) - \frac{d}{dt}\left(m_e h_e + W_{cv}\right)$$

$$\left(\frac{dU}{dt}\right)_{cv} = \frac{d}{dt} m_i h_i + \dot{Q} - \frac{d}{dt}(m_e h_e) + \dot{W}_{cv} \quad \text{—(1)}$$

$\Big[\because$ $E = KE + PE + U$
$dE = d(KE) + d(PE) + dU$
If $d(KE)$ & $d(PE)$ are neglected, then
$dE = dU\Big]$

Let h_i & h_e are assumed to be constant with respect to time, then

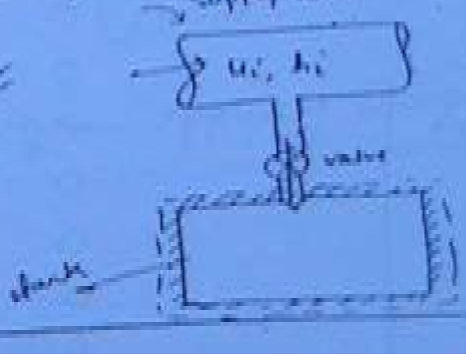
$$\left(\frac{dU}{dt}\right)_{cv} = h_i \frac{dm_i}{dt} + \dot{Q} - h_e \frac{dm_e}{dt} - \dot{W}_{cv}$$

or, $$\boxed{\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv}} \quad \text{—(2)}$$

PROBLEM

Q: An insulated storage tank that is initially evacuated is connected to a supply pipeline carrying a fluid at specific internal energy 'u_i' and specific enthalpy 'h_i'. The valve is opened and fluid flows into the tank from the supply line and reaches the pressure same as that of supply pipe line. Show that the final specific internal energy of the fluid in the tank is equal to 'h_i'.

Soln:



$m_1 = 0$ (evacuated)

Applying mass conservation :-

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e$$

$\dot{m}_e = 0$ (because no mass is leaving C.V)

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i \quad \text{——} \quad ①$$

70

Now Energy conservation :

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i + \underbrace{\cancelto{0}{\dot{Q}}}_{(\text{insulated})} - \underbrace{\cancelto{0}{\dot{m}_e h_e}}_{(\because \dot{m}_e = 0)} - \underbrace{\cancelto{0}{\dot{W}}}_{(\text{No work done})}.$$

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i \quad \text{——} \quad ②$$

$$\left(\frac{dU}{dt}\right)_{cv} = h_i \left(\frac{dm}{dt}\right)_{cv}$$

Integrating!, we have.

$$(dU)_{cv} = h_i \, (dm)_{cv}$$

$$U_2 - U_1 = h_i (m_2 - m_1)$$

$$m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

$$\because m_1 = 0$$

$$\therefore \cancel{m_2} u_2 = h_i (\cancel{m_2})$$

$$\therefore \boxed{u_2 = h_i}$$ proved.

Note :- If the fluid flowing is an ideal gas,

$$u_2 = h_i$$

$$C_v T_2 = C_p T_i$$

$$T_2 = \frac{c_p}{c_v} T_i$$

$$\therefore \boxed{T_2 = \gamma T_i}$$

Therefore 'T_2' will always be greater than $\underline{T_i}$.

$U = mC_v dT$

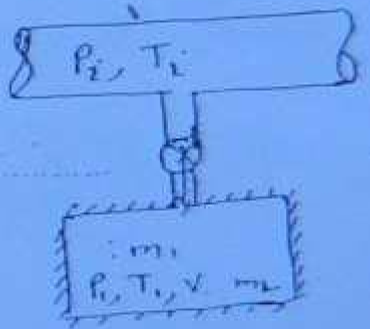
$u = \underline{C_v T}$

$h = C_p T$

ESE-2011

Q. The Pressure-cylinder of volume 'V' contains air at a pressure 'P_1' and temperature 'T_1'. It is to be filled from a supply pipeline maintained at a constant pressure 'P_i' and temperature 'T_i'. Show that the final temp^n of air in the cylinder after it has been charged to the pressure same as the supply line pressure is given by $T_2 = \dfrac{\gamma T_i}{1 + \frac{P_1}{P_i}\left(\frac{\gamma T_i}{T_1} - 1\right)}$. The tank is insulated.

Sol:- Applying mass conservation:



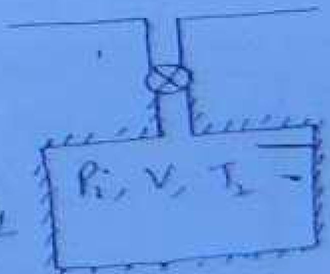
Initially

$$\left(\frac{dm}{dt}\right)_{CV} = \dot{m}_i - \dot{m}_e$$

$PV = mRT$

$m = \dfrac{PV}{RT}$ $\therefore$ $m_2 = \dfrac{P_i V}{RT_2}$ & $m_1 = \dfrac{P_1 V}{RT_1}$

$(\dot{m}_e) = 0$ (because no mass is leaving control volume).



finally

$$\therefore \left(\frac{dm}{dt}\right)_{CV} = \dot{m}_i \quad \text{——} ①$$

Again, Applying Energy conservation:

(P.T.O)

Finally we get; ϕ

$$U_2 - U_1 = (m_2 - m_1) h_i$$ (as from last problem).

$$m_2 u_2 - m_1 u_1 = (m_2 - m_1) h_i$$

$$\Rightarrow m_2 C_v T_2 - m_1 C_v T_1 = (m_2 - m_1) C_p T_i$$

$$\Rightarrow C_v (m_2 T_2 - m_1 T_1) = C_p T_i (m_2 - m_1).$$

$$\Rightarrow C_v \left(\frac{P_i V}{R} - \frac{P_1 V}{R} \right) = C_p T_i \left(\frac{P_i V}{R T_2} - \frac{P_1 V}{R T_1} \right)$$

$$\Rightarrow \frac{P_i V}{R} - \frac{P_1 V}{R} = \frac{C_p}{C_v} T_i \left(\frac{P_i V}{R T_2} - \frac{P_1 V}{R T_1} \right)$$

$$\Rightarrow (P_i - P_1) = \gamma T_i \left(\frac{P_i}{T_2} - \frac{P_1}{T_1} \right).$$

$$\Rightarrow (P_i - P_1) = \frac{\gamma T_i P_i}{T_2} - \frac{\gamma T_i P_1}{T_1}$$

$$\Rightarrow \cancel{P_i + P} \quad P_i - P + \frac{\gamma T_i P_1}{T_1} = \frac{\gamma T_i P_i}{T_2}$$

$$\Rightarrow T_2 = \frac{\gamma T_i P_i}{P_i - P_1 + \frac{\gamma T_i P_1}{T_1}}$$

$$T_2 = \frac{\gamma T_i P_i}{P_i \left[1 - \frac{P_1}{P_i} + \frac{\gamma T_i P_1}{P_i T_1} \right]}$$

$$\therefore \boxed{T_2 = \frac{\gamma T_i}{1 + \frac{P_1}{P_i} \left(\frac{\gamma T_i}{T_1} - 1 \right)}}$$ (Proved).

UNIT-III

← : SECOND LAW OF THERMODYNAMICS : →

(Directional Law) ↦ (Concept of entropy)

first law of thermodynamics simply says that energy is conserved. It does not give any direction for a particular process. It is the second law of thermodynamics which gives direction for a particular process through the concept of entropy - and hence second law is known as directional law. (73)

Work is known as high grade energy. **Heat** is known as low grade energy.

It is found that, for a cycle, complete conversion of low grade energy (heat) into high grade energy (work) is impossible.

Thermal Energy Reservoirs (TER) :-

Source :- Source is a ~~res~~ reservoir which supplies thermal energy without undergoing any temperature change.

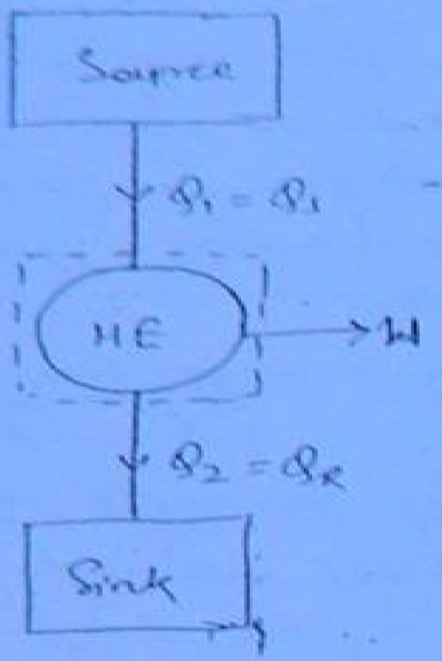
Sink :- It is a reservoir which absorbs thermal energy without undergoing any temperature change.

Statements of 2nd law of thermodynamics :-

Ist > KELVIN - PLANK STATEMENT :-

It is impossible to develop a device operating on a cycle which produces work while exchanging heat with a single reservoir. This device is known as PMM-II and the efficiency of PMM-II is 100% and therefore 100% efficiency is impossible from second law of thermodynamics.

* CONCEPT OF HEAT ENGINE :-



$$Q_1 = W + Q_2$$

$$W = Q_1 - Q_2$$

$$\eta = \frac{O}{I} = \frac{W}{Q_1}$$ (74)

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\therefore \boxed{\eta = 1 - \frac{Q_R}{Q_S}} \text{ --- } ①$$

Heat Engine is a device which converts part of heat into work and rejects remaining to sink or surrounding.

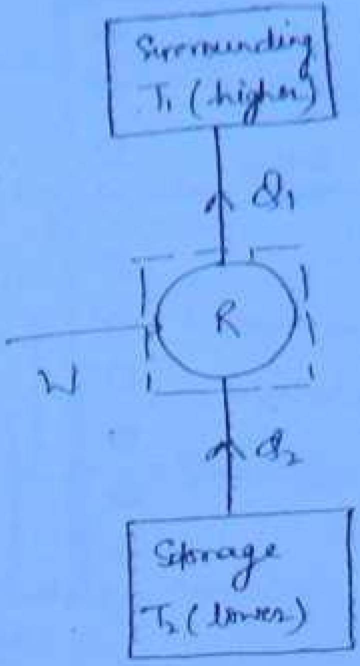
Equation ① is valid both for reversible and irreversible cycle.

2:- CLAUSIUS STATEMENT:-

It is impossible to transfer heat from lower temperature to the higher temperature without any external input.

* REFRIGERATOR:-

A refrigerator is a device which maintains lower temperature compared to surrounding. As lower temperatures are to be maintained continuously, refrigerators must operate on a cycle.



$$COP = \frac{\text{Desired Effect}}{\text{Energy Input}}$$

$$(COP)_R = \frac{Q_2}{W}$$

$$\boxed{(COP)_R = \frac{Q_2}{Q_1 - Q_2}} \quad \text{—②}$$

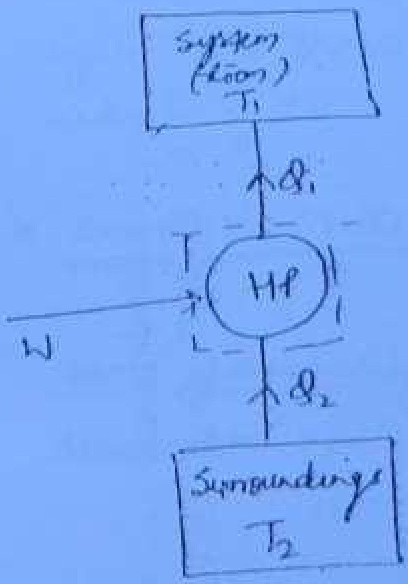
(75)

$\because W + Q_2 = Q_1$

$\therefore W = Q_1 - Q_2$

Equation ② is valid for reversible and irreversible refrigerators.

* HEAT PUMP :- Heat pump is a device which maintains higher temperature compared to surrounding.



$$(C.O.P)_{HP} = \frac{Q_1}{W}$$

$W + Q_2 = Q_1$

$W = Q_1 - Q_2.$

$$\boxed{(COP) = \frac{Q_1}{Q_1 - Q_2}} \quad \text{—③}$$

This equation is valid for reversible as well as irreversible heat pump.

23.a 4.3) RELATIONSHIP BETWEEN COP OF A HEAT PUMP AND COP OF A REFRIGERATOR OPERATING BETWEEN SAME TEMPERATURES :-

$$(COP)_{HP} = \frac{Q_1}{Q_1 - Q_2}$$

96

$$(COP)_R = \frac{Q_2}{Q_1 - Q_2}$$

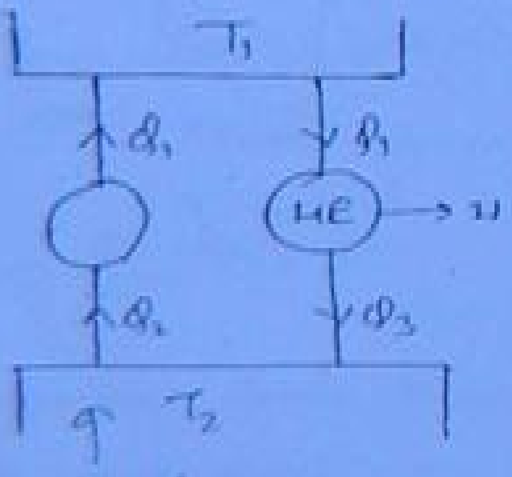
$$(COP)_{HP} - (COP)_R = \frac{Q_1}{Q_1 - Q_2} - \frac{Q_2}{Q_1 - Q_2}$$

$$(COP)_{HP} - (COP)_R = 1.$$

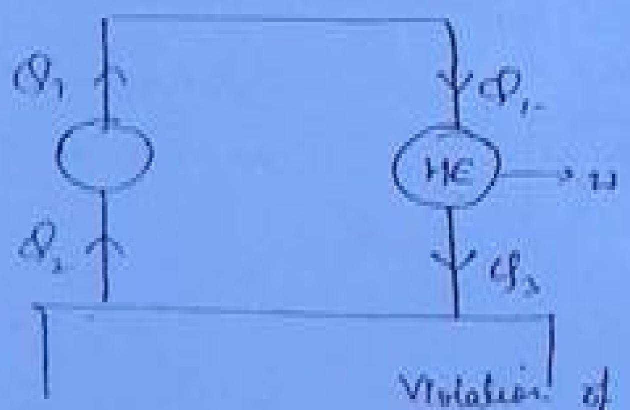
$$\text{or,} \quad \boxed{(COP)_{HP} = 1 + (COP)_R} \quad \text{——} ④$$

TO SHOW THAT A KELVIN - PLANK AND CLAUSIS STATEMENTS ARE PARALLEL STATEMENTS OF 2ND LAW OF THERMODYNAMICS :-

CASE - 1) Violation of Clausius Statement :-



Violation of Clausius Statement



Violation of Kelvin - Plank Statement.

⇒* CARNOT CYCLE :- (Reversible Cycle) :-

A cycle is said to be a reversible cycle when each process in a cycle is reversible.

slow

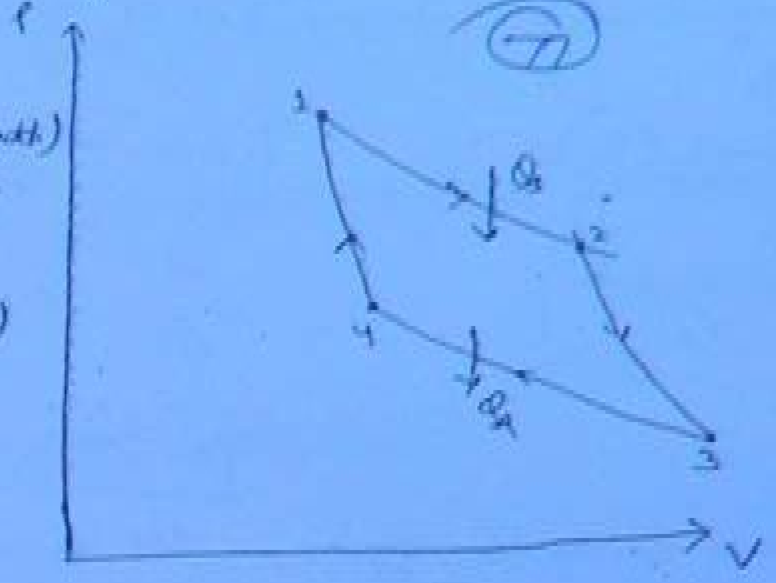
1-2 → Isothermal expansion (heat add)

2-3 → Adiabatic expansion

3-4 → Isothermal compression (-heat rejection)

4-1 → Adiabatic compression.

fast



Isothermal → slow
Adiabatic → fast. }

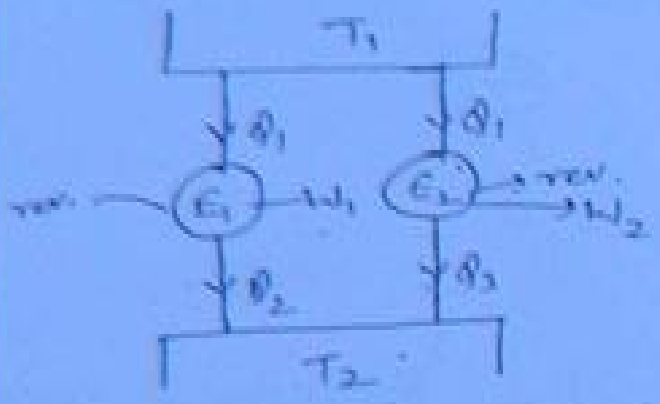
Carnot cycle consist of two isothermal processes and two adiabatic processes. Isothermal process is a slow process and adiabatic process is a fast process and hence, these two combinations in a cycle are not possible, therefore Carnot's cycle is a theoritical cycle and it is used for comparing other actual cycles.

* CARNOT'S THEOREM :-

for various cycles operating between same temperature limits, none has efficiency greater than reversible cycle efficiency

(P.T.O)

Let us consider two engines 'E1' and 'E2'. Let 'E1' be irreversible engine and 'E2' be reversible engine.



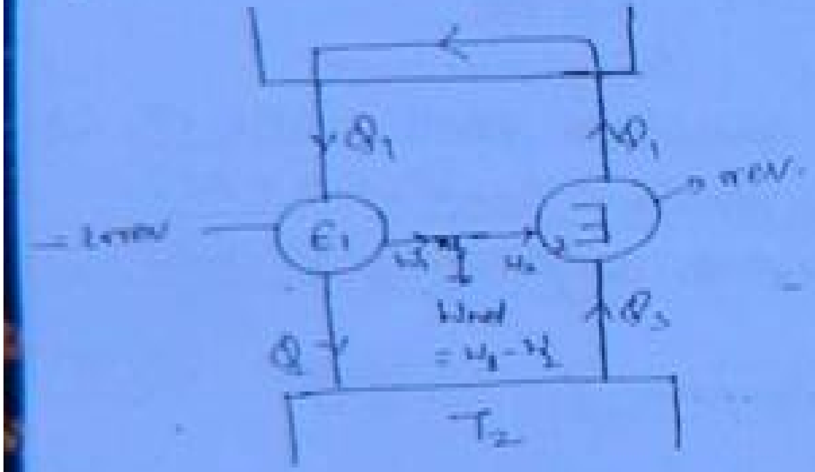
Let us assume, $\eta_{irrev} > \eta_{rev}$.

$$\eta_{irrev} = \frac{W}{Q_1} \; ; \; \eta_{rev} = \frac{W_2}{Q_1}$$

$$\frac{W_1}{\cancel{Q_1}} > \frac{W_2}{\cancel{Q_1}}$$

$$\therefore \boxed{W_1 > W_2}$$

As 'E2' is a reversible engine, let us reverse the engine.



-- violation of K-P statement.

$(\eta_{irrev} > \eta_{rev} \rightarrow \text{wrong})$

is is the violation of Kelvin-Plank statement and hence
assumption is wrong. Similarly if we take the
fficiency of both the engines to be same, then it would
to violation of Clausius statement. Therefore this is
possible. And hence, efficiency of a reversible
cle is always greater than efficiency of irreversible
cle operating between same temperature limits.

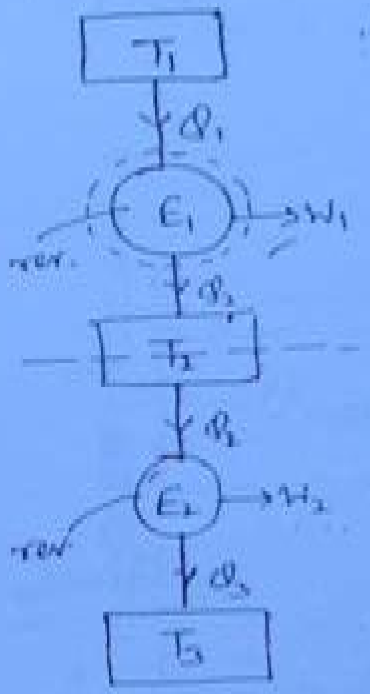
* Important Points with respect to reversible cycle :-

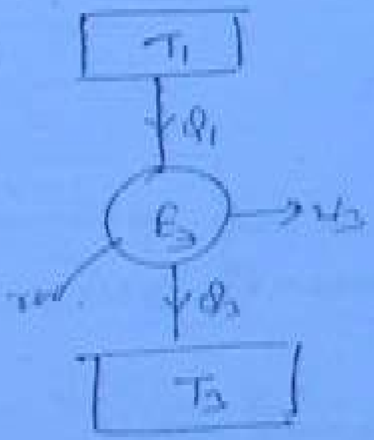
① Efficiency of all reversible cycles is equal if they operate between same temperature limits.

② Efficiency of a reversible cycle is independent of working fluid.

*** ③ Efficiency of a reversible cycle depends only on temperature limits.

⇒ * THERMODYNAMIC TEMPERATURE SCALE :- (79)





$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$\eta_1 = 1 - \frac{Q_2}{Q_1}$$

$$\eta_1 = f(T_1, T_2)$$

$$1 - \frac{Q_2}{Q_1} = f_1(T_1, T_2)$$

$$1 - f_1(T_1, T_2) = Q_2/Q_1$$

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f_1(T_1, T_2)}$$

$$Q_1/Q_2 = \phi_1(T_1, T_2)$$

Similarly :-

$$\frac{Q_2}{Q_3} = \phi(T_2, T_3)$$

and $\frac{Q_1}{Q_3} = \phi(T_1, T_3)$.

$$\frac{Q_1}{Q_2} = \frac{\frac{Q_1}{Q_3}}{\frac{Q_2}{Q_3}} \quad \text{or} \quad \frac{Q_1}{Q_2} = \frac{\phi_3(T_1, T_3)}{\phi_2(T_2, T_3)}$$

$$\frac{Q_1}{Q_2} = \frac{\psi_1(T_1)}{\psi_2(T_2)}$$

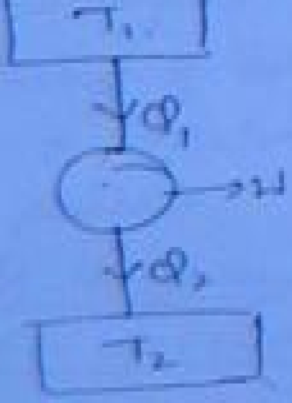
(88)

Based on kelvin's experiments.

$$\boxed{\frac{Q_1}{Q_2} = \frac{T_1}{T_2}}$$ → This is only valid for reversible cycle.

$\left(\frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right)$ is valid for a reversible cycle.

* EFFICIENCY OF A REVERSIBLE ENGINE :-



$$\eta = 1 - \frac{Q_2}{Q_1}$$

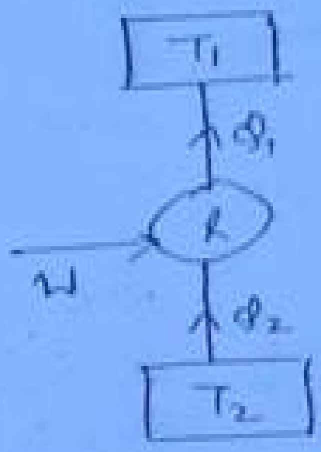
If the cycle is reversible,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}.$$

$$\boxed{\eta_{rev} = 1 - \frac{T_2}{T_1}}$$

$$\boxed{\eta_{rev} = \eta_{max} = 1 - \frac{T_L}{T_H}}$$

⇒ * COP OF A REVERSIBLE REFRIGERATOR :-



$$COP_R = \frac{Q_2}{Q_1 - Q_2}$$

$$COP_R = \frac{Q_2}{Q_2\left[\frac{Q_1}{Q_2} - 1\right]}$$

$$(COP)_R = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

for reversible refrigerator,

$$(COP)_{rev,R} = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$\boxed{(COP)_{rev,R} = \frac{T_2}{T_1 - T_2}}$$

or $$\boxed{(COP)_{max,R} = (COP)_{rev,R} = \frac{T_L}{T_H - T_L}}$$

Similarly.

for reversible heat pump:-

~~$(COP)_{HP}$~~

$$\boxed{(COP)_{rev,HP} = \frac{T_H}{T_H - T_L}}$$

[redacted]

⇒* CLAUSIUS INEQUALITY:-

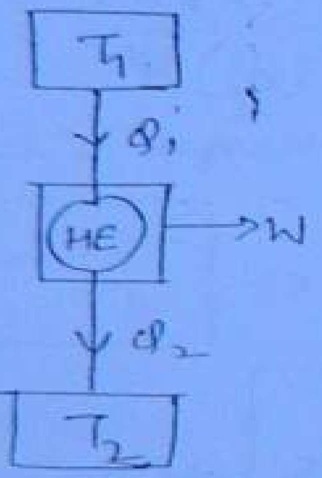
The cyclic integral of $\frac{dQ}{T}$ is less than or equal to zero.

$$\oint \frac{dQ}{T} \leq 0$$

$\oint \frac{dQ}{T} = 0 \Rightarrow$ Cycle is reversible

$\oint \frac{dQ}{T} < 0 \Rightarrow$ cycle is irreversible.

se I:- Reversible Cycle:-



$$\oint_{rev} \frac{dQ}{T} = \frac{Q_1}{T_1} + \left(-\frac{Q_2}{T_2}\right) \quad \text{—①}$$

we know that for a reversible process,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

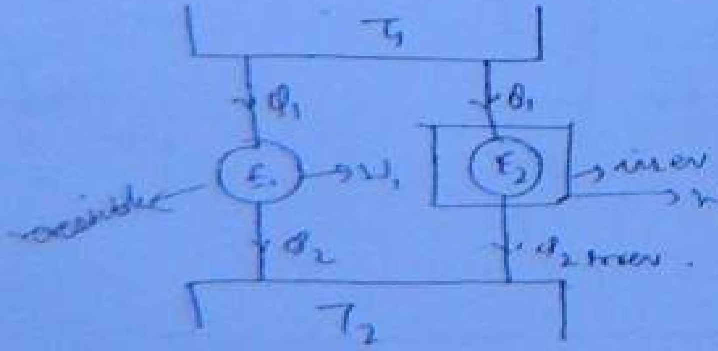
or. $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$ —②

substituting in eqⁿ ①, we have

$$\therefore \oint \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_1}{T_1}$$

$$\Rightarrow \boxed{\oint \frac{dQ}{T} = 0}$$

-II: Irreversible Cycle:-



$\eta_1 = \frac{W_1}{Q_1}$; $\eta_2 = W_2/Q_1$

$\eta_1 > \eta_2$

$$\frac{W_1}{\cancel{Q_1}} > \frac{W_2}{\cancel{Q_1}}$$

$\Rightarrow W_1 > W_2$.

for reversible process

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}.$$

$$\oint_{irrev.} \frac{dQ}{T} = \cancel{\oint} \frac{Q_1}{T_1} + \left(-\frac{Q_{2\,irrev.}}{T_2}\right)$$

$$\oint_{irrev.} \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_{2\,irrev}}{T_2}$$

$$\oint_{irrev.} \frac{dQ}{T} = \frac{Q_2}{T_2} - \frac{Q_{2\,irrev}}{T_2}$$

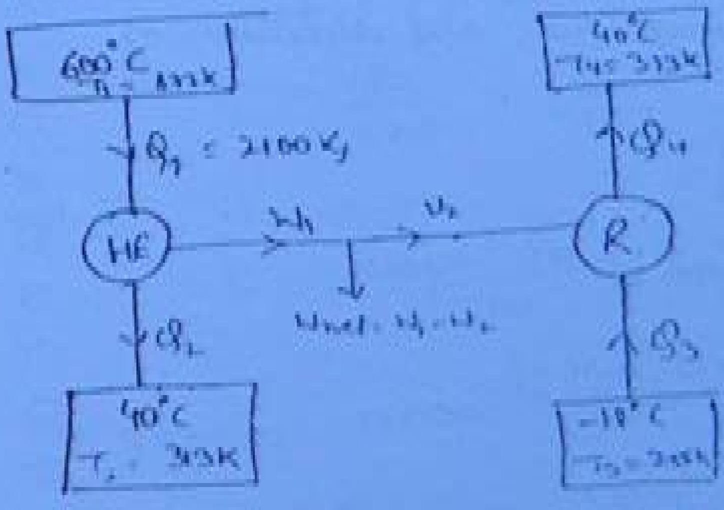
(83)

$$\oint_{irrev.} \frac{dQ}{T} = \frac{Q_2 - Q_{2\,irrev.}}{T_2}.$$

$$\boxed{\oint \frac{dQ}{T} < 0 \;;\; (Q_2 < Q_{2\,irrev.})}$$

PROBLEM

Q:- A reversible heat engine operates between 600°C and
40°C., ~~data~~ This engine drives a reversible refrigerator
operating between 40°C and −18°C. Still there is a net
work output of 370kj while the heat recieved by the
engine is 2100 kjoule. Determine the cooling effect.



$$(COP)_R = \frac{T_3}{T_4 - T_3}$$

$$\cancel{\frac{Q_3}{W_2}} >$$

$$(COP)_R = \frac{Q_3}{W_2} = \frac{255}{313 - 255}$$

$$\therefore \frac{Q_3}{W_2} = 4.396$$

$W_1 = Q_1 - Q_2$

Also ∵ It is reversible cycle.

$$\therefore \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow Q_2 = \frac{Q_1 \times T_2}{T_1} = \frac{2100 \times 313}{873}$$

$$\therefore Q_2 = 752.9 \text{ kj}$$

$$\therefore W_1 = (2100 - 752.9) = 1347.1 \text{ kj}$$

(84)

$W_{net} = W_1 - W_2 = 370$

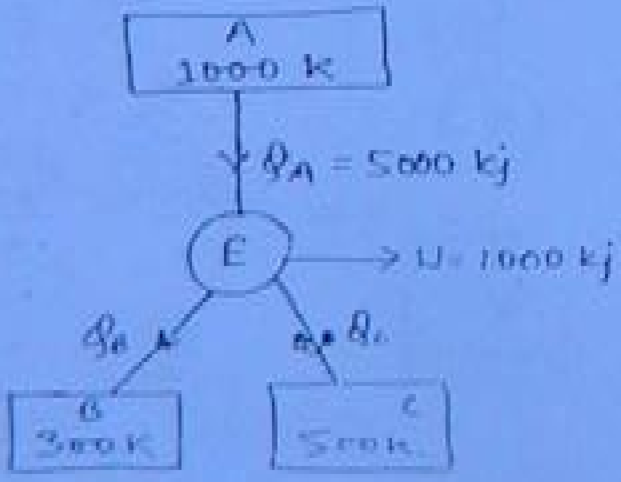
$$\therefore W_2 = 977.13 \text{ kj}$$

$$\frac{Q_3}{977.1} = 4.396$$

$$\boxed{Q_3 \text{ (Cooling Effect)} = 4295.2 \text{ kj}}$$ (Ans)

***

figure shows a reversible cycle during which, it exchanges heat with three thermal reservoirs and develops 1000 kj of work. find the magnitude and direction of Q_B and Q_C.



~~let Q_B and Q_C are the heat rejected by the system~~

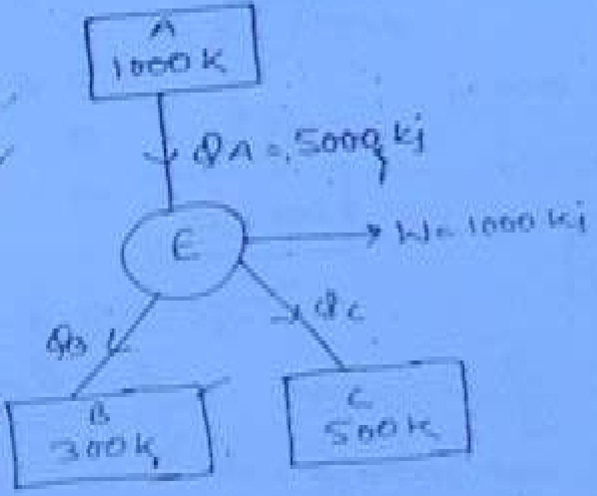
$$\cancel{W = Q_A - (Q_B + Q_C)} \quad —①$$

$$\cancel{\frac{Q_A}{T_A} = \frac{Q_B}{T_B}}$$

$$\cancel{\Rightarrow \frac{5000}{1000} = \frac{Q_B}{300} \Rightarrow Q_B = 1500\text{ kj}}$$

$$\cancel{\frac{Q_A}{T_A} = \frac{Q_C}{T_C} \Rightarrow Q_C = \frac{5000}{1000} \times 500 = 2500\text{ kj}}$$

let the direction of Q_B and Q_C are downwards.



<u>Using Energy conservation</u>:

$$Q_A = W + Q_B + Q_C$$

$$\Rightarrow 5000 = 1000 + Q_B + Q_C$$

$$\therefore Q_B + Q_C = 4000 \quad —①$$

Again, using clausius inequality.

$$\oint_{rev} \frac{dQ}{T} = 0$$

$$\frac{5000}{1000} + \left(-\frac{Q_C}{500}\right) + \left(\frac{-Q_B}{300}\right) = 0$$

$$5 = \frac{Q_C}{500} + \frac{Q_B}{300} \quad —②$$

from eqⁿ ①,

$$5 = \frac{4000 - Q_B}{500} + \frac{Q_B}{300}$$

$$7500 = 12000 - 300\,Q_B + 500\,Q_B$$

$$\Rightarrow 200\,Q_B = -4500$$

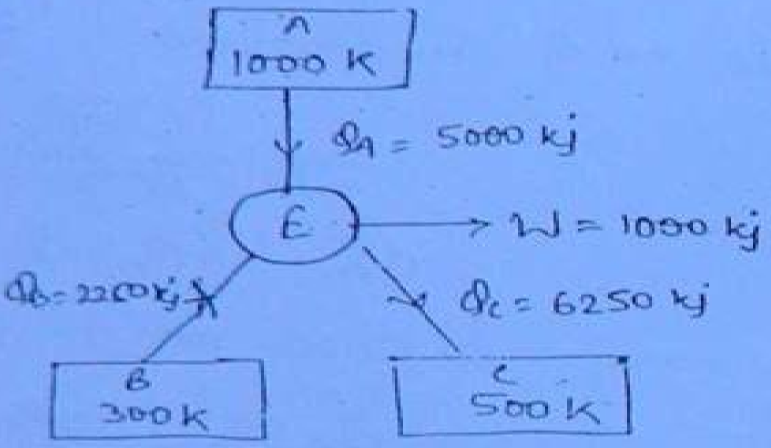
$$\therefore Q_B = \underline{-2250\text{ kj}}$$

$Q_B + Q_C = 4000$

$\Rightarrow -2250 + Q_C = 4000$

$\therefore Q_C = \underline{6250\ kj}$

∵ The value of 'Q_B' is negative, it means that the direction assumed by us is wrong, hence the direction of 'Q_B' is upward.



Two reversible heat engines 'A' and 'B' are arranged in series. Heat Engine 'A' rejects heat directly to 'B'. Engine 'A' recieves 300 kj of heat at a temperature of 427°C from a high temperature source while engine 'B' rejects heat to a sink at 7°C. If the work output of 'A' is 2 times that of 'B', find :-

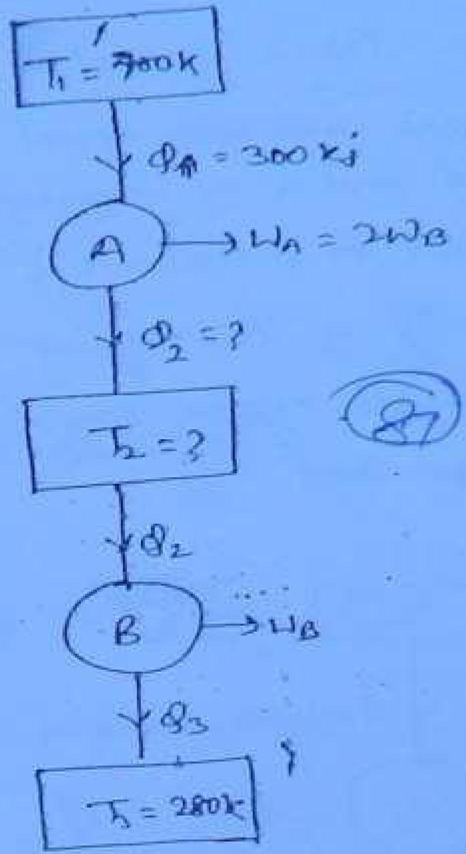
Intermediate temperature of 'A' and 'B'.

Efficiencies of each engine

Heat rejected by Engine 'A' i.e. heat recieved by Engine 'B'.

Heat rejected to sink.

Solⁿ:



~~$W_B = Q_2 - Q_3$~~

~~$2W_B = Q_1 - Q_2$~~

~~$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$~~

~~$W_A = 2Q_2 - 2Q_3$~~

~~$2Q_2 - 2Q_3 = Q_1 - Q_2$~~

~~$\Rightarrow 3Q_2 = 300 + 2Q_3$~~

~~$Q_2 = 100 + \frac{2Q_3}{3}$~~

~~$\frac{Q_2}{T_2} = \frac{Q_3}{T_3}$~~

~~$= \frac{100 + \frac{2Q_3}{3}}{} = \frac{Q_3}{T_3}$~~

~~$= 300 + 2Q_3 = \frac{3Q_3}{T_3}$~~

~~$= 300 \times 280 +$~~

for a reversible engine,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad -①$$

Also $\frac{Q_2}{T_2} = \frac{Q_3}{T_3}$ —②

from ① & ②

$$\frac{Q_1}{T_1} = \frac{Q_3}{T_3} \Rightarrow Q_3 = \frac{300 \times 280}{700} = 120\,kJ.$$

$W_A = 2W_B$

$\therefore Q_1 - Q_2 = 2Q_2 - 2Q_3 \Rightarrow 3Q_2 = Q_1 + 2Q_3$

$\Rightarrow Q_2 = \frac{300 + 240}{3} = 180\,kJ$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow T_2 = \frac{Q_2 \times T_1}{Q_1} = \frac{180 \times 700}{300} = 420\,K.$$

$$\eta_A = 1 - \frac{T_2}{T_1} = 0.4 = 40\%.$$

$$\eta_B = 1 - \frac{T_3}{T_2} = 0.33 = 33.3\%$$

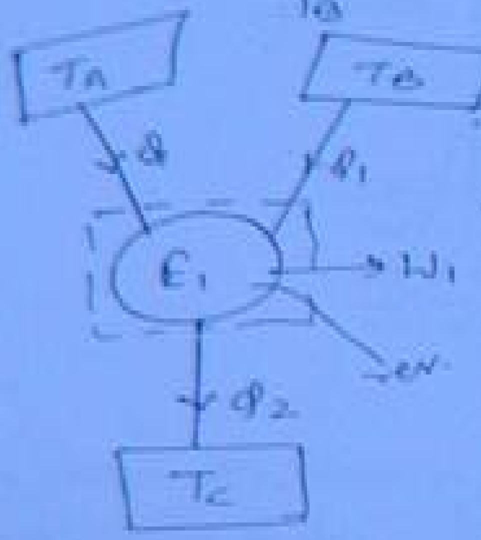
*****
Q.
V.V.I

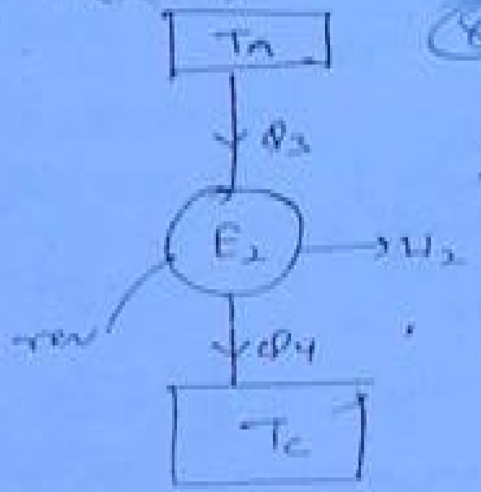
A reversible engine works between three thermal reservoirs 'A', 'B' and 'C'. The engine receives equal amount of heat from reservoirs 'A' and 'B' at temperatures T_A and T_B respectively and rejects heat to a reservoir 'C' at a temperature 'T_C'. If the efficiency of this engine is α times the efficiency of a reversible engine operating between two reservoirs 'A' and 'C', show that

$$\frac{T_A}{T_B} = 2(1-\alpha)\frac{T_A}{T_C} + (2\alpha-1)$$

(88)

Soln





$$\eta_1 = 1 - \frac{Q_2}{2Q_1} \quad \text{—①}$$

$$\oint \frac{dQ}{T} = 0 \quad (\text{for rev. cycle})$$

$$\therefore \frac{Q_1}{T_A} + \frac{Q_1}{T_B} - \frac{Q_2}{T_C} = 0$$

$$Q_1\left[\frac{1}{T_A} + \frac{1}{T_B}\right] = \frac{Q_2}{T_C}$$

$$\frac{Q_2}{Q_1} = \frac{T_C}{T_A} + \frac{T_C}{T_B}$$

$$\eta_1 = 1 - \frac{1}{2}\left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right) \quad \text{——②}$$

Now,

$$\eta_2 = 1 - \frac{Q_4}{Q_3} \quad \text{——} ③$$

Again $\oint \frac{dQ}{T} = 0$

$$\frac{Q_3}{T_A} - \frac{Q_4}{T_C} = 0 \Rightarrow \frac{Q_3}{T_A} = \frac{Q_4}{T_C}$$

or ~~Q_3~~ $\frac{Q_4}{Q_3} = \frac{T_C}{T_A}$

$$\therefore \eta_2 = 1 - \frac{T_C}{T_A} \quad \text{——} ④$$

(89)

But it is given that,

$$\eta_1 = \alpha \eta_2$$

$$\therefore \quad 1 - \frac{1}{2}\left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right) = \alpha\left(1 - \frac{T_C}{T_A}\right) \quad \text{——} ⑤$$

multiplying both sides with T_A

∴ Eq: ⑤ becomes,

$$T_A - \frac{T_A}{2}\left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right) = \alpha T_A\left(1 - \frac{T_C}{T_A}\right)$$

$$\Rightarrow T_A - \frac{T_C}{2} - \frac{T_C}{2}\cdot\frac{T_A}{T_B} = \alpha T_A - \alpha T_C$$

$$\Rightarrow T_A - \frac{T_C}{2} - \alpha T_A + \alpha T_C = \frac{T_C}{2}\cdot\frac{T_A}{T_B}$$

$$\Rightarrow \frac{2}{T_C}\left[T_A - \frac{T_C}{2} - \alpha T_A + \alpha T_C\right] = \frac{T_A}{T_B}$$

$$\Rightarrow \frac{2T_A}{T_C} - 1 - 2\alpha\frac{T_A}{T_C} + 2\alpha = \frac{T_A}{T_B}$$

or, $\boxed{\frac{T_A}{T_B} = \frac{2T_A}{T_C}(1-\alpha) + 2\alpha - 1}$

Work Book

Chapter - 4

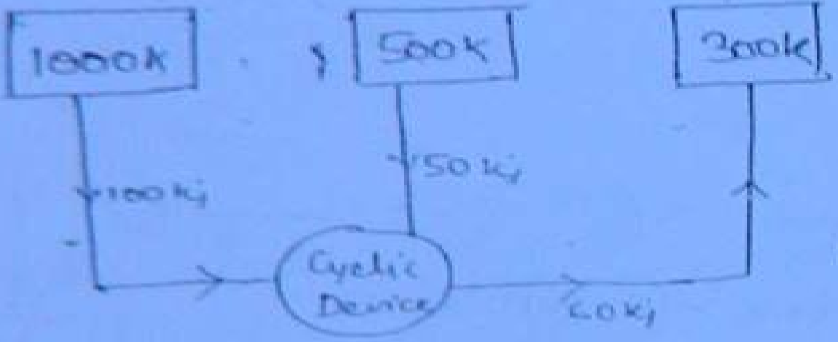
① — (a)
② — (b)
③ — (c)
④ — (a)
⑤
⑥ — (d)
⑦ — (d)
⑧ — (b)
⑨ — (d)

⑩ — (b)
⑪
⑫
⑬
⑭
⑮

⑯
⑰
⑱
⑲

(96)

①



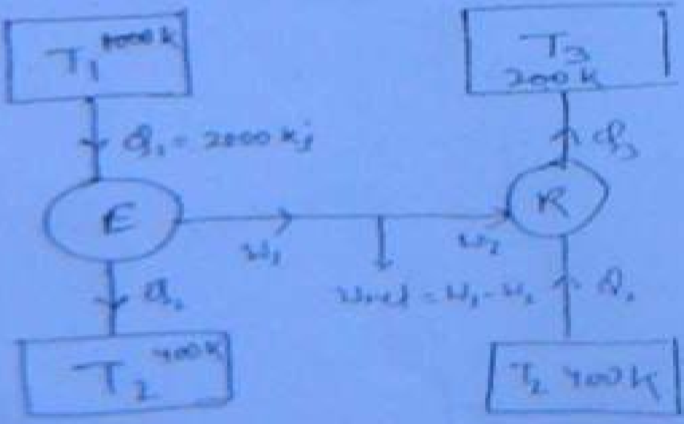
∵ Heat supplied > Heat Rejected.

∴ There will work output. & If there is work output, it is an engine.

Also. $\oint \frac{dQ}{T} = 0$

∴ It is reversible.

③



$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow Q_2 = \frac{Q_1 \times T_2}{T_1} = \frac{2000 \times 400}{1000}$$

$$= 800\ kJ$$

$$W_1 = Q_1 - Q_2 = (2000 - 800) = 1200\ kJ$$

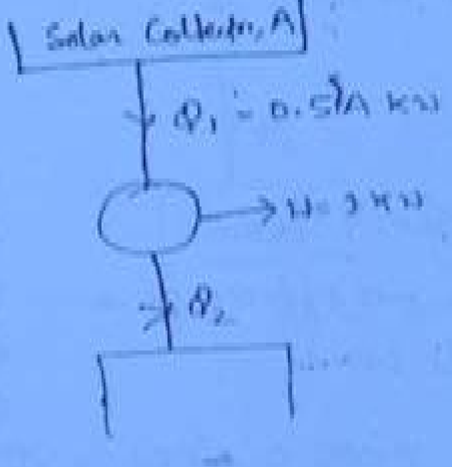
$$W_2 = \text{~~Wnet~~}\ W_1 - W_{net} = (1200 - 300) = 900\ kJ$$

$$W_2 = Q_2 - Q_3 = 90$$

$$\Rightarrow Q_3 = Q_2 - W_2$$

97

③



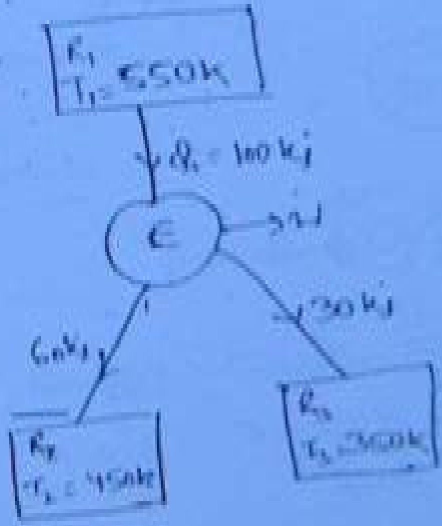
$0.5\ kW/m^2 \times A\ m^2$

$$\eta = W/Q_1$$

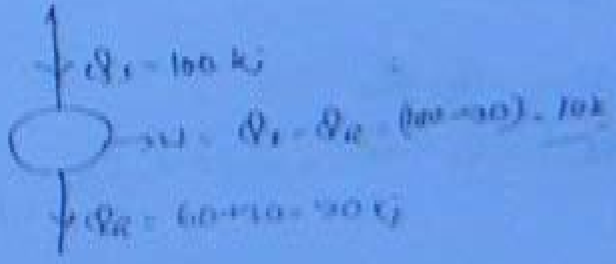
$$\Rightarrow 0.2 = \frac{1}{0.5A}$$

$$\Rightarrow \boxed{A = 10}$$

④



$$\eta = \frac{\text{Output}}{\text{Input}}$$



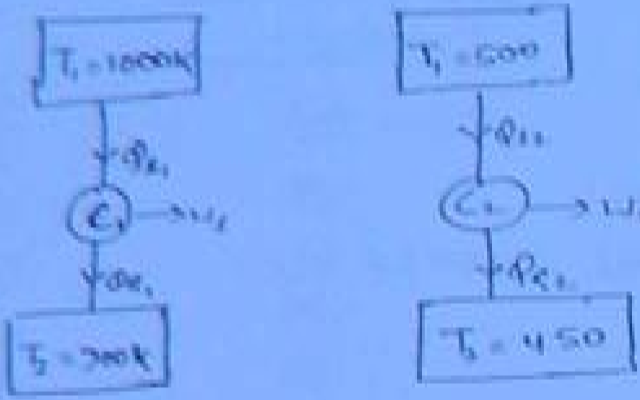
$Q_1 = 100$ kJ

$W = Q_1 - Q_R = (100 - 90) = 10k$

$Q_R = 60 + 30 = 90$ kJ

$$\eta = \frac{10}{100} = 0.1 = 10\%$$

⑥



$W_1 = W_2 = W.$

$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{900}{1000}$

$= 0.1$

$\eta_1 = \frac{O}{I} = \frac{W}{Q_{S_1}}$

$\eta_2 = 1 - \frac{\overset{9}{\cancel{450}}}{\underset{10}{\cancel{500}}} = 0.1$

91

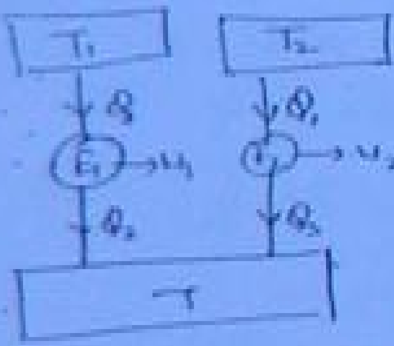
$\eta_2 = \frac{O}{I} = \frac{W}{Q_{S_2}}$

But $\eta_1 = \eta_2 \Rightarrow \frac{W}{Q_{S_1}} = \frac{W}{Q_{S_2}}$

$\therefore \underline{Q_{S_1} = Q_{S_2}}$ $\therefore Q_{R_1} = Q_{R_2}.$

∵ Supply will be same, work is same and efficiency is same, therefore rejection will also be same.

⑩



$\left(\frac{T_1}{T_1 - T_2}\right)$

$\left(\frac{1}{9-1}\right) = \frac{1}{}$

$\eta_1 \left(1 - \frac{T_2}{T_1}\right) = \eta_2 \left(1 - \frac{T}{T_2}\right)$

$\eta_1 = 1 - \frac{T}{T_1} = \frac{W_1}{Q}$

$\eta_2 = 1 - \frac{T}{T_2} = \frac{W_2}{Q}$

$W_1 = \underline{Q_1} - Q_2$

$W_2 = \underline{Q_1} - Q_3$

$\therefore W_1 = Q\left(1 - T/T_1\right)$

$W_2 = Q\left(1 - T/T_2\right)$

∵ T_2 is less ∴ W_2 will be less.

or, $\underline{W_1 > W_2}$

(12) $(\eta_{max})_E = \left(1 - \frac{T_2}{T_1}\right) = \left(\frac{T_1 - T_2}{T_1}\right)$

$\frac{T_1}{T_1 - T_2} - 1$

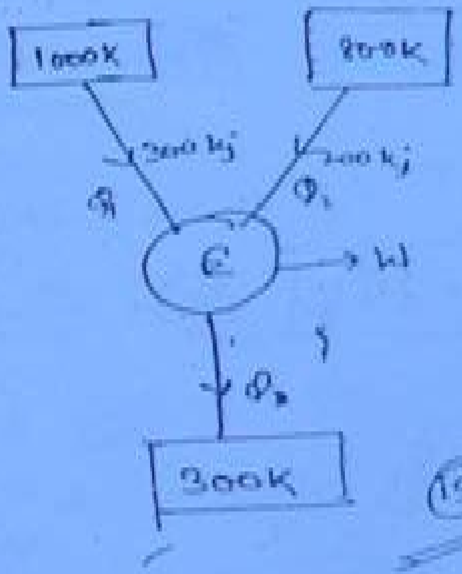
$(\eta_{max})_R = \left(\frac{T_2}{T_1 - T_2}\right) =$

$\frac{\cancel{T_1} - \cancel{T_1} + T_2}{T_1 - T_2} = \left(\frac{T_2}{T_1 - T_2}\right)$

$$\boxed{(\eta_{max})_R = \frac{1}{(\eta_{max})_E} - 1}$$

(93)

(14)



$\frac{300}{1000} + \frac{200}{800} = \frac{Q}{300}$

$\Rightarrow 0.3 + 0.25 = \frac{Q}{300}$

$\therefore Q = 165$ kJ ⑤

(15) $Q_1 + Q_2 = W + Q$

$\Rightarrow W = Q_1 + Q_2 - Q$

$= 500 - 165 = 335$ kJ ④

(13) $T_h = 627°C = 900$ K.

$T_L = 27°C + 273 = 300$ K.

$\eta_{max} = \eta_{rev} = 1 - \frac{300}{900} = 0.6667 = 66.67\%$

$\eta_{inventor} = ?$

$\eta_{inventor} = O/I$

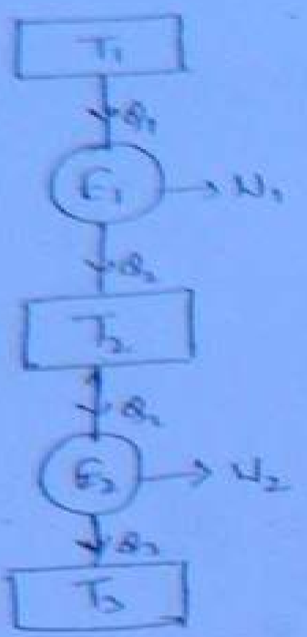
Output = 50 kW = 50 kJ/sec.

Input = $\frac{75000 \times 3}{3600}$ kJ/sec = 62.5 kJ/sec

$\therefore \eta_{inventor} = \frac{50}{62.5} = 0.8 = 80\%$

∵ Reversible efficiency is max^m efficiency and $\eta_{inventor}$ is exceeding that, hence it is not possible

⑯



$\eta = \frac{W_1}{Q_1}$

$\eta_{E_1} = 1 - \frac{T_2}{T_1}$

$\eta_{E_2} = 1 - \frac{T_3}{T_2}$

$\frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2}$ (99)

$\Rightarrow \frac{\cancel{T_2 T_1} - T_2^2 - \cancel{T_1 T_2} + T_1 T_3}{T_1 T_2} = 0$

$\Rightarrow T_1 T_3 = T_2^2 \quad \therefore \boxed{T_2 = \sqrt{T_1 T_3}}$ ⓐ

$W_1 = Q_1 - Q_2$

$W_2 = Q_2 - Q_3$

$W_1 = W_2 \Rightarrow Q_1 - Q_2 = Q_2 - Q_3$

$\Rightarrow$ ~~$Q_1 - Q_2 = Q_2 + Q_3$~~

$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$\frac{Q_2}{T_2} = \frac{Q_3}{T_3}$

$\frac{Q_1}{T_1} = \frac{Q_3}{T_3}$

$\eta_1 = \frac{W_1}{Q_1} \Rightarrow 1 - \frac{T_2}{T_1} = \frac{W}{Q_1}$

$\eta_2 = \frac{W_2}{Q_2} \Rightarrow 1 - \frac{T_3}{T_2} = \frac{W}{Q_2}$

$Q_1\left(1 - \frac{T_2}{T_1}\right) = Q_2\left(1 - \frac{T_3}{T_2}\right)$

⑰ $W_1 = W_2$ (given)

$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3 \Rightarrow Q_1 + Q_3 = 2Q_2$ — ①

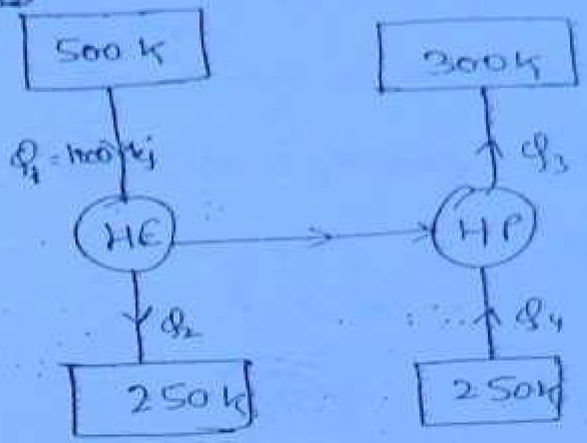
② $\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3} = K \Rightarrow Q_1 = KT_1,\ Q_2 = KT_2$

$Q_3 = KT_3$

$\cancel{K}T_1 + \cancel{K}T_3 = 2\cancel{K}T_2$

$\Rightarrow T_2 = (T_1 + T_3)/2$ ⓑ

(18) (19)



$(COP)_{HP} = \frac{Q_3}{W} = \frac{T_h}{T_h - T_l}$

$= \frac{300}{300-250} = 6.$

①

(95)

(18) $W = (1000 - Q_2)$

$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$\Rightarrow \frac{1000}{500} = \frac{Q_2}{250}$

$\Rightarrow Q_2 = 500$ kJ

$\therefore W = (1000 - 500) = 500$ kJ

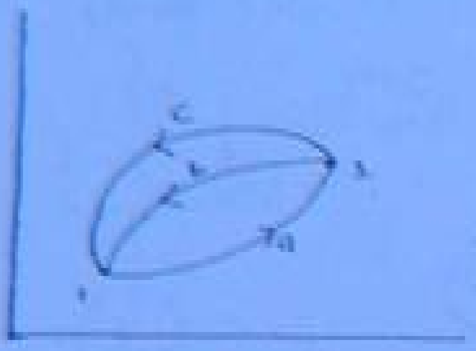
$Q_3 = W \times 6 = 500 \times 6 = 3000$ kJ

①

← : ENTROPY : →

Case - I :-

REVERSIBLE CYCLE :-



1 - a - 2 - b - 1 → reversible cycle

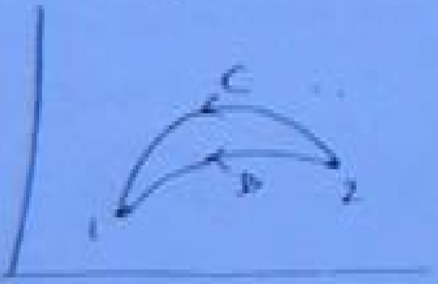
1 - a - 2 - c - 1 → reversible cycle

$$\oint_{rev} \frac{dQ}{T} = 0 \Rightarrow \cancel{\left(\frac{dQ}{T}\right)_{1a2}} + \left(\frac{dQ}{T}\right)_{2b1} = 0$$

$$\cancel{\left(\frac{dQ}{T}\right)_{1a2}} + \left(\frac{dQ}{T}\right)_{2c1} = 0$$

$$\left(\frac{dQ}{T}\right)_{2b1} - \left(\frac{dQ}{T}\right)_{2c1} = 0$$

or, $$\boxed{\left(\frac{dQ}{T}\right)_{\substack{2b1 \\ (rev)}} = \left(\frac{dQ}{T}\right)_{2c1\ (rev.)}}$$



$\left(\frac{dQ}{T}\right)_{rev}$ is same for both paths 'b' and 'c' and it depends only on end points (2 and 1), therefore $\left(\frac{dQ}{T}\right)_{rev}$ must be a property and this property is known as entropy.

$$\left(\frac{dQ}{T}\right)_{2b1} = \left(\frac{dQ}{T}\right)_{2c1} = dS.$$

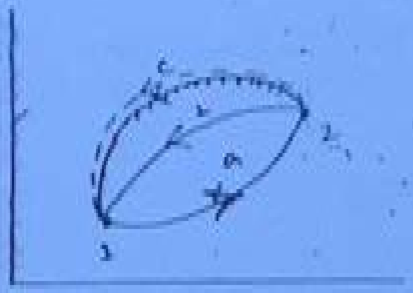
$$\boxed{\left(\frac{dQ}{T}\right)_{\underset{rev}{2b1}} = dS}$$ *** This is the second law of thermodynamics for equation for a process.

(97)

$$\boxed{dS = \left(\frac{dQ}{T}\right)_{REV}}$$ ***

CASE-II :-

IRREVERSIBLE CYCLE :-



1-a-2-b-1 (reversible cycle)
1-a-2-c-1 (irreversible ")

$$\oint_{rev} \frac{dQ}{T} = 0$$

$$\left(\frac{dQ}{T}\right)_{1a2} + \left(\frac{dQ}{T}\right)_{2b1} = 0$$

$$\left(\frac{dQ}{T}\right)_{1a2} = -\left(\frac{dQ}{T}\right)_{2b1} \quad \text{——} ①$$

$$\oint_{\text{irrev.}} \frac{dQ}{T} < 0 = \left(\frac{dQ}{T}\right)_{1a2} + \left(\frac{dQ}{T}\right)_{2c1} < 0 \quad \text{—②}$$

Substituting the value from eqⁿ ① to eqⁿ ⑤.

$$-\left(\frac{dQ}{T}\right)_{2b1} + \left(\frac{dQ}{T}\right)_{2c1} < 0$$

$$\left(\frac{dQ}{T}\right)_{\underbrace{2c1}_{\text{irrev.}}} < \left(\frac{dQ}{T}\right)_{\underbrace{2b1}_{\text{rev.}}}$$

(98)

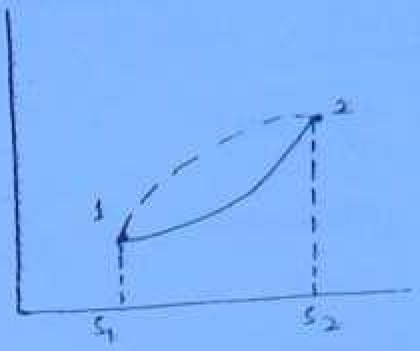
$$\left(\frac{dQ}{T}\right)_{\text{irrev.}} < ds$$

or,
$$\boxed{ds > \left(\frac{dQ}{T}\right)_{\text{irrev.}}}$$

$$ds = \left(\frac{dQ}{T}\right)_{\text{rev}}$$

$$ds > \left(\frac{dQ}{T}\right)_{\text{irrev.}}$$

$$\boxed{ds \geq \left(\frac{dQ}{T}\right)}$$

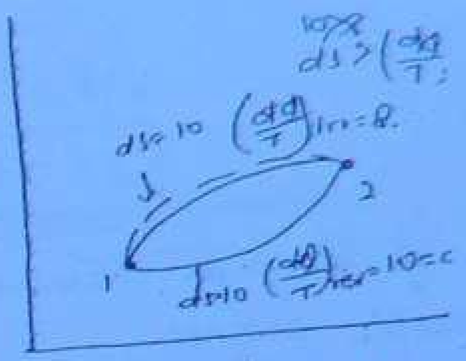


$$(dS)_{irrev} = S_2 - S_1$$

$$\boxed{(dS)_{rev} = (dS)_{irrev}}$$

(99)

$$(dS)_{rev} = S_2 - S_1$$



As entropy is a property, it depends only on ~~po~~ end points and hence as long as the end points are same, the entropy change remains same for reversible as well as irreversible process.

In order to find the entropy change for irreversible process, it must be replaced by a reversible process between same end points.

-1 ENTROPY CHANGE OF SYSTEM FOR REVERSIBLE PROCESS:-

Case-1:- ~~When heat is app~~ When heat is supplied to the system:-

$$dS = \left(\frac{dQ}{T}\right) \quad \overset{\rightarrow +ve}{} \quad \underset{\rightarrow +ve}{}$$

$$dS = +ve$$

$$dS > 0.$$

The entropy of system increases when heat is supplied in a reversible process.

Case-II:-

When heat is rejected from the system:-

$$ds = \frac{\overset{-ve}{dQ}}{\underset{+ve}{T}}$$

$$ds = -ve.$$

$$\boxed{ds < 0}$$

Entropy of a system decreases when heat is rejected in a reversible process.

se-III:-

Reversible Adiabatic Process:-

$$ds = \frac{dQ}{T}$$

1) reversible
2) Adiabatic ⟶ $dQ = 0$

$$ds = \frac{0}{T}$$

$$ds = 0$$

$$S_2 - S_1 = 0 \rightarrow \text{Isot Entropy}$$

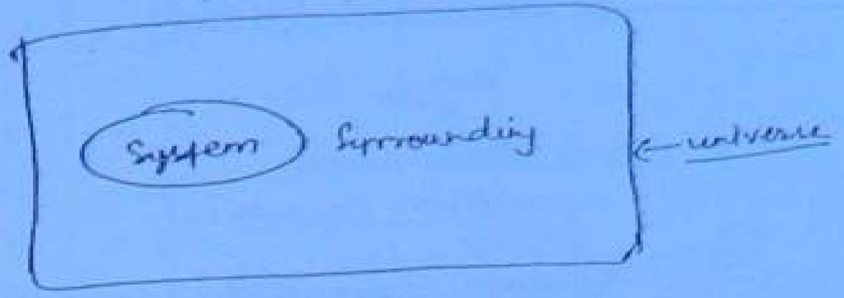
$$S_2 = S_1 = \text{constant}.$$

↘ Isentropic

7 reversible adiabatic process is always Isentropic process.

:- System entropy can increase, ~~dect~~ system entropy can decrease or it can remain constant in a reversible process depending upon heat transfer.

-: ENTROPY CHANGE OF UNIVERSE :-



01

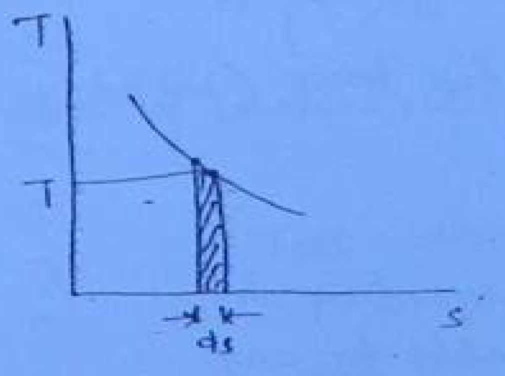
$$ds \geq \frac{dQ}{T}$$

$$dS_{(universe)} \geq 0$$ (∵ universe is isolated system. ∴ dQ=0)

$$(dS)_{system} + (dS)_{surrounding} \geq 0.$$

Note Entropy of system can increase, can decrease or it can remain constant. Similarly, entropy of surroundings can increase, can decrease or can remain constant but the overall change in entropy of universe can never decrease. and this is known as the principle of increase of entropy.

← T-S DIAGRAM :→



$$\frac{dQ}{T} = dS$$

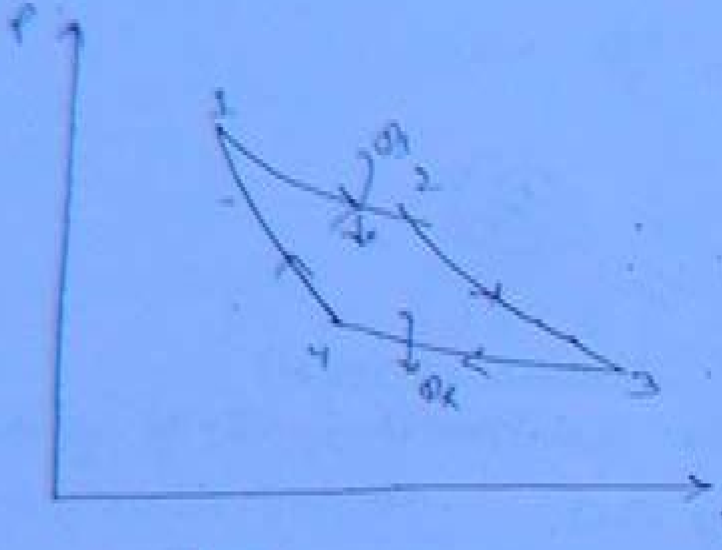
$$dQ = TdS. \text{ —②}$$

$$\boxed{dQ = \text{Area}}$$

Area = TdS —①

Area under the curve when projected on entropy axis gives reversible heat transfer.

REPRESENTATION OF CARNOT CYCLE ON T-S DIAGRAM

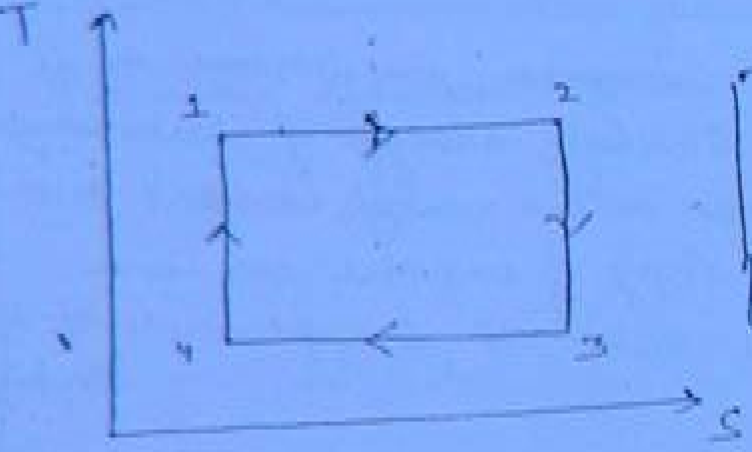


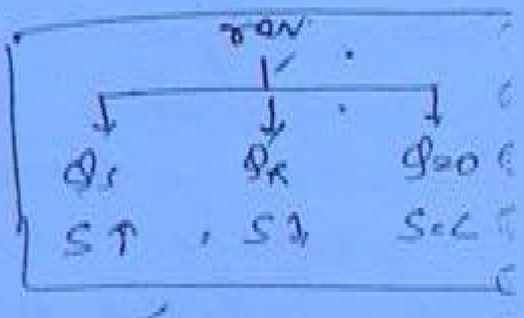
1-2 → Isothermal heat addition (expansion) $\overset{\text{by rev}}{}$

2-3 → Adiabatic Expansion (rev.)

3-4 → Isothermal heat rejection (rev) (compression)

4-1 → Adiabatic comp. (rev)





COMBINED FIRST AND SECOND LAW OF THERMODYNAMI

$dQ = dU + PdV$ → rev. process (1st law)

$\frac{dQ}{T} = dS \Rightarrow dQ = TdS$ → rev. process (2nd law)

Combining both, we have

$$\boxed{TdS = dU + PdV} \quad -①$$

→ This equation is applicable for any process (reversible or irreversible) because it connects various properties.

$h = u + Pv$

$dh = du + Pdv + vdP$

$dh = dQ + vdP \quad (\because dQ = du + Pdv)$

$dh = Tds + vdP \quad (\because dQ = Tds)$

(103)

$$\therefore \boxed{Tds = dh - vdP} \quad \text{—②}$$

This equation is applicable for reversible as well as irreversible process. because it connects various properties

Equation ① and Eq: ② are combined first and second law equations.

⇒* REPRESENTATION OF CONSTANT VOLUME AND CONSTANT PRESSURE LINES ON T-S DIAGRAM:- (for ideal gas):-

$Tds = du + Pdv$

$v = c \Rightarrow dv = 0.$

$du = C_v dT$

$Tds = C_v dT + P(0)$

$\Rightarrow Tds = C_v dT$

$$\Rightarrow \boxed{dT/ds = T/C_v}$$

Slope of constant volume lines of T-S diagram = $\underline{\underline{T/C_v}}$

$Tds = dh - vdP$

$P = C.$

$dP = 0$

$dh = C_p dT$

$Tds = C_p dT - v(0)$

$\Rightarrow Tds = C_p dT$

$$\Rightarrow \boxed{\frac{dT}{ds} = T/C_p}$$

Hence,

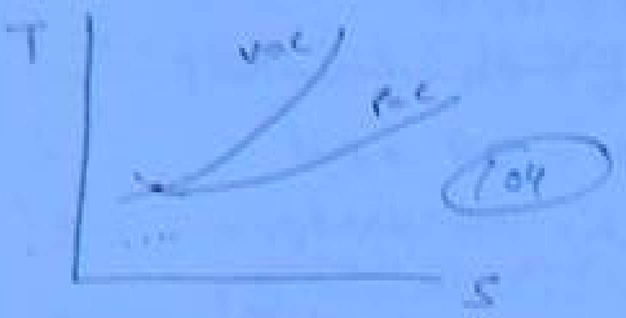
Slope of constant pressure lines on T-S diagram = $\underline{T/C_p}$

(P.T.O)

$P = C \rightarrow T/C_p$

$V = C \rightarrow T/C_v$

$\therefore C_p > C_v$



As 'C_p' is greater than 'C_v' ($C_p > C_v$), the slope of constant volume line is greater than the slope of constant pressure lines on T-S diagram. ~~Entropy change~~ of

→ ENTROPY CHANGE FOR IDEAL GAS :-

$$Tds = dU + PdV$$

$$ds = \frac{dU}{T} + \frac{P}{T}dV$$

ideal gas

$$dU = C_v dT; \quad PV = RT$$

$$\Rightarrow \frac{P}{T} = \frac{R}{V}$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_v dT}{T} + \int_{V_1}^{V_2} \frac{R}{V} dV$$

$$\boxed{(S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}} \quad \text{—— ①}$$

$$Tds = dh - vdP$$

$$ds = \frac{dh}{T} - \frac{V}{T}dP$$

for ideal gas,

$$dh = C_p dT; \quad PV = RT$$

$$\Rightarrow \frac{V}{T} = R/P$$

$$\therefore ds = \frac{C_p dT}{T} - \frac{R}{P}dP$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{C_p dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dP$$

$$\Rightarrow \boxed{(S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}} \quad \text{—— ②}$$

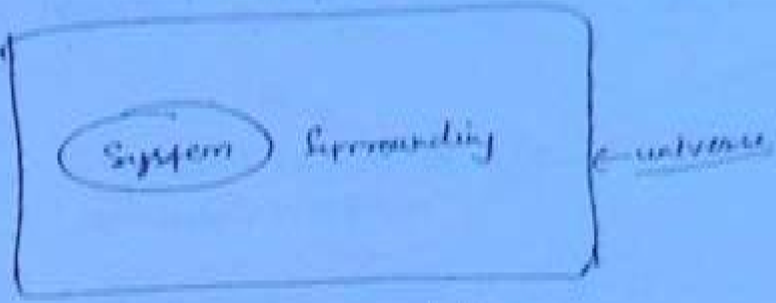
:- Show that for an ideal gas entropy change ~~($S_2 - S_1$)~~ =

$$(S_2 - S_1) = C_p \ln \frac{V_2}{V_1} + C_p \ln \frac{P_2}{P_1}.$$

~~$Tds = dh - vdP$~~

~~$dh = du + vdP + PdV$~~

ENTROPY CHANGE OF UNIVERSE



$$dS \geq \frac{dQ}{T}$$

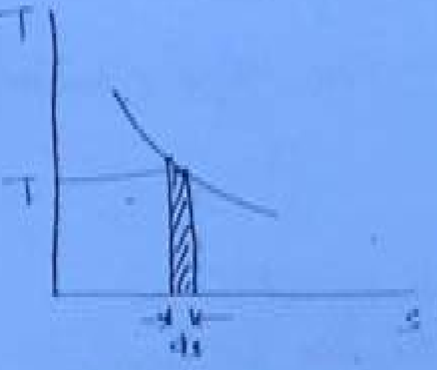
$$dS_{(universe)} \geq 0 \qquad (\because \text{universe is isolated system} \therefore dQ = 0)$$

$$(dS)_{system} + (dS)_{surrounding} \geq 0.$$

Note

Entropy of system can increase, can decrease or it can remain constant. Similarly entropy of surroundings can increase, can decrease or can remain constant but the overall change in entropy of universe can never decrease. and this is known as the principle of increase of entropy.

T-S DIAGRAM :



$$\frac{dQ}{T} = dS$$

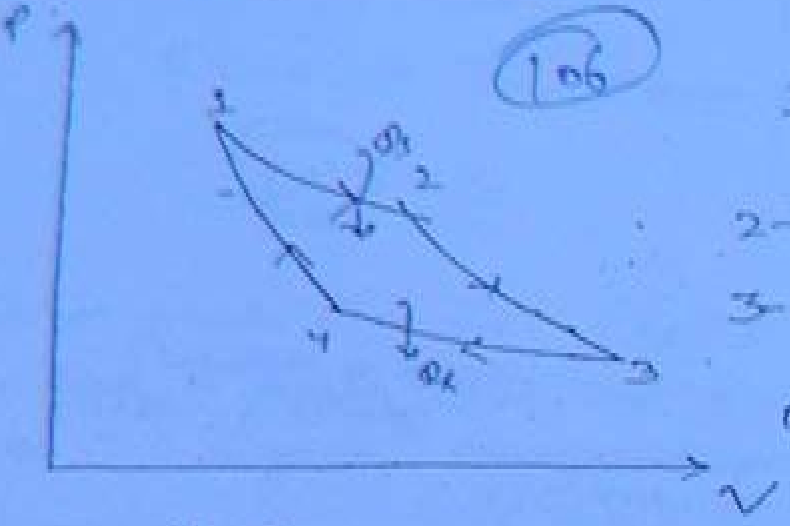
$$dQ = TdS \quad \text{—②}$$

$$\boxed{dQ = Area}$$

Area = TdS —①

Area under the curve when projected on entropy axis gives reversible heat transfer.

REPRESENTATION OF CARNOT CYCLE ON T-S DIAGRAM.

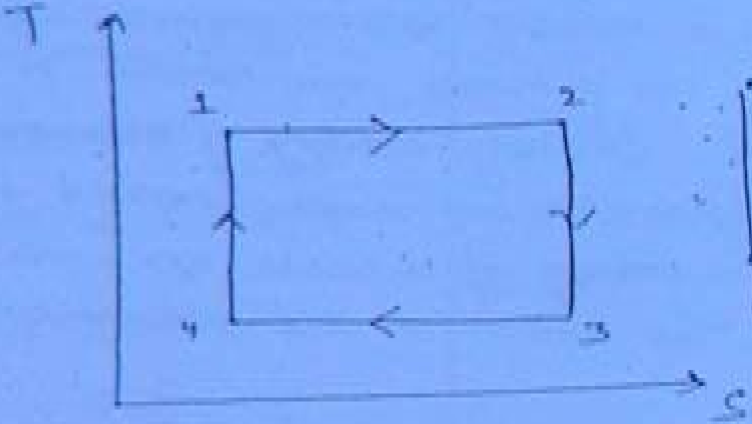


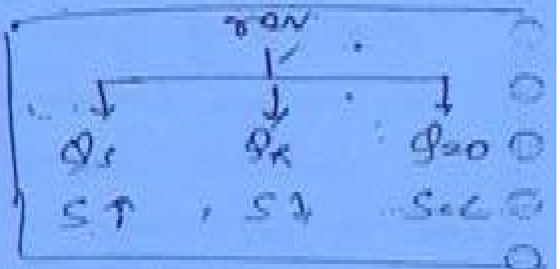
1-2 → Isothermal heat addition (expansion) (rev)

2-3 → Adiabatic Expansion (rev.)

3-4 → Isothermal heat rejection (rev) (compression)

4-1 → Adiabatic comp. (rev)





* COMBINED FIRST AND SECOND LAW OF THERMODYNAMICS

$dQ = dU + PdV$ → rev. process (1st law)

$\frac{dQ}{T} = dS \Rightarrow dQ = TdS$ → rev. process (2nd law)

Combining both, we have

$$\boxed{TdS = dU + PdV} \quad \text{①}$$

→ This equation is applicable for any process (reversible or irreversible) because it connects various properties.

$$(S_2-S_1) = C_p \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1}$$

Soln. ~~$(S_2-S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$~~

Also, ~~$(S_2-S_1) = \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$~~

~~$\Rightarrow \frac{P_1V_1}{P_2V_2} = \frac{T_1}{T_2}$~~

$$(S_2-S_1) = C_v \ln \frac{T_2}{T_1} + (C_p - C_v) \ln \frac{V_2}{V_1} \quad (\because C_p - C_v = R)$$

$$= C_v \ln \frac{T_2}{T_1} + C_p \ln \frac{V_2}{V_1} - C_v \ln \frac{V_2}{V_1}$$

$$(S_2-S_1) = C_v \left[\ln \frac{T_2}{T_1} - \ln \frac{V_2}{V_1}\right] + C_p \ln \frac{V_2}{V_1}$$

$$(S_2-S_1) = C_v \ln \left[\frac{T_2/T_1}{V_2/V_1}\right] + C_p \ln V_2/V_1$$

$$(S_2-S_1) = C_v \ln \left(\frac{T_2}{T_1} \cdot \frac{V_1}{V_2}\right) + C_p \ln \frac{V_2}{V_1}$$

$$PV = mRT$$

$$\frac{PV}{T} = mR \Rightarrow \frac{PV}{T} = C$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$\therefore (S_2-S_1) = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1}$$

$$\boxed{(S_2-S_1) = C_p \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1}}$$

$$S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$

$$S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right).$$

$$S_2 - S_1 = \cancel{C_p \ln\left(\frac{P_2}{P_1}\right)}\ C_p \ln\left(\frac{V_2}{V_1}\right) + C_v \ln\left(\frac{P_2}{P_1}\right).$$

Work Book

Chapter - 5

① d
② c
③ b
④ a
⑤ d
⑥ c
⑦ a
⑧ b
⑨ ~~d~~ c
⑩ ~~c~~ d
⑪ ~~a~~ d.
⑫ ~~a~~ c
⑬ a
⑭ ~~d~~ a
⑮ a
⑯ ~~d~~ d
⑰ a
⑱ b
⑲ b
⑳ d
㉑ b
㉒ d
㉓ d
㉔ b
㉕ c
㉖ a
㉗ b
㉘ a
㉙ b
㉚ b
㉛ c
㉜ a
㉝ b
㉞ d

03/08/2011

* $dS = \left(\frac{dQ}{T}\right)_{rev} + (dS)_{gen}$; $dS = \left(\frac{dQ}{T}\right)_{rev} + 0$

↓ external ↓ internal (irreversibilities)

Entropy change occurs due to <u>external interactions</u> and <u>internal irreversibilities</u>. In a reversible process as there are no irreversibilities the entropy change occurs only due to <u>external interactions</u>.

* In an adiabatic process as there is no interaction of system with surroundings in the form of heat transfer, therefore in an adiabatic process, there is no entropy change for surroundings.

$h = u + Pv$

$dh = du + Pdv + vdp$ $\quad (\because dQ = du + Pdv)$

$dh = dQ + vdp$ $\quad (\because dQ = Tds)$

$dh = Tds + vdp$

$$\therefore \boxed{Tds = dh - vdp} \quad — ②$$

This equation is applicable for reversible as well as irreversible process. because it connects various properties

∴ Equation ① and Eq^n ② are combined first and second law equation.

⇒* REPRESENTATION OF CONSTANT-VOLUME AND CONSTANT PRESSURE LINES ON T-S DIAGRAM:- (for ideal gas):-

$$Tds = du + Pdv$$

$$V = C \Rightarrow dV = 0.$$

$$du = c_v dT$$

$$Tds = c_v dT + P(0)$$

$$\Rightarrow Tds = c_v dT$$

$$\Rightarrow \boxed{dT/ds = T/c_v}$$

Slope of constant volume lines of T-S diagram = T/c_v.

$$Tds = dh - vdp$$

$$P = C.$$

$$dP = 0.$$

$$dh = C_p dT$$

$$Tds = C_p dT - v(0)$$

$$\Rightarrow Tds = C_p dT$$

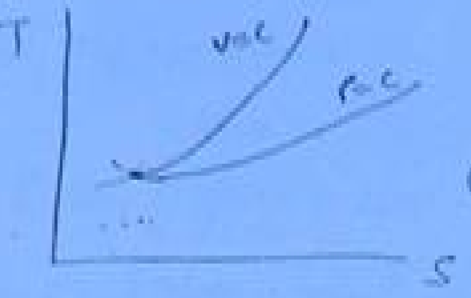
$$\Rightarrow \boxed{\frac{dT}{ds} = T/C_p}$$

Hence,
Slope of constant pressure lines on T-s diagram = T/C_p

$P = c \rightarrow T/C_p$

$V = c \rightarrow T/C_v$

$\therefore C_p > C_v$



As 'C_p' is greater than 'C_v' ($C_p > C_v$), the slope of constant volume line is greater than the slope of constant pressure lines on T-S diagram. ~~Entropy change~~ ~~of~~

→ ENTROPY CHANGE FOR IDEAL GAS ←

$T\,dS = dU + P\,dV$

$\therefore dS = \frac{dU}{T} + \frac{P}{T}dV$

ideal gas

$dU = C_v dT$; $PV = RT$

$\Rightarrow \frac{P}{T} = \frac{R}{V}$

$$\int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} \frac{C_v dT}{T} + \int_{V_1}^{V_2} \frac{R}{V} dV$$

$$(S_2 - S_1) = C_v \ln\frac{T_2}{T_1} + R \ln\frac{V_2}{V_1} \quad \text{—— ①}$$

$T\,dS = dh - v\,dP$

$dS = \frac{dh}{T} - \frac{V}{T}dP$

for ideal gas

$dh = C_p dT$; $PV = RT$

$\Rightarrow \frac{V}{T} = R/P$

$\therefore dS = \frac{C_p dT}{T} - \frac{R}{P}dP$

$$\int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} \frac{C_p dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dP$$

$$\Rightarrow \boxed{(S_2 - S_1) = C_p \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1}} \quad \text{—— ②}$$

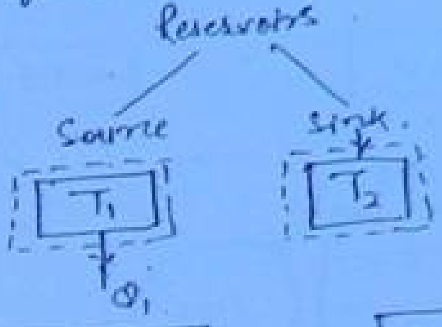
Show that for an ideal gas entropy change ~~$(S_2 - S_1)$~~ =

$$(S_2 - S_1) = C_p \ln\frac{V_2}{V_1} + C_v \ln\frac{P_2}{P_1}.$$

~~$T\,dS = dh - v\,dP$~~

~~$dh = du + v\,dP + P\,dv$~~

* Entropy change for Reservoirs



$$\boxed{(\Delta S)_{source} = -\frac{Q_1}{T_1}}$$

$$\boxed{(\Delta S)_{sink} = \frac{Q_2}{T_2}}$$

All temp must be in kelvin.

* Entropy change for a finite body :-

$$ds = \frac{dQ}{T}$$

$$\& \quad dQ = mc\,dT$$

$$\int_{S_i}^{S_f} ds = \int_{T_i}^{T_f} mc \frac{dT}{T}$$

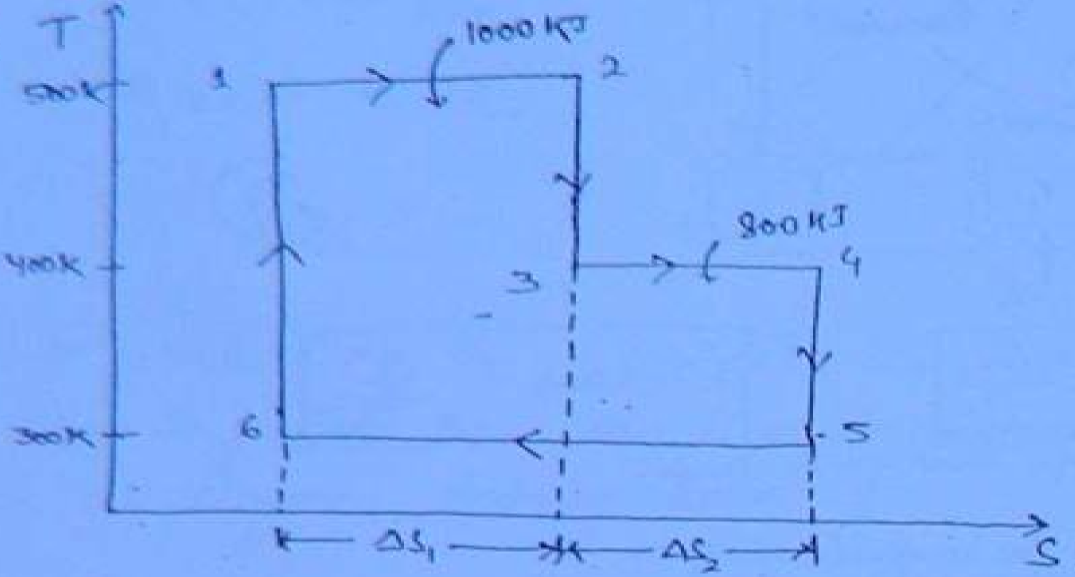
$$\Rightarrow \boxed{S_f - S_i = mc \ln \frac{T_f}{T_i}}$$

Problem :

Q. A closed system executes a reversible cycle 1-2-3-4-5-6-1 consisting of 6 processes. During process 1-2, 1000 kj of heat is added at const. temp. of 500 K. 2-3 is adiabatic expansion in which the temp decreases from 500 to 400 K. 3-4 is a constant temp. heat ~~rejection~~ addition in which 800 kj of heat is added at 400 k. 4-5 is adiabatic expansion in which the temp. decreases from 400 k to 300 k. In 5-6, heat is rejected at a constant temp. of 360 K. 6-1 is adiabatic compression. Represent the cycle on T-S diagram and calculate the net work transfer and efficiency of cycle.

Sol^n:

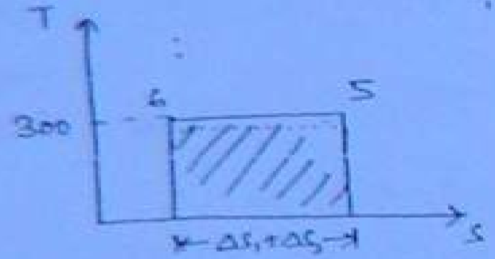
1/2



$$\Sigma W = \Sigma Q$$

$$\Rightarrow W = Q_S - Q_R$$

$$Q_S = 1000 + 800 = 1800 \text{ kj}$$



$$Q_R = 300 \times (\Delta S_1 + \Delta S_2)$$

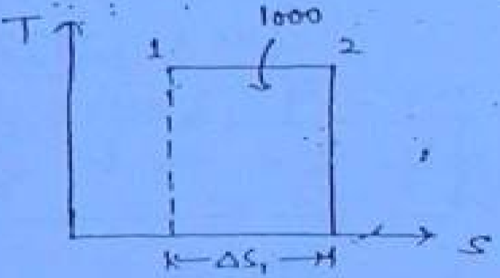
$$= 300 \times (2+2)$$

$$= 1200.$$

$$W = Q_S - Q_R$$

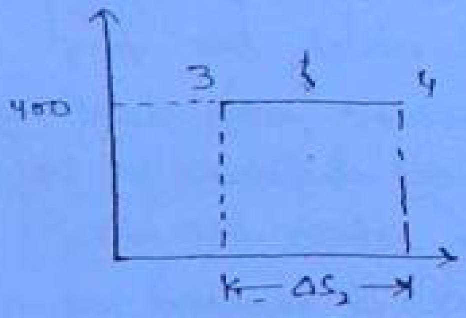
$$= 1800 - 1200$$

$$\therefore W = 600 \text{ kj}$$



$$1000 = 500 \times \Delta S_1$$

$$\Rightarrow \Delta S_1 = 2.$$



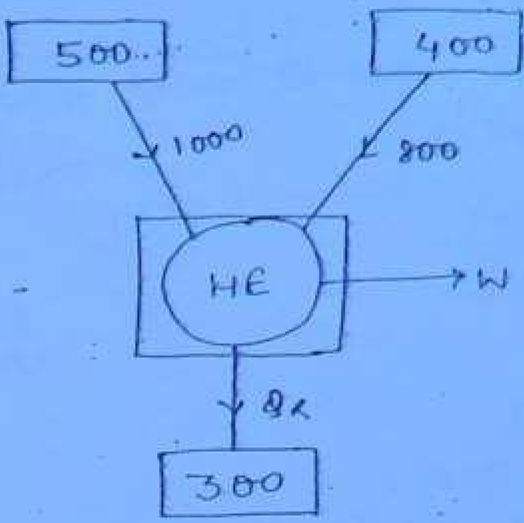
$$800 = 400 \times \Delta S_2$$

$$\therefore \Delta S_2 = 2.$$

Now efficiency,

$$\eta = W/Q_S = \frac{600}{1800} = 33.3\%$$

Another Method:



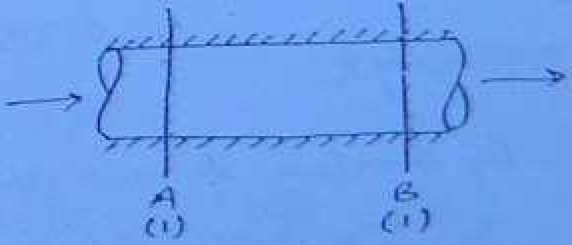
$$\oint \frac{dQ}{T} = 0 \Rightarrow \frac{1000}{500} + \frac{800}{400} - \frac{Q_R}{300} = 0.$$

$$\Rightarrow Q_R = 4\times 300 = \underline{\underline{1200\,kJ}}.$$

Q:- Air is flowing steadily in an insulated pipe, the pressure and tempr at two stations A and B are given in table, Establish the direction of flow of air. Take $C_p = 1.005\ kJ/kg\cdot K$ and $R = 0.287\ kJ/kg\cdot K$.

	A	B
Pressure	130 kPa	100 kPa
Tempr	50°C	13°C

* Pressure alone can never decide the direction. It is total energy which decides the direction



Soln: $(\Delta S)_{univ.} \geq 0$

$\Rightarrow (\Delta S)_{system} + (\Delta S)_{surr.} \geq 0$

as the system is insulated

$\Rightarrow (\Delta S)_{surr} = 0.$

$(\Delta S)_{system} \geq 0$

Let us assume, flow from A to B

Now, $(\Delta S)_{system} = S_B - S_A = C_p \ln \frac{T_B}{T_A} - R \ln \frac{P_B}{P_A}$

$= -0.0469\ kJ/kg.$

$\Rightarrow (\Delta S)_{system} < 0$.

As the entropy change is less than 0, so it is not possible hence direction is from B to A.

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Work Book
Chapter :5

(2)ⓐ

adiabatic

B - Rev.

$$dS = \frac{dQ}{T}$$

A. Irrev.

$$dS = \left(\frac{dQ}{T}\right)_{irr} + (\delta S)_{gen}.$$

$$= 0 + (\delta S)_{gen}$$

$$(\delta S)_{gen} > 0.$$

So, $\delta S > 0$.

③ $m = 10$ kg.

$C_v = \frac{5R}{2}$

$P_1 = 100$

$T_1 = 300$

$P_2 = 200$

$T_2 = 500$

$C_p - C_v = R.$

$$\Rightarrow C_p = R + \frac{5}{2}R$$

$$= \frac{7}{2}R$$

$$= \frac{7}{2}(0.287)$$

$$= 1.0045 \text{ kj/kg k}$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= 1.0045 \ln \frac{500}{300} - 0.287 \ln \frac{200}{100}$$

$$= 0.314 \text{ kj/kg k.}$$

Pay Attention to unit always.

$S_2 - S_1 = 0.314$ kj/kg-k × 10 = 3.14 kj/k.

⑥

⑤ - ⓐ All are properties, hence valid for all processes.

⑦ - (c)

$h = U + PV$

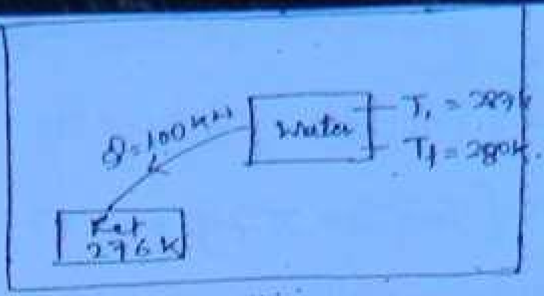
$= f(T) + mRT$

$h = f(T)$

ⓒ as $S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

hence, it is function of pressure also.

(7).



$$\Delta S = (\Delta S)_{H_2O} + (\Delta S)_{ref}$$

$$(\Delta S)_{ref} = \frac{Q}{T} = \frac{100}{7}\ \text{kW/K}.$$

$$(\Delta S)_{H_2O} = mc \ln \frac{T_f}{T_1}$$

$$= \frac{100}{7} \ln \frac{280}{287}$$

$$Q = mc\Delta T$$

$$mc = \frac{Q}{\Delta T} = \frac{100}{7}.$$

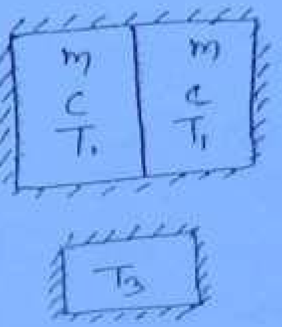
$$\text{So, } (\Delta S) = (\Delta S)_{H_2O} + (\Delta S)_{ref}.$$

$$= \frac{100}{7} \ln \frac{280}{287} + \frac{100}{7}\ \frac{\text{kW}}{\text{K}}.$$

$$= 9.55 \times 10^{-3}\ \text{kW/K} = \underline{\underline{9.55\ \text{W/K}}}$$

\underline{\underline{(d)}}.

(8) Show that mixing is irreversible ?



$\Delta S = \Delta S_1 + \Delta S_2$ — as this is extensive

$$\text{So, } \Delta S = mc \ln \frac{T_3}{T_1} + mc \ln \frac{T_3}{T_2}$$

$$= mc \ln \frac{T_3^2}{T_1 T_2}$$

$$= mc \ln \left(\frac{T_3}{\sqrt{T_1 T_2}} \right) \text{ — ①}$$

Now, heat loss = heat gain

$$mc(T_1 - T_3) = mc(T_3 - T_2)$$

$$\Rightarrow T_1 - T_3 = T_3 - T_2$$

$$\Rightarrow T_3 = \frac{T_1 + T_2}{2}$$

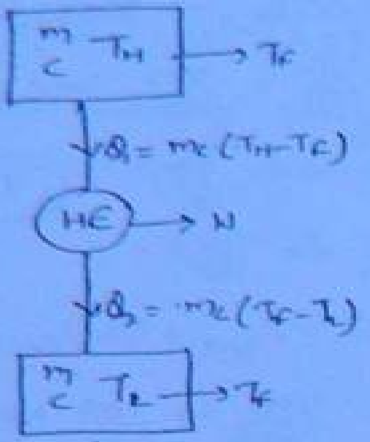
substituting it into ①, we get :-

$$\Delta S = 2mc \ln \left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right) \text{ — ② } \underline{\text{Ans.}}$$

We know that AM > GM and hence this entropy change is +ve but as the system is insulated the surrounding entropy change is 0 and hence $(\Delta S)_{universe} = (\Delta S)_{system}$. Therefore in this case $(\Delta S)_{universe} > 0$ and hence <u>mixing is an</u> irreversible process.

$AM = \frac{T_1 + T_2}{2}$ $\quad GM = \sqrt{T_1 T_2}$

⑨



So, $W = Q_1 - Q_2$

$$= mc\left[(T_H - T_F) - (T_F - T_L)\right]$$

$$= mc\left[T_H + T_L - 2T_F\right] \text{ ——— } ①$$

* As the system undergoes cycle the $(\Delta S)_{system} = 0$ because for a cycle initial and final points are same entropy being property depends only on points.

Now $(\Delta S)_{univ} \geq 0$

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$(\Delta S)_{univ.} = 0$ (rev. cycle)

$$\underset{\substack{\downarrow \\ \text{cycle}}}{(\Delta S)_{system}} + \underset{\substack{\downarrow \\ \text{surrounding}}}{(\Delta S)_{surrounding}} = 0.$$

$$0 + (\Delta S)_{surr.} = 0.$$

$$(\Delta S)_{surr.} = 0.$$

$$mc \ln \frac{T_F}{T_H} + mc \ln \frac{T_F}{T_L} = 0.$$

$$\Rightarrow mc \ln \frac{T_F}{T_H} + mc \ln \frac{T_F}{T_L} = 0.$$

$$\Rightarrow mc\left[\ln \frac{T_F^2}{T_H T_L}\right] = 0.$$

$$\Rightarrow \ln \frac{T_F^2}{T_H T_L} = 0 = \ln 1.$$

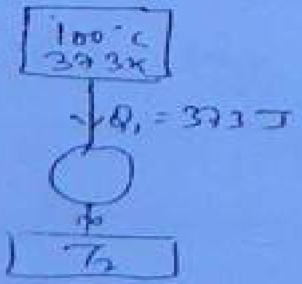
$$T_F^2 = T_H T_L$$

$$\Rightarrow T_F = \sqrt{T_H T_L}$$

and putting it into ①, we get :-

$$\boxed{W = mc\left[T_H + T_L - 2\sqrt{T_H T_L}\right]}$$

⑩ → ⓓ , ⑪ → ⓓ ⑫ – ⓒ



$$(\Delta S)_{sys} = \frac{-Q}{T}$$

$$= \frac{-373}{373}$$

$$= \underline{-1}.$$

⑬ (a) Same as 8th problem.

⑭ $ds = \frac{dq}{T}$ & $dq = mc\,dT$

$\Rightarrow ds = mc\frac{dT}{T}$

$\Rightarrow \int_{s_1}^{s_2} ds = m\int_{T_1}^{T_2} c\frac{dT}{T} = \int_{T_1}^{T_2} \left(\frac{a+bT^2}{T}\right) dT$

$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{a\,dT}{T} + \int_{T_1}^{T_2} bT\,dT$

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$= a\ln\frac{T_2}{T_1} + b\,\frac{T_2^2 - T_1^2}{2}$

$= a\ln\frac{T_2}{T_1} + 0.5b\,(T_2^2 - T_1^2)$

Ans is (a)

⑮ $Tds = dh - vdP$

$\Rightarrow Tds = dh$ as const. P

$\Rightarrow \frac{dh}{ds} = T$

So, as T ↑ $\frac{dh}{ds}$ ↑

(a) Ans.

⑯ → (d)

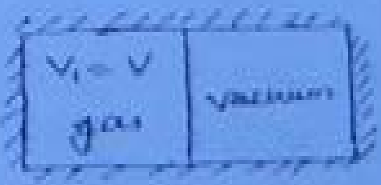
⑰ $\frac{dT}{ds} = \frac{T}{C_p} = \frac{T}{\gamma R/\gamma-1} = \frac{(\gamma-1)T}{\gamma R}$

(a) Ans

⑱ $mc\ln\frac{T_f}{T_1} \Rightarrow 1\times1\times\ln\frac{(30+273)}{(600+273)}$

$= -1.05\ kj/k.$

⑲ (i)



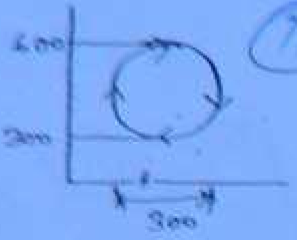
$s_2 - s_1 = C_v\ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}$

As ideal gas undergoes free expansion

So, $T_2 = T_1$

$\Delta S = C_v \ln 1 + R \ln \frac{2V}{V} = R \ln 2$ as $V_2 = 2V$
$V_1 = V$.

(20) → (a) $1 - \frac{300}{600}$



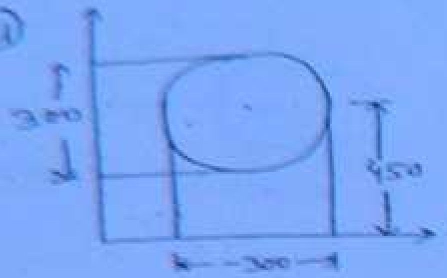
$\boxed{718}$

πr^2

$300 \times \pi \times (150)^2$ W.

$\underline{\Sigma Q = \Sigma W}$.

(21)



W = Area under curve
$= \pi/4\, d^2 = \pi/4 \times (300)^2$

$\eta = W/Q_s$

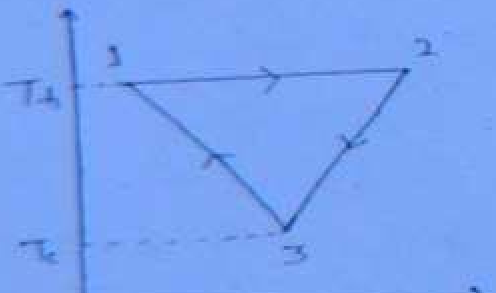
Q_s = area under curve
$= \frac{\pi}{4} \times (300)^2 \times 1/2 + 450 \times 300$

$$\eta = \frac{W}{Q_s} = \frac{\pi/4 \times (300)^2}{\pi/4 \times \frac{1}{2} \times (300)^2 + 450 \times 300}$$

$\eta = 41.4\%$ $\underline{(\text{Ans})}$.

(22) - (a). (23) - (d).

(24) Increase in entropy means heat supplied. So, to find Q_s on TS diagram take the area under curve upon which 's' is increasing.



$\eta = W/Q_s$

W = Area of Δ
$= \frac{1}{2} \times (\Delta S)(T_h - T_c)$

$Q_s = T_h \times \Delta S$

(25) - (c)

(26) - (a) $S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$

Rigid container V = const.

$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \cancel{\ln \frac{V}{V}}^{0}$

$S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$. as $T_2 < T_1$.

$(\Delta S) = -ve$. Hence Ans is ⓐ.

㉗ ⓑ

A – 2
B – 3
C – 4
D – 5

During expansion there is reduction in temperature.

㉘ – ⓐ As long as end points are same, property are remain changed.

㉙ – Ⓑ , ㉚ – Ⓑ, ㉛ – ⓒ

㉜ $S_2 - S_1 = W/S_1$ $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

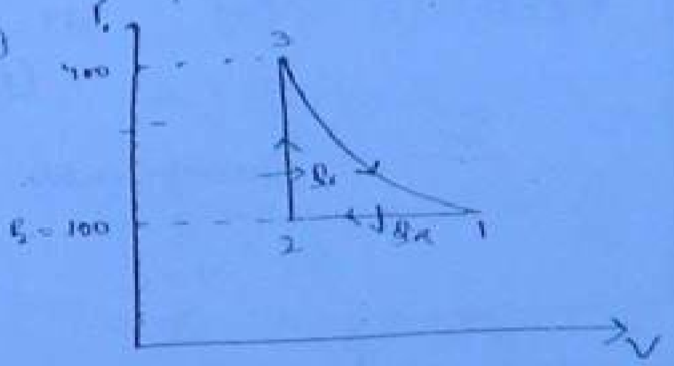
$$P_1V_1^{1.4} = P_2V_2^{1.4}$$

$$\Rightarrow \frac{400 \times 1}{100} = P_2V_2^{1.4}$$

$$\ln 4 = 1.4 \ln V_2 = \underline{2.69\ m^3}$$

$$W = \frac{P_1V_1 - P_2V_2}{1.4 - 1} = 377.5\ kJ$$

㉝ ⓐ



$$Q_v = dU = mC_v dT$$

$$Q_P = dH = mC_p dT$$

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$Q_S = mC_v(T_3 - T_2)$$

$$Q_{12} = -mC_p(T_1 - T_2)$$

$$-Q_{12} = mC_p(T_1 - T_2)$$

$$Q_R = mC_p(T_1 - T_2)$$

$$\eta = 1 - \frac{\dot m c_p (T_1 - T_2)}{\dot m c_v (T_3 - T_2)}$$

$$= 1 - \frac{\gamma \cancel{T_2}\left(\frac{T_1}{T_2} - 1\right)}{\cancel{T_2}\left(\frac{T_3}{T_2} - 1\right)}$$

(120)

$$= 1 - \frac{\gamma (T_1/T_2 - 1)}{(T_3/T_2 - 1)}$$

$$= 1 - \frac{\gamma\left(\frac{v_1}{v_2} - 1\right)}{\left(\frac{P_3}{P_2} - 1\right)}$$

$$\eta = 1 - \frac{1.4\left(\frac{2.69}{1} - 1\right)}{\left(\frac{400}{100} - 1\right)}$$

$$\therefore \eta = 0.211 = 21.1\%$$

53 — ⓑ.

4) $P_1 = 100 \quad V_1 = 0.002$

$P_2 = 125 \quad V_2 = 0.003$

$$S_2 - S_1 = C_p \ln \frac{V_1}{V_2} + C_v \ln \frac{P_2}{P_1}$$

$$= 1.005 \times \ln \frac{0.003}{0.002} + 0.718 \ln\left(\frac{125}{100}\right)$$

$$= 0.567.$$

ⓓ Ans.

⑨ — ⓐ. Isentropic

↓

Incompressible

↓

$\rho = \text{const} \Rightarrow v = \text{const}.$

$\rho = m/\text{vol}.$

$\Rightarrow v = C$

$T\,dS = dU + P\,dV$

$v = \text{const}$, so $dV = 0$.

$\therefore T\,dS = dU$

$s = \text{const}$ (given)

$ds = 0$.

$T(0) = dU$

$\Rightarrow dU = 0 \longrightarrow dU = C(dT)$

$U = \text{const} \longrightarrow 0 = C(dT)$

$U = C(T) \qquad dT = 0$

(Ans)

So.

Incompressible

↓

$V = C$

↓

$S = C$

↓

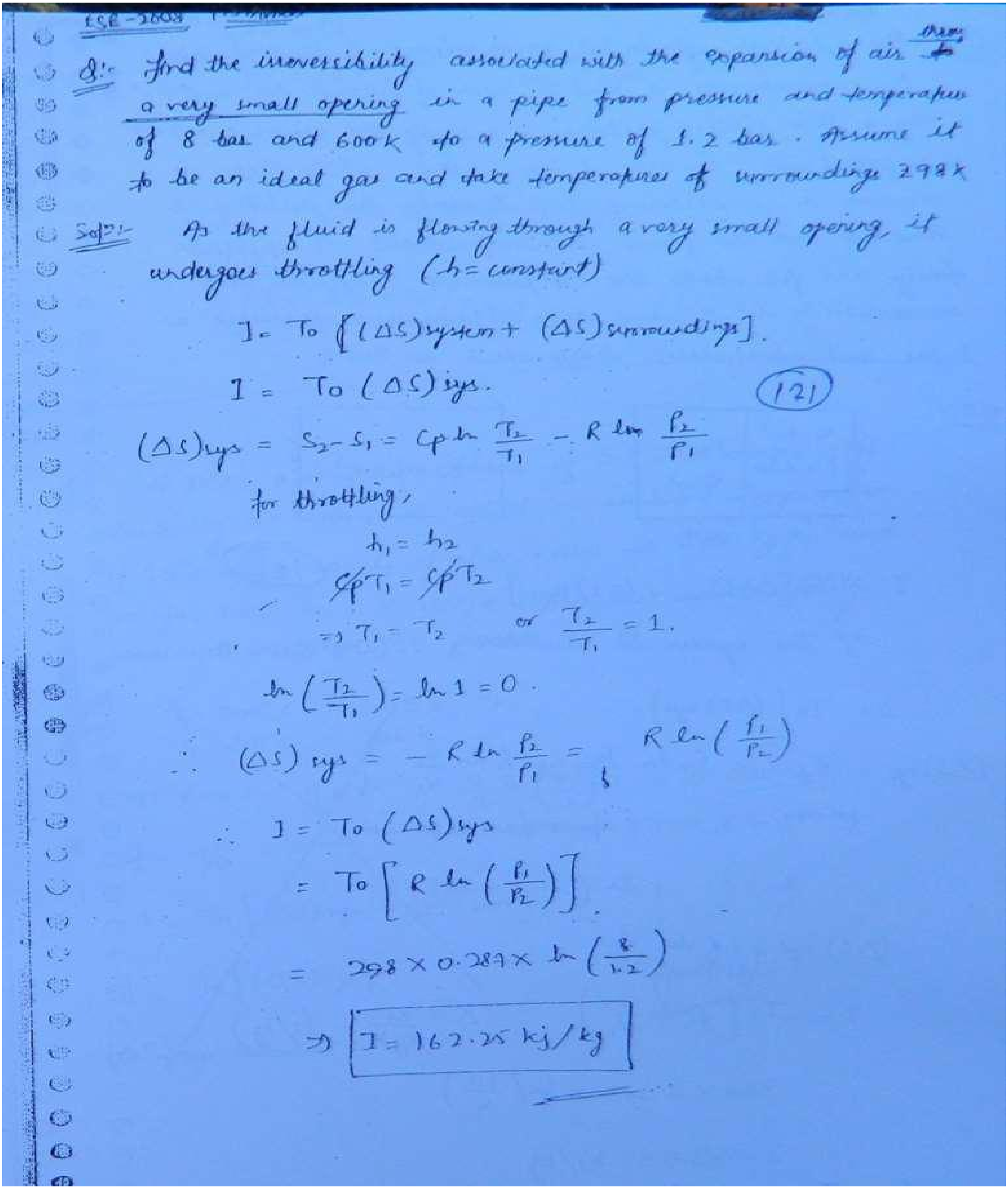
$U = C$

↓

$T = C$

⑤ → ⓒ (Ans).

UNIT -IV



ESE-2003

Q:- Find the irreversibility associated with the expansion of air ~~to~~ through a very small opening in a pipe from pressure and temperature of 8 bar and 600 K to a pressure of 1.2 bar. Assume it to be an ideal gas and take temperatures of surroundings 298 K

Sol:- As the fluid is flowing through a very small opening, it undergoes throttling (h = constant)

$$I = T_0 [(\Delta S)_{system} + (\Delta S)_{surroundings}].$$

$$I = T_0 (\Delta S)_{sys}.$$

(121)

$$(\Delta S)_{sys} = S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

for throttling,

$$h_1 = h_2$$

$$\cancel{C_p} T_1 = \cancel{C_p} T_2$$

$$\Rightarrow T_1 = T_2 \quad \text{or} \quad \frac{T_2}{T_1} = 1.$$

$$\ln \left(\frac{T_2}{T_1}\right) = \ln 1 = 0.$$

$$\therefore (\Delta S)_{sys} = -R \ln \frac{P_2}{P_1} = R \ln \left(\frac{P_1}{P_2}\right)$$

$$\therefore I = T_0 (\Delta S)_{sys}$$

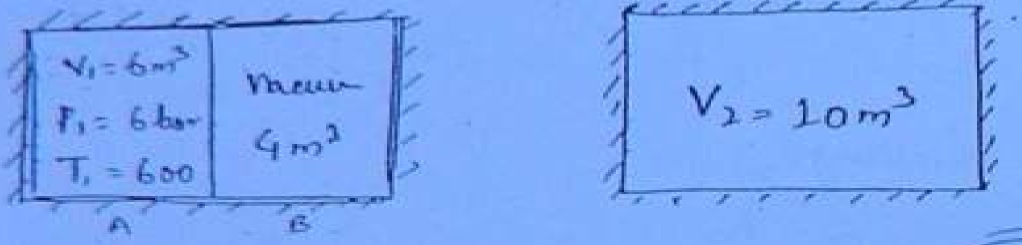
$$= T_0 \left[R \ln \left(\frac{P_1}{P_2}\right)\right].$$

$$= 298 \times 0.287 \times \ln \left(\frac{8}{1.2}\right)$$

$$\Rightarrow \boxed{I = 162.25 \text{ kJ/kg}}$$

Q:- An adiabatic cylinder of 10 m^3 volume is divided into two compartments 'A' and 'B', each of volume 6m^3 and 4m^3 by a thin sliding partition. Initially the compartment 'A' is filled with air at 6 bar, 600 K, while there is vacuum in compartment B. Suddenly, the partition is removed and the fluid in compartment 'A' expands freely and fills both the compartments. Calculate the irreversibility in kJ and take atmospheric pressure as 1 bar and atmospheric temperature as 300 K.

Sol:-



$I = T_0[(\Delta S)_{sys} + (\Delta S)_{surr.}]$

∵ The system is insulated, ∴ $(\Delta S)_{surr} = 0$.

$\therefore I = T_0[(\Delta S)_{sys}]$

$(\Delta S)_{sys} = S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$;

$T_1 = T_2$ (from free expansion)

$\therefore \frac{T_2}{T_1} = 1 \rightarrow \ln\left(\frac{T_2}{T_1}\right) = \ln 1 = 0$.

$(\Delta S)_{sys} = R \ln \frac{V_2}{V_1}$

$I = T_0\left(R \ln \frac{V_2}{V_1}\right)$ $\quad K \times \frac{kJ}{kg K} = \underline{kJ/kg)}$

$= 300 \times 0.287 \times \ln\left(\frac{10}{6}\right)$

$\therefore I = 43.98$ kJ/kg.

$PV = mRT$

$m = PV/RT = \frac{6 \times 100 \times 6}{0.287 \times 600}$

$\therefore m = 20.9\ kg,$

$\therefore I = (43.98 \times 20.9)\ \frac{kj}{\cancel{kg}} \times \cancel{kg}$

$\therefore \boxed{I = 919.2\ kj}$ (Ans) (123)

Q:- A solid aluminium sphere of 0.1 m diameter and initially at 200°c is cooled to ambient temperature of 25°c. The density of aluminium is 2700 kg/m³ and specific heat is 0.9 kj/kg K. find the irreversibility associated with ~~pr~~ this process in kj.

Sol:- ~~$T_1 = 200°C = 473\ k.$~~

~~$T_2 = 25°C = 298\ k,$~~

~~$Vol^m\ Area = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{0.1}{2}\right)^3 = 0.00052\ m^3$~~

~~$\rho = m/v \Rightarrow m = v \times \rho = 0.00052 \times 2700 = 1.414\ kg.$~~

~~$I = T_0\left[(\Delta S)_{sys} + (\Delta S)_{surr}\right]$~~

~~$I = T_0\left[(\Delta S)_{sys}\right]$~~

~~$(\Delta S)_{sys} =$~~

(P.T.O)

ESE -

Q: Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kJ/kg and at the exit is 2585 kJ. The values of flow availability at the entrance and exit are 1787 kJ and 140 kJ/kg respectively. The ambient temperature is 300K.

find :- ① actual work

② max^m work

③ change in entropy of the system. (Neglect K.E & P.E changes).

125

Sol: $W_{max} = \phi_1 - \phi_2 = 1787 - 140 = 1647$ kJ/kg.

$h_1 = 4142$ kJ/kg, $h_2 = 2585$ kJ/kg.

$\phi_1 = 1787$ kJ/kg, $\phi_2 = 140$ kJ/kg

Steady flow equation:

$$h_1 + \cancel{\frac{c_1^2}{2}} + \cancel{z_1 g} + \cancel{q} = h_2 + \cancel{\frac{c_2^2}{2}} + \cancel{z_2 g} + w_{cv}$$

(adiabatic)

(Neglecting K.E & P.E changes).

ⓐ $\therefore \boxed{W_{(actual)} = h_1 - h_2 = 1557 \text{ kJ/kg}}$

ⓑ $W_{max} = \phi_1 - \phi_2$ ($\because$ the system is open).

$= (1787 - 140) = 1647$ kJ/kg.

ⓒ $W_{max} = h_1 - h_2 - T_0(S_1 - S_2)$ [$\because$ the system is open].

$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$

$\Rightarrow (S_1 - S_2) = - 0.3$ kJ/kg-K

$\therefore (\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = \underline{\underline{0.3 \text{ kJ/kg K}}}$.

(Ans)

ESE-

Q:- Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kJ/kg and at the exit is 2585 kJ. The values of flow availability at the entrance and exit are 1787 k and 140 kJ/kg respectively. The ambient temperature is 300K.

(125)

find :- ① actual work
② max^m work
③ change in entropy of the system. (Neglect K.E & P.E changes).

Sol:- $W_{max} = \phi_1 - \phi_2 = 1787 - 140 = 1647$ kJ/kg.

$h_1 = 4142$ kJ/kg, $h_2 = 2585$ kJ/kg.
$\phi_1 = 1787$ kJ/kg, $\phi_2 = 140$ kJ/kg

Steady flow equation:-

$$h_1 + \cancel{\frac{c_1^2}{2}} + \cancel{z_1 g} + \overset{0}{\cancel{Q}} = h_2 + \cancel{\frac{c_2^2}{2}} + \cancel{z_2 g} + W_{cv}$$

(adiabatic) (Neglecting K.E & P.E changes).

ⓐ $\therefore \boxed{W_{(actual)} = h_1 - h_2 = 1557 \text{ kJ/kg}}$

ⓑ $W_{max} = \phi_1 - \phi_2$ ($\because$ The system is open).

$= (1787 - 140) = 1647$ kJ/kg.

ⓒ $W_{max} = h_1 - h_2 - T_0(S_1 - S_2)$ [$\because$ The system is open].

$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$

$\Rightarrow (S_1 - S_2) = -0.3$ kJ/kg-K.

$\therefore (\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = 0.3$ kJ/kgK.

(Ans)

ESE-

Q: Steam flows steadily in an adiabatic turbine, the enthalpy at the entrance is 4142 kJ/kg and at the exit is 2585 kJ. The values of flow availability at the entrance and exit are 1787 kJ and 140 kJ/kg respectively. The ambient temperature is 300K.

find :- ① actual work

125

② max^m work

③ change in entropy of the system. (Neglect K.E & P.E changes).

Sol: $W_{max} = \phi_1 - \phi_2 = 1787 - 140 = 1647$ kJ/kg.

$h_1 = 4142$ kJ/kg, $h_2 = 2585$ kJ/kg.

$\phi_1 = 1787$ kJ/kg, $\phi_2 = 140$ kJ/kg

Steady flow equation:

$$h_1 + \cancel{\frac{C_1^2}{2}} + \cancel{Z_1 g} + \underset{\text{(adiabatic)}}{\cancel{Q}} = h_2 + \cancel{\frac{C_2^2}{2}} + \cancel{Z_2 g} + W_{CV}$$

(Neglecting K.E & P.E changes).

ⓐ ∴ $\boxed{W_{(actual)} = h_1 - h_2 = 1557 \text{ kJ/kg}}$

ⓑ $W_{max} = \phi_1 - \phi_2$ (∵ The system is open).

$= (1787 - 140) = 1647$ kJ/kg.

ⓒ $W_{max} = h_1 \ h_2 - T_0(S_1 - S_2)$ [∵ The system is open].

$\Rightarrow 1647 = 4142 - 2585 - 300(S_1 - S_2)$

$\Rightarrow (S_1 - S_2) = -0.3$ kJ/kg-K

∴ $(\Delta S) = (S_2 - S_1) = -(S_1 - S_2) = 0.3$ kJ/kg K.

(Ans)

Q:- Find the maximum work per kg of air that can be obtained in a piston cylinder arrangement. Air expands from initial state of 9 bar, 400 K to a final state of 1.5 bar, 300 K. Assume atmospheric pressure to be 1 bar and atmospheric temperature to be 288 K. Also find maximum useful work.

(126)

Ans:- $W_{max} = U_1 - U_2 - T_0(S_1 - S_2)$ [for closed system].

∵ Air can be treated as ideal gas

$\therefore U = mC_vT \quad \because m = 1\ kg.$

$\therefore U = C_vT.$

$\therefore W_{max} = C_vT_1 - C_vT_2 - T_0(S_1 - S_2)$

$= C_v(T_1 - T_2) - T_0(S_1 - S_2)$

Now,

$P_1 = 9\ bar \qquad T_1 = 400\ K$

$P_2 = 1.5\ bar \qquad T_2 = 300\ K.$

$T_0 = 288\ K.$

Now,

$$(S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= 1.005 \ln\left(\frac{300}{400}\right) - 0.287 \ln\left(\frac{1.5}{9}\right)$$

$\therefore (S_2 - S_1) = 0.225\ kJ/kgK.$

$\therefore (S_1 - S_2) = -0.225\ kJ/kg\ K.$

$\therefore W_{max} = C_v(T_1 - T_2) - T_0(S_1 - S_2)$

$= 0.718(400 - 300) - 288(-0.225)$

$\Rightarrow \boxed{W_{max} = 136.6 \text{ kj/kg}}$ (Ans)

Maximum useful work,

$W_{max\ useful} = W_{max} - W_{atm}.$

$W_{atm} = P_0(V_2 - V_1)$

Now, $PV = mRT$

$$\Rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.287 \times 400}{9 \times 100} = 0.1275 \text{ m}^3.$$

Similarly.

$$V_2 = \frac{mRT_2}{P_2} = \frac{1 \times 0.287 \times 300}{1.5 \times 100} = 0.574 \text{ m}^3.$$

$$\therefore W_{atm} = \cancel{1\times100}\ P_0(V_2 - V_1)$$

$$= 1 \times 100(0.574 - 0.1275)$$

$$= 44.65 \text{ kj/kg}$$

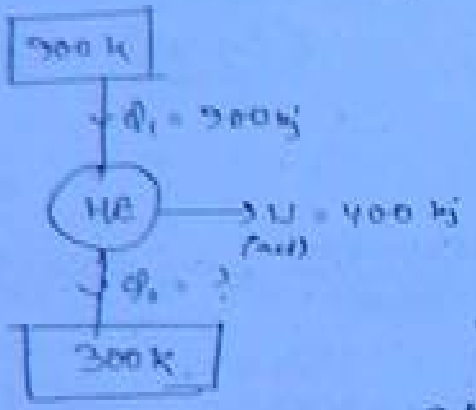
$$\therefore W_{max\ useful} = W_{max} - W_{atm}$$

$$= (136.6 - 44.65) \text{ kj/kg}$$

$\boxed{W_{max\ useful} = 91.95 \text{ kj/kg.}}$ (Ans)

Q:- 7.1) A heat engine operates between two thermal reservoirs at 900 k and 300 k. The heat supplied to the engine is 900 kj. The work output is 400 kj. Find the irreversibility. Surrounding temp is 300 K. (128)

Infr



~~$I = T_0 [(\Delta S)_{sys} + (\Delta S)_{surr}]$.~~

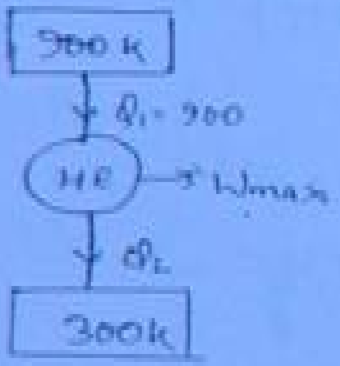
~~$W = Q_1 - Q_2$~~

~~$\Rightarrow Q_2 = Q_1 - W = 500 kj$~~

$\Rightarrow W_{act} = 400 kj$

$I = W_{max} - W_{act}$.

Let us assume the engine to be reversible.



Wmax is only for reversible engine.

for reversible engine,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow \frac{900}{900} = \frac{Q_2}{300}$$

$$\Rightarrow \underline{Q_2 = 300 kj}$$

$\therefore W_{max} = Q_1 - Q_2 = (900 - 300) = 600 kj$

Now, $I = W_{max} - W_{act}$

$= (600 - 400)$

$\therefore \boxed{I = 200 kj}$ (Ans)

→ MIXTURE OF IDEAL GASES ←

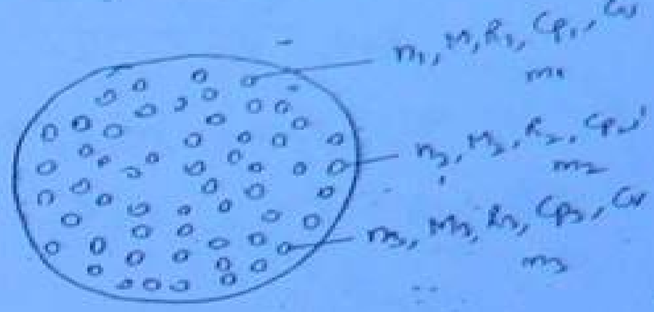
⇒ **Mole fraction (x)** :- It is defined as the ratio of no. of moles of a gas to the total number of moles.

$$x = \frac{n}{\Sigma n}$$

$$x_1 = \frac{n_1}{n_1 + n_2 + \dots}$$

$$x_2 = \frac{n_2}{n_1 + n_2 + \dots}$$

$$\boxed{x_1 + x_2 + \dots = 1}$$



$V_1 = V_2 = V_3 = V$

$T_1 = T_2 = T_3 = T$

$P = P_1 + P_2 + P_3$.

$PV = mRT$

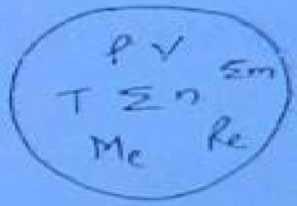
$PV = n\bar{R}T$

$P_1 V_1 = n_1 \bar{R} T_1$

but $V_1 = V_2 = V_3 = V$ & $T_1 = T_2 = T_3 = T$

$\therefore P_1 V = n_1 \bar{R} T$ —①

$PV = \Sigma n \bar{R} T$ —②



Dividing ① by ② we have

$$\frac{P_1 \cancel{V}}{P \cancel{V}} = \frac{n_1 \cancel{\bar{R}T}}{\Sigma n \cancel{\bar{R}T}} \quad \therefore \frac{P_1}{P} = \frac{n_1}{\Sigma n}$$

But, $\frac{n_1}{\Sigma n} = x_1$

$$\therefore \frac{P_1}{P} = x_1$$

∴ Partial pressure = Mole fraction × Total pressure.

or $P_1 = x_1 P$

Similarly, $P_2 = x_2 P$

⋮

⇒* EQUIVALENT GAS CONSTANT (R_e) ! →

$$PV = mRT$$

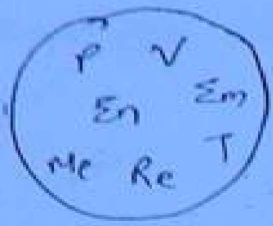
$$P_1V_1 = m_1R_1T_1$$

But $V_1 = V_2 = V_3 = \cdots = V$ & $T_1 = T_2 = \cdots = T$.

$$\therefore P_1V = m_1R_1T$$

Similarly $P_2V = m_2R_2T$

$$\vdots$$



$$(P_1+P_2+\cdots)V = (m_1R_1+m_2R_2+\cdots)T$$

But, $(P_1+P_2+\cdots) = P$.

$$\therefore PV = (m_1R_1+m_2R_2+\cdots)T \quad \text{—①}.$$

But

$$PV = \Sigma m R_e T \quad \text{—②}$$

from ① & ②.

$$\Sigma m R_e \cancel{T} = (m_1R_1 + m_2R_2+\cdots)\cancel{T}$$

$$\therefore R_e = \frac{m_1R_1 + m_2R_2 + \cdots}{\Sigma m}$$

or,

$$\boxed{R_e = \frac{m_1R_1 + m_2R_2 + \cdots}{m_1+m_2+\cdots}}$$

Similarly,

$$\boxed{C_{pe} = \frac{m_1C_{p_1} + m_2C_{p_2} + \cdots}{m_1+m_2+\cdots}}$$

$$\boxed{C_{v_e} = \frac{m_1 C_{v_1} + m_2 C_{v_2} + \cdots}{m_1 + m_2 + \cdots}}$$

⇒ * EQUIVALENT MOLECULAR WEIGHT (M_e) :-

P V

T Σm

Σn Re

Me

$$n = \frac{m}{M}$$

m ~~$= N$~~

$$m = nM$$

$$m_1 = n_1 M_1$$

$$m_2 = n_2 M_2$$

$$\vdots$$

$$m_1 + m_2 + \cdots = n_1 M_1 + n_2 M_2 + \cdots$$

$$\Rightarrow \Sigma m = (n_1 M_1 + n_2 M_2 + \cdots) \quad \text{———} ①$$

Also

$$\Sigma m = \Sigma n M_e \quad \text{———} ②$$

from ① & ②

$$\Sigma n M_e = (n_1 M_1 + n_2 M_2 + \cdots)$$

$$\Rightarrow M_e = \frac{(n_1 M_1 + n_2 M_2 + \cdots)}{\Sigma n}$$

$$\text{or,} \quad M_e = \frac{n_1}{\Sigma n} \cdot M_1 + \frac{n_2}{\Sigma n} \cdot M_2 + \cdots$$

$$\boxed{M_e = x_1 M_1 + x_2 M_2 + \cdots}$$

⇒ Equations to be Remembered:

1. $PV = mRT$; $PV = n\bar{R}T$

2. $x = \frac{n}{\sum n}$; 3. $P_1 = x_1 P$; $P_2 = x_2 P$. and so on

4. $$R_e = \frac{m_1R_1 + m_2R_2 + \cdots}{m_1 + m_2 + \cdots}$$

5. $$C_{pe} = \frac{m_1C_{p_1} + m_2C_{p_2} + \cdots}{m_1 + m_2 + \cdots}$$

6. $$C_{ve} = \frac{m_1C_{v_1} + m_2C_{v_2} + \cdots}{m_1 + m_2 + \cdots}$$

7. $M_e = x_1 M_1 + x_2 M_2 + \cdots$

8. $R = \frac{\bar{R}}{M}$.

:- A mixture of ideal gases consists of 3 kg of nitrogen and 5 kg of Carbon dioxide at a pressure of 300 kPa and temperature of 20°C. find ① Mole fraction of each constituent

② Equivalent molecular weight of the mixture

③ Equivalent gas constant

④ Partial pressure of each constituent

⑤ Total volume of the mixture

⑥ Density of the mixture

⑦ 'Cp' and 'Cv' of the mixture

⑧ If the mixture is heated at constant volume to 40°C, find changes in internal energy, enthalpy and entropy of the mixture.

(1) If the mixture is heated at constant pressure to 40°C, find changes in internal energy, enthalpy and entropy of the mixture.

Solⁿ: Take 'γ' for nitrogen as 1.4 & 'γ' for CO_2 as 1.286

$N_2 - 1$; $CO_2 \rightarrow 2$. (133)

$m_1 = 3\,kg$, $m_2 = 5\,kg$.

$M_1 = 28$; $M_2 = 44$.

$n = \frac{m}{M}$ ∴ $n_1 = \frac{m_1}{M_1} = \frac{3}{28} = 0.$~~107142~~

$n_2 = \frac{m_2}{M_2} = \frac{5}{44} =$

Now

① $x = \frac{n}{\Sigma n}$; ∴ $x_1 = \frac{n_1}{n_1+n_2} = \frac{3/28}{3/28 + 5/44} =$

∴ $\boxed{x_1 = 0.485}$

$x_2 = \frac{n_2}{n_1+n_2} = \frac{5/44}{3/28 + 5/44} = \boxed{0.515}$ (An)

② $M_e = x_1 M_1 + x_2 M_2$

$= 0.485 \times 28 + 0.515 \times 44 =$

$\Rightarrow \boxed{M_e = 36.25}$

③ $R_e = \frac{\bar{R}}{M_e} = \frac{8.314}{36.25}$

$\boxed{R_e = 0.229\ kJ/kgK}$

④ $P_1 = x_1 P \Rightarrow P_1 = 0.485 \times 300$

$\Rightarrow \boxed{P_1 = 145.5\ kPa}$

$P_2 = x_2 P \Rightarrow P_2 = 0.515 \times 300 \Rightarrow \boxed{P_2 = 154.5}$

⑤ $PV = \Sigma m R_c T$

$$\Rightarrow V = \frac{8 \times 0.229 \times 293}{300}$$

$$\therefore \boxed{V = 1.789 \text{ m}^3}$$

⑥ Density, $\rho = \frac{\Sigma m}{V} = \frac{8}{1.789} = 4.47 \text{ kg/m}^3$.

$$\therefore \boxed{\rho = 4.47 \text{ kg/m}^3}$$

⑦ $$C_{pc} = \frac{m_1 C_{p1} + m_2 C_{p2}}{m_1 + m_2}$$

$$C_{p1} = \frac{\gamma R}{\gamma - 1}$$

$$R_1 = \frac{\bar{R}}{M_1} = \frac{8.314}{28} = 0.2969$$

$$\Rightarrow C_{p1} = \frac{1.4 \times 0.2969}{(1.4 - 1)}$$

$$\therefore C_{p1} = 1.039$$

$$R_2 = \frac{8.314}{44} = 0.18895$$

$$C_{p2} = \frac{1.286 \times .189}{1.286 - 1}$$

$$\therefore C_{p2} = 0.849 \simeq 0.85$$

$$\therefore C_{pc} = \frac{3 \times 1.039 + 5 \times 0.849}{8}$$

$$\boxed{C_{pc} = 0.92}$$ (Ans)

Now,

[illegible] $C_{pe} - C_{ve} = R_e$. [$\because C_p - C_v = R$]

$\Rightarrow C_{ve} = C_{pe} - R_e$

$= (0.92 - 0.229)$

$\therefore \boxed{C_{ve} = 0.6916 \text{ kj/kgK.}}$

⑧. $T_2 = 40°C$, $T_1 = 20°C$

(a) $dU = m C_{ve} dT$

$= 8 \times 0.6916 \times 20 = 110.656$ kj

(b) $dH = m C_{pe} dT$

$= 8 \times 0.9206 \times 20$

$\Rightarrow dH = 147.296$ kj/~~kg~~

(c) $dS = (S_2 - S_1) = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$

$\because$ vol^m is contant, $\therefore V_1 = V_2$

$\therefore \ln \frac{V_2}{V_1} = \ln 1 = 0.$

$\therefore (S_2 - S_1) = C_v \ln \frac{T_2}{T_1}$

$= 0.6916 \ln\left(\frac{313}{293}\right)$

$\therefore dS = 0.0459$ kj/kg K

$\therefore$ Total Entropy change $= (0.045 \times 8)$ kj/k

$dS = \boxed{0.364 \text{ kj/k}}$

⑨ $dU = mC_v dT = 110.656$ kj

$dH = mC_p dT = 147.296$ kj

<u>Ans.</u> $dS = (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

$\because$ Pressures are constant.

$\therefore P_1 = P_2$.

$\therefore \ln \frac{P_2}{P_1} = \ln 1 = 0$.

$\therefore (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} = 0.9106 \times \ln \left(\frac{313}{293}\right)$

$\therefore dS = (S_2 - S_1) = 0.0607$ kj/kg k

$\therefore$ Total Entropy change $= 8 \times 0.0607$

$$\boxed{dS_{(Total)} = 0.486 \text{ kj/k}}$$ (Ans)

⇒ THERMODYNAMIC RELATIONS :-

<u>Theorem : 1</u> The equation $(dZ = Mdx + Ndy)$ is exact when

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

<u>Theorem :- 2</u>

$$f = \phi(x, y, z)$$

$$\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1.$$

<u>Theorem :- 3</u>

$$z = \phi(x, y)$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1.$$

⇒ * MAXWELL'S EQUATIONS :→

① $Tds = dU + PdV$

$dU = Tds - PdV$

$dz = Mdx + Ndy$

$M = T;\ x = S;\ N = -P;\ y = V.$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V}$$

This is first Maxwell's equation.

② $Tds = dh - VdP$

$\Rightarrow dh = Tds + VdP$

$dz = Mdx + Ndy$

$M = T;\ x = S;\ N = V;\ y = P$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P}$$

This is second Maxwell's equation

③ $G = H - TS$ (Gibb's function)

$dG = dH - (Tds + SdT)$

$\left[\because Tds = dH - VdP \Rightarrow dH - Tds = VdP\right]$

or, $dG = \underbrace{dH - Tds}_{} - SdT$

$\therefore dG = VdP - SdT$

$dz = Mdx + Ndy$

$M = V;\ x = P;\ N = -S;\ y = T.$

$(\partial M/\partial y)_x = (\partial N/\partial x)_y$

$$\therefore \boxed{\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial T}\right)_T}$$

This is third Maxwell's equation

④ From Helmholtz's function :

~~[illegible]~~

$$F = U - TS$$

$$dF = dU - (TdS + SdT)$$

$$\Rightarrow dF = \underbrace{dU - TdS}_{} - SdT \quad \because \left[\begin{array}{l} TdS = dU + PdV \\ \Rightarrow dU - TdS = -PdV \end{array} \right)$$

$$\Rightarrow dF = -PdV - SdT$$

$$dz = Mdx + Ndy \qquad ; M = -P;\ x = V;\ N = -S;\ y = T.$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\therefore \boxed{+\left(\frac{\partial P}{\partial T}\right)_V = +\left(\frac{\partial S}{\partial V}\right)_T}$$

$$\Rightarrow \boxed{\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$$

This is 4th Maxwell's equation

Maxwell's Equation :

① $\boxed{\left(\frac{\partial T}{\partial V}\right)_S = \left(-\frac{\partial P}{\partial S}\right)_V}$

② $\boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P}$

③ $\boxed{\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T}$

④ $\boxed{\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$

⇒* Tds Equations :-

$$S = f(T, V)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_V dT + \left(\frac{\partial s}{\partial V}\right)_T dV$$

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_V dT + T\left(\frac{\partial s}{\partial V}\right)_T dV \quad \text{—— ①}$$

Now At constant pressure, i.e. $P=C$

$$\frac{dT}{ds} = T/C_p$$

$$\left(\frac{\partial T}{\partial s}\right)_p = T/C_p$$

$$C_p = T\left(\frac{\partial s}{\partial T}\right)_p \quad \text{—— ⓐ}$$

Similarly, $C_v = T\left(\frac{\partial s}{\partial T}\right)_v$ —— ⓑ

Also, from Maxwell's Equation :-

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v \quad \text{—— ⓒ}$$

$$\& \quad \left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P \quad \text{—— ⓓ}$$

Substituting from ⓑ & ⓒ in eq: ①, we have -

$$\boxed{Tds = C_v\,dT + T\left(\frac{\partial P}{\partial T}\right)_v dv} \quad \text{—— Ⓐ}$$

This is known as first Tds Equation

$S = f(T,P)$

$$dS = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial P}\right)_T dP \ \ldots$$

$$TdS = T\left(\frac{\partial s}{\partial T}\right)_P dT + T\left(\frac{\partial s}{\partial P}\right)_T dP \quad \text{——} ②$$

substituting from ③ & ④ in eqⁿ ②, we have :-

$$\boxed{TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP} \quad \text{——} Ⓑ$$

This is known as second Tds equation.

Now.

Equating both T-ds equations A & B, we have

$$TdS = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_V dV = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$$

$$\text{or, } T\left(\frac{\partial V}{\partial T}\right)_P dP + T\left(\frac{\partial P}{\partial T}\right)_V dV = (C_p - C_v)\, dT$$

$$\text{or, } dT = \boxed{\frac{T}{(C_p - C_v)}\left(\frac{\partial V}{\partial T}\right)_P} dP + \boxed{\frac{T}{(C_p - C_v)}\left(\frac{\partial P}{\partial T}\right)_V} dV \quad \text{——} ①$$

$$T = \phi(P,V)$$

$$dT = \boxed{\left(\frac{\partial T}{\partial P}\right)_V} dP + \boxed{\left(\frac{\partial T}{\partial V}\right)_P} dV \quad \text{——} ②$$

Comparing eqⁿ ① & ②, we have :-

$$\frac{T}{C_p - C_v}\left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial T}{\partial P}\right)_V$$

$$(C_p - C_v) = T\left(\frac{\partial V}{\partial T}\right)_P \boxed{\left(\frac{\partial P}{\partial T}\right)_V} \quad \text{—— ③}$$

Again, $T = \phi(P, V)$

from third theorem we have :-

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

or, $-\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial P}{\partial T}\right)_V$ —— ④

Substituting eqn ④ in eqn ③ we have :-

$$(C_p - C_v) = T\left(\frac{\partial V}{\partial T}\right)_P \left[-\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P\right]$$

$$\boxed{C_p - C_v = -T\left[\left(\frac{\partial V}{\partial T}\right)_P^2\right]\left(\frac{\partial P}{\partial V}\right)_T} \quad \text{—— (C)}$$

This equation has been asked 9 times.

$$C_p - C_v = -\overset{(+)ve}{T}\left[\overset{(+)ve}{\left(\frac{\partial V}{\partial T}\right)_P}\right]^2 \overset{(-)ve}{\left(\frac{\partial P}{\partial V}\right)_T}$$

$$\boxed{\frac{dP}{dV} = -P/V}$$

$C_p - C_v = +ve$

or, $C_p - C_v > 0$

$$\boxed{C_p > C_v} \quad \text{—— (D)}$$

for an isothermal process $\left(\frac{\partial P}{\partial V}\right)$ is always negative

square of a quantity is always positive

Temperature can never be negative

⇒* CO-EFFICIENT OF VOLUME EXPANSIVITY (β):-

It shows variation of volume with respect to temperature under isobaric conditions.

$$\boxed{\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}$$ —— Ⓔ

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⇒* ISOTHERMAL COMPRESSIBILITY (K_T):-

This shows variation of volume with respect to pressure under isothermal conditions.

$$\boxed{K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T}$$ —— Ⓕ

**

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \Rightarrow \left(\frac{\partial P}{\partial V}\right)_T = -\frac{1}{K_T V}$$

Again $$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \beta V$$

Now $$C_P - C_V = -T\left[\left(\frac{\partial V}{\partial T}\right)_P\right]^2\left(\frac{\partial P}{\partial V}\right)_T$$

$$C_P - C_V = -T\left[\beta^2 V^{\cancel{2}}\right] \times \frac{-1}{K_T \cancel{V}}$$

****

$$\boxed{C_P - C_V = \frac{TV\beta^2}{K_T}}$$ —— Ⓖ

This has been asked 13 times.

$\Rightarrow$ * ENERGY EQUATION :→

$$TdS = dU + PdV$$

$$dU = TdS - PdV$$

$$\Rightarrow dU = \underbrace{C_v dT + T\left(\frac{\partial P}{\partial T}\right)_V dV}_{\text{from 1st T-ds Equation}} - PdV$$

$$\Rightarrow dU = \boxed{C_v} dT + \boxed{\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]} dV \quad \text{—— ①}$$

Now $U = f(T, V)$.

$$dU = \boxed{\left(\frac{\partial U}{\partial T}\right)_V} dT + \boxed{\left(\frac{\partial U}{\partial V}\right)_T} dV \quad \text{—— ②}$$

Comparing equation ① & ②, we have.

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V \; ; \quad \& \quad \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Now for an ideal gas:

$$PV = mRT$$

$$P = \frac{mR}{V} \cdot T$$

for constant volume, this is of the form $y = cx$ ($\because \frac{mR}{V} = c$

$$\therefore \frac{dy}{dx} = c$$

$$\therefore \left(\frac{\partial P}{\partial T}\right)_V = \frac{mR}{V}$$

Also, $\frac{mR}{V} = \frac{P}{T}$

$$\therefore \left(\frac{\partial P}{\partial T}\right)_V = \frac{mR}{V} = \frac{P}{T}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = P/T$$

$$T\left(\frac{\partial P}{\partial T}\right)_V = P$$

$$\text{or } T\left(\frac{\partial P}{\partial T}\right)_V - P = 0 \quad \text{——} \ ③$$

Putting the value of ③ in eq^n ①:-

$$\boxed{dU = C_v\, dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] dV.} \quad \text{——} \ Ⓗ$$

This is energy equation.

for ideal gas,

$$\left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] = 0.$$

$$\therefore \boxed{dU = C_v\, dT} \quad \text{——} \ ①$$

This is for an ideal gas.

for an ideal gas:-

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P = 0.$$

It means $\boxed{\left(\frac{\partial U}{\partial V}\right)_T = 0.}$ —— Ⓑ Ⓠ

i.e. ((The internal ~~change due~~ energy of an ideal gas does not change even if the volume changes.

It means internal energy is independent of volume for an ideal gas.))

With Respect to (α):-

For an ideal gas under isothermal conditions, there is no change of internal energy with respect to volume. i.e. internal energy of an ideal gas is independent of volume.

x

$$U = f(T, P, V)$$

$$T = \phi(U, P, V)$$

from 2nd theorem, we have:-

$$\left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial U}\right)_T = 1$$

$$\left(\frac{\partial U}{\partial V}\right)_T = \left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T$$

for an ideal gas, $\left(\frac{\partial U}{\partial V}\right)_T = 0$.

$$0 = \left(\frac{\partial U}{\partial P}\right)_T \cdot \left(\frac{\partial P}{\partial V}\right)_T$$

$\hookrightarrow$ This is always negative.

$$\boxed{\left(\frac{\partial U}{\partial P}\right)_T = 0} \quad \text{——} \; (\beta)$$

With respect to (β):-

This shows that the internal energy of an ideal gas does not vary with respect to pressure under isothermal conditions. Therefore, internal energy of an ideal gas is a function of temperature.

⇒* JOULE THOMSON COEFFICIENT (μ) :-

Joule Thomson co-efficient shows variation of temperature with respect to pressure under throttling or isenthalpic conditions.

$$\boxed{\mu = \left(\frac{\partial T}{\partial P}\right)_h} \text{ —— (I)}$$

⇒*

$$T\,ds = dh - v\,dP$$

$$dh = T\,ds + v\,dP$$

$$\Rightarrow dh = C_p\,dT - T\left(\frac{\partial v}{\partial T}\right)_P dP + v\,dP$$

(from 2nd Tds Equation)

$$\Rightarrow dh = C_p\,dT - \left[T\left(\frac{\partial v}{\partial T}\right)_P \cancel{+} - v\right] dP \text{ —— (I)}$$

for an ideal gas:-

$$PV = mRT$$

$$V = \frac{mR}{P} \cdot T$$

This is of the form $y = cx$ ($\because P = c$).

$$\therefore \left(\frac{\partial V}{\partial T}\right)_P = \frac{mR}{P}$$

But, $\frac{mR}{P} = \frac{V}{T}$

$$\therefore \left(\frac{\partial V}{\partial T}\right)_P = \frac{mR}{P} = \frac{V}{T}$$

or, $T\left(\frac{\partial V}{\partial T}\right)_P - V = 0$

$$\therefore\ dh = C_p dT - \left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right] dP$$

$$\Rightarrow dh = C_p dT - 0 \qquad (\text{for ideal gas})$$

$$\Rightarrow \boxed{dh = C_p dT} \quad \text{——} \ (k)$$

for an ideal gas $dh = C_p dT$.

for throttling, h = const, $dh = 0$.

$$\therefore\ 0 = C_p dT - \left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right] dP$$

$$\text{or, } C_p dT = \left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right] dP$$

$$\frac{dT}{dP} = \frac{1}{C_p}\left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right]$$

$$\left(\frac{\partial T}{\partial P}\right)_h = \frac{1}{C_p}\left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right]$$

$$\Rightarrow \mu = \frac{1}{C_p}\left[T\left(\frac{\partial v}{\partial T}\right)_P - v\right].$$

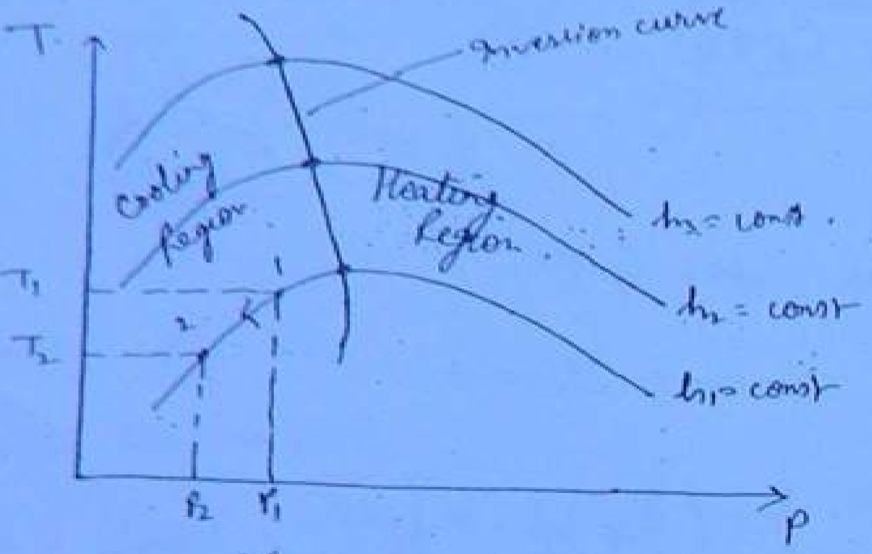
for an ideal gas,

$$T\left(\frac{\partial v}{\partial T}\right)_P - v = 0$$

$$\therefore\ \boxed{\mu = 0} \quad (\text{for an ideal gas}) \ \text{——} \ (l)$$

Joule-Thomson co-efficient for an ideal gas is zero.

For an ideal gas, as enthalpy is a function of temperature, under throttling conditions, as enthalpy remains constant, temperature also remains constant.



Slope → $\left(\frac{\partial T}{\partial P}\right)_h$

$\mu = \left(\frac{\partial T}{\partial P}\right)_h$ = Slope of isenthalpic curves on T-P diagram.

For cooling Region, μ = ⊕ve.

For heating Region, μ = ⊖ve.

Joule Thomson co-efficient is positive or ~~negat~~ slope of isenthalpic curves on T-P diagram is positive in cooling region. Similarly Joule Thomson co-efficient or slope of isenthalpic curves is negative in heating region.

⇒ PROPERTIES OF PURE SUBSTANCES :⇐

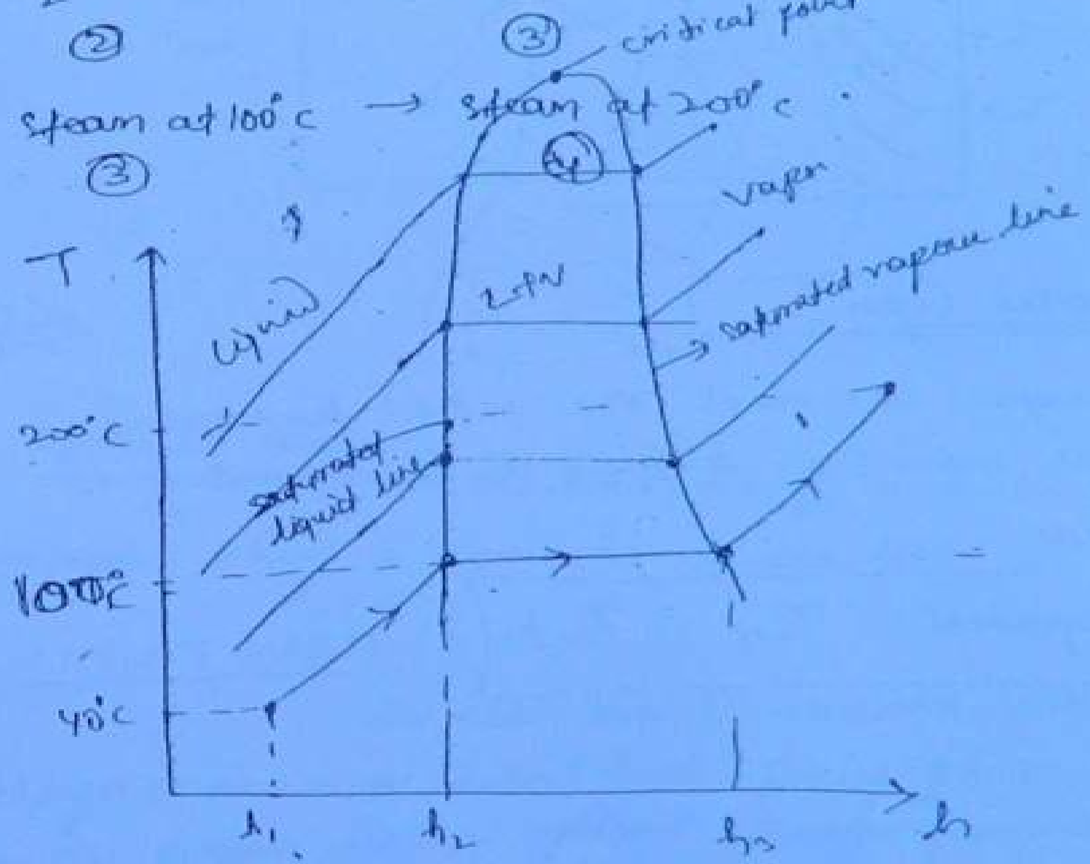
H_2O at 40°C ⟶ Steam at 200°C

when pressure is atmospheric.

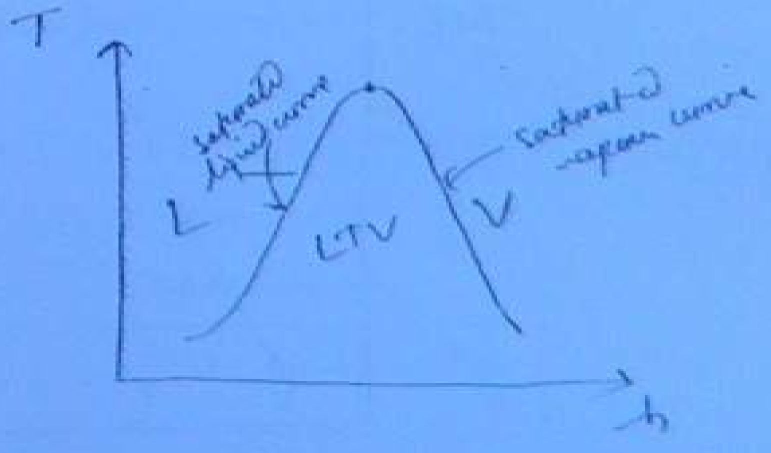
I. H_2O at 40°C ⟶ H_2O at 100°C
① ②

II. H_2O at 100°C ⟶ ~~H_2O~~ Steam at 100°C
② ③

III. Steam at 100°C ⟶ Steam at 200°C .
③ ④



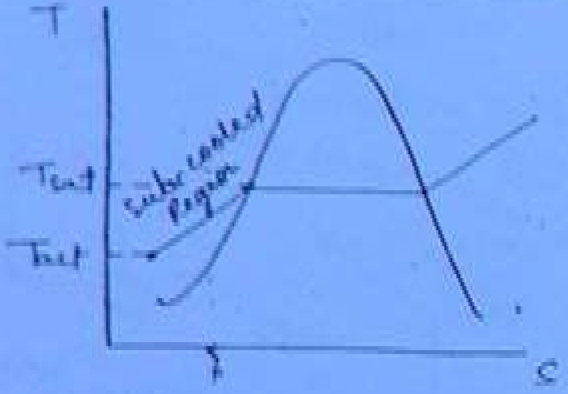
Critical point is the point at which saturated liquid and saturated vapour meets.



⇒* Sub-cooled or under cooled Region :-

It is a region in which the actual temperature is less than saturation temperature corresponding to that pressure.

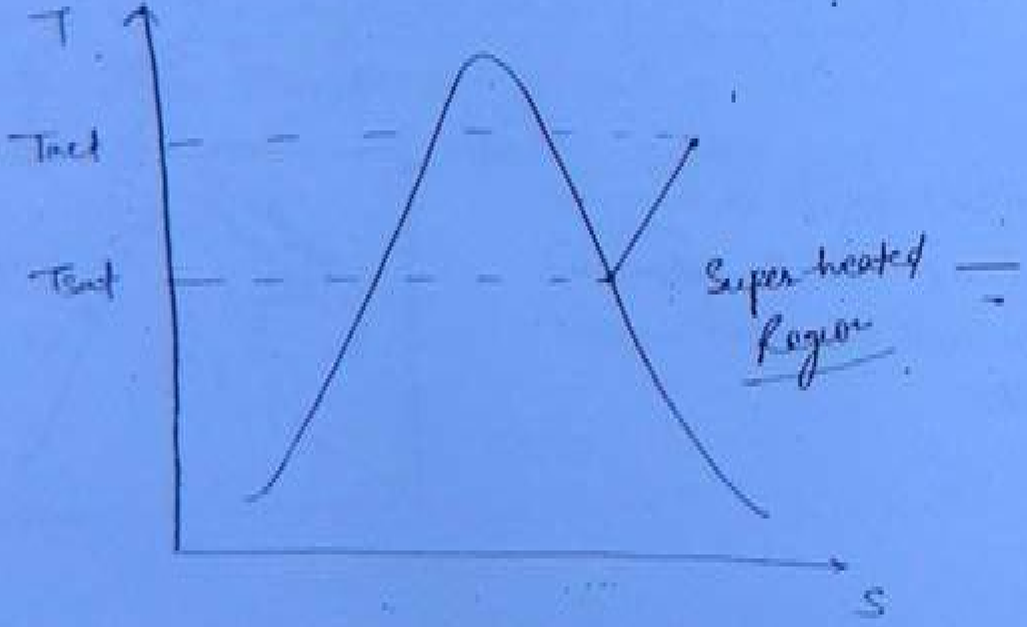
$$\boxed{\text{Degree of subcooling} = T_{saturation} - T_{actual}}$$



⇒* Super-heated Region :-

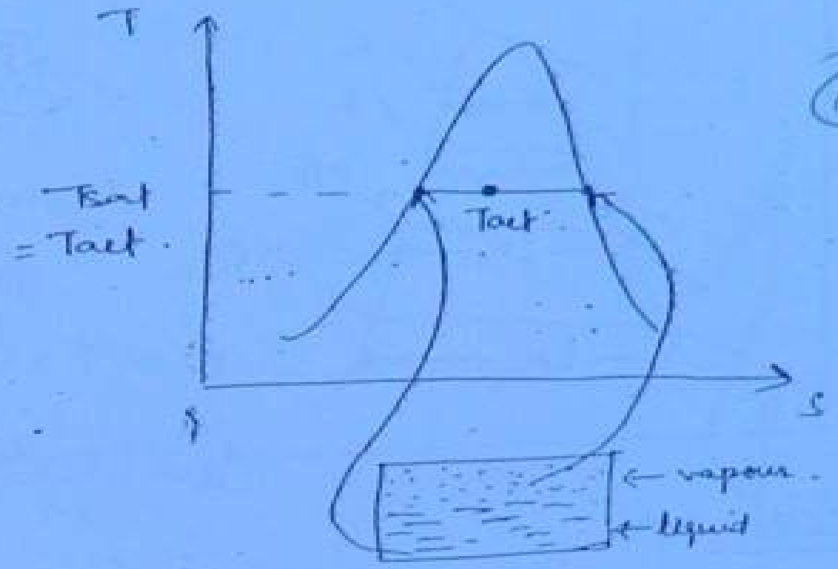
It is a region in which the actual temperature greater than saturation temperature corresponding to that pressure.

$$\boxed{\text{Degree of superheat} = T_{act} - T_{sat}}$$



⇒* Wet Region :-

It is a region in which both liquid and vapour exist in equilibrium. For wet region, the actual temperature is equal to saturation temperature i.e. ($T_{act} = T_{sat}$).

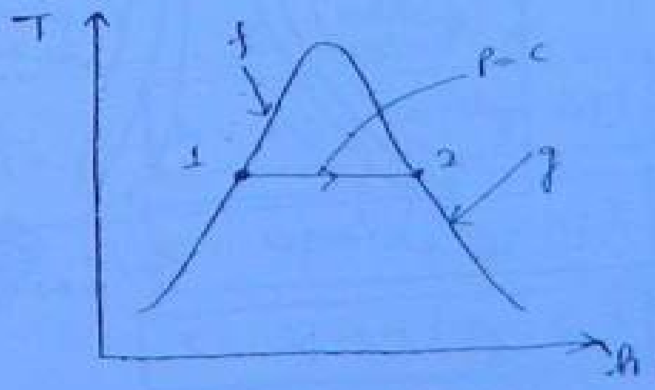


⇒* SENSIBLE HEAT :-

The heat transfer associated with temperature change is known as sensible heat.

⇒* LATENT HEAT :-

The heat transfer associated with phase change is known as latent heat. During phase change as the pressure remains constant, therefore latent heat is equal to change in enthalpy.



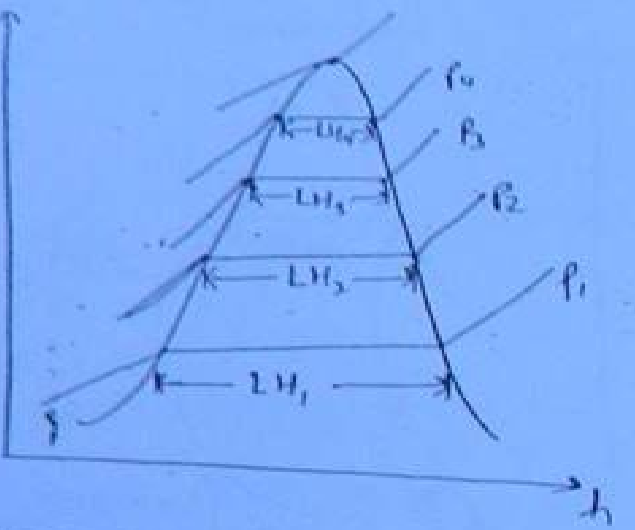
$$LH = Q_{12} = Q_p = dH$$

$$LH = dh$$

$$LH = h_2 - h_1$$

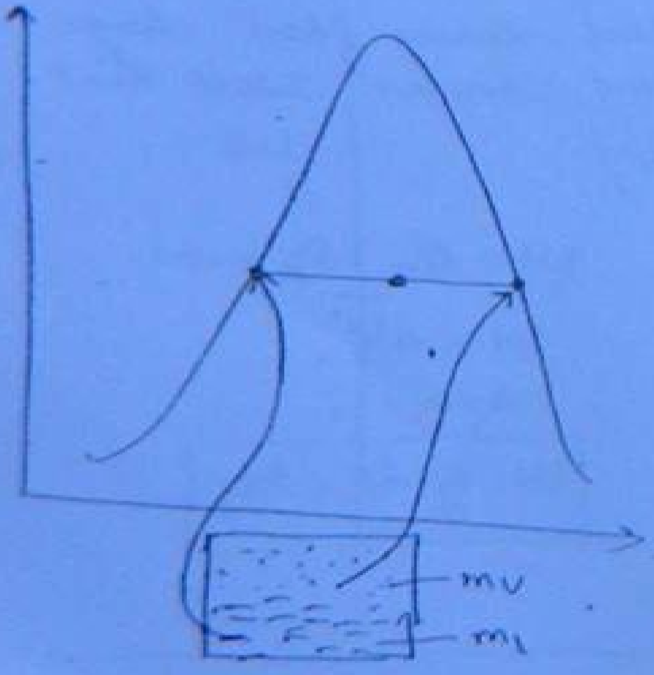
$$\boxed{LH = h_g - h_f = h_{fg}}$$

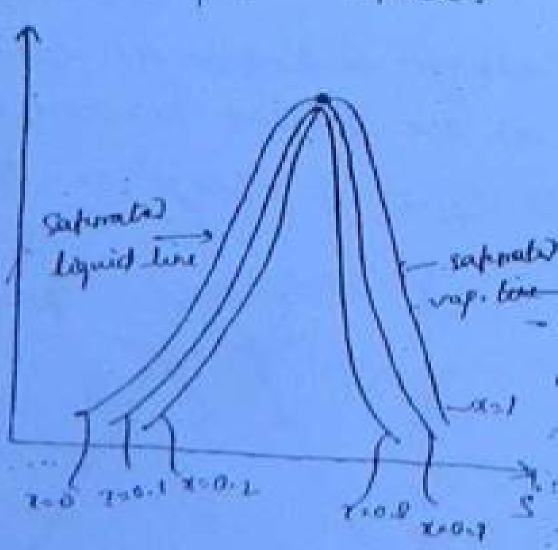
Note :- With increase in pressure, the latent heat of vaporization decreases and at critical point latent heat of vaporization is equal to zero.



⇒ * DRYNESS FRACTION :→

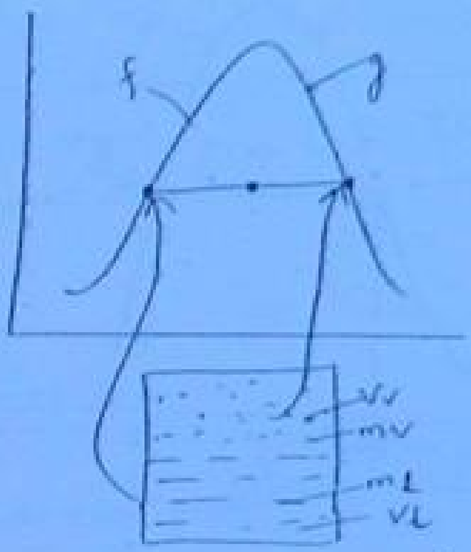
It is defined as the ratio of mass of vapour to the total mass of the mixture. The dryness fraction along saturated liquid curve = 0 and the dryness fraction along saturated vapour curve is equal to 1. The dryness fraction is also known as quality of mixture.





$$\boxed{x = \frac{m_v}{m_v + m_l}}$$

⇒* SPECIFIC VOLUME OF MIXTURE :-



$$v = V/m$$

$$V = vm.$$

$$m = m_v + m_l$$

$$V = V_v + V_l$$

$$mv = m_v v_v + m_l v_l$$

$$v = \frac{m_v v_v + m_l v_l}{m}$$

$$v = \frac{m_v}{m} v_v + \frac{m_l}{m} v_l$$

$$v = \frac{m_v}{m_v + m_l} v_v + \frac{m_l}{m_v + m_l} v_l$$

$$x = \frac{m_v}{m_v + m_l}.$$

$$1 - x \Rightarrow 1 - \frac{m_v}{m_v + m_l}.$$

$$1 - x = \frac{m_l}{m_v + m_l}.$$

$$v = x v_v + (1 - x) v_l$$

$$v = x v_v + v_l - x v_l$$

$$v = v_l + x(v_v - v_l)$$

$$\boxed{v = v_f + x(v_g - v_f)}$$

Similarly

$$\boxed{h = h_f + x(h_g - h_f)}$$

$$\boxed{u = u_f + x(u_g - u_f)}$$

$$\boxed{s = s_f + x(s_g - s_f)}$$

These equations must be used when the point is in wet region.

-: ENTHALPY AT VARIOUS POINTS :- (V.V.I)

CASE-I:- When the point is on saturated vapour curve.

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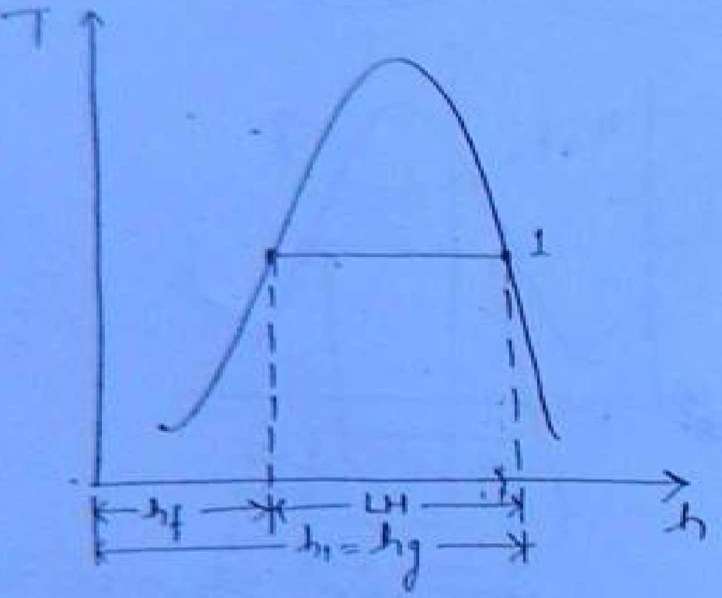
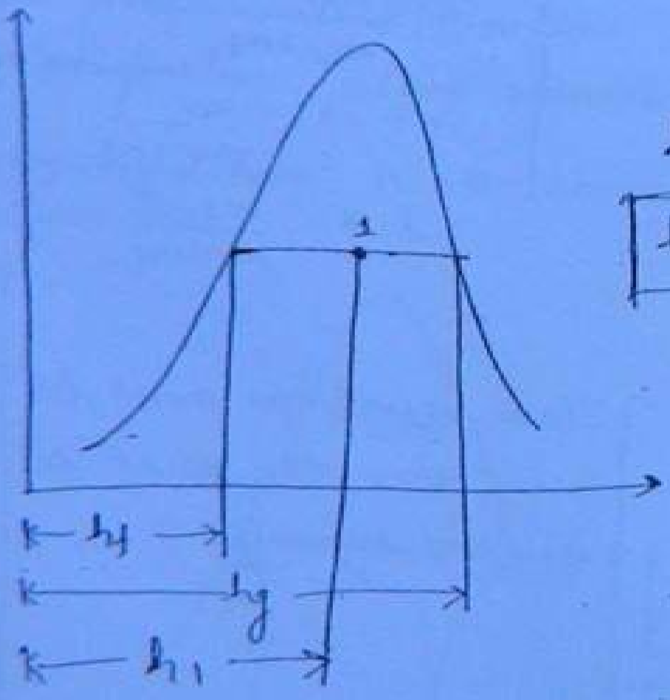


$$h_g = h_1 = h_f + LH$$

$$h_g = h_f + LH$$

~~LH =~~ $\boxed{h_g - h_f = LH}$

Case-II: When the point is in wet region.

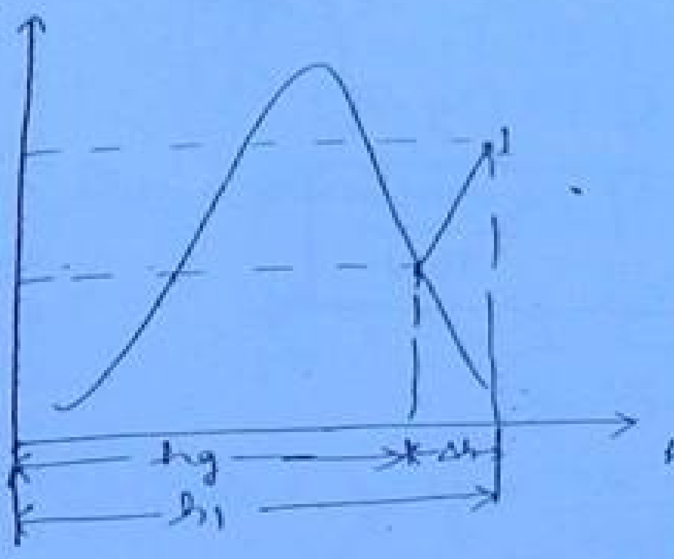


$$h_1 = h_f + x(h_g - h_f)$$

$$\boxed{h_1 = h_f + x(LH)}$$

$$\because (h_g - h_f) = h_{fg} = LH.$$

Case - III : when the point is in superheated region.



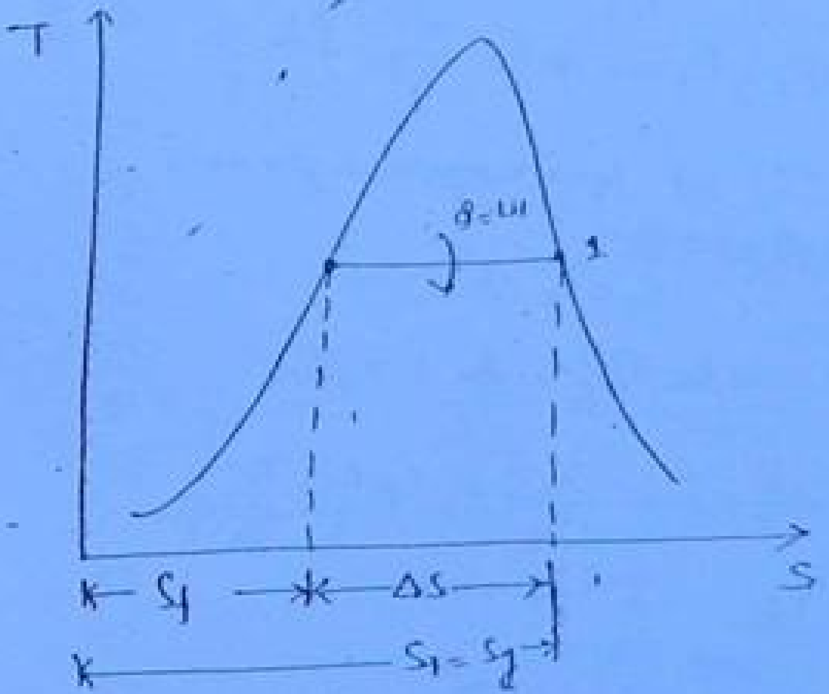
$$h_1 = h_g + \Delta h$$

$$\Delta h = C_{p\,vap.}(T_1 - T_{sat})$$

$$\boxed{h_1 = h_g + C_{p\,vap.}(T_1 - T_{sat})}$$

⇒* ENTROPY AT VARIOUS POINTS :-

Case-I : when the point is on saturated vapour curve.



$$S_1 = S_f + \Delta S$$

$$dS = \frac{dQ}{T}$$

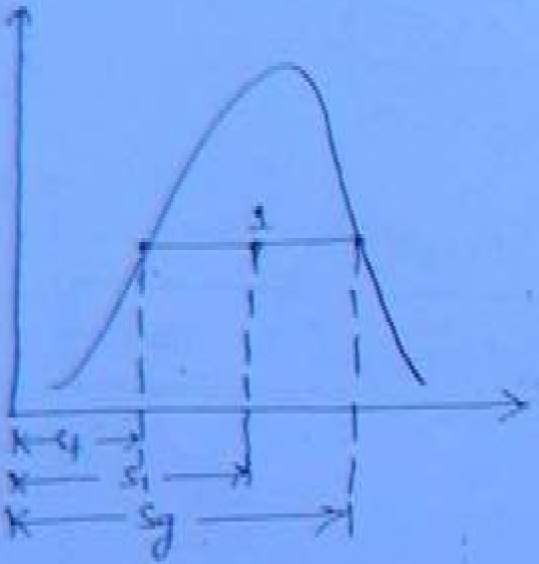
$$\Delta S = \frac{LH}{T_{sat}}$$

$$S_1 = S_f + \frac{LH}{T_{sat}}$$

$$S_g = S_f + \frac{LH}{T_{sat}}$$

$$\boxed{S_g - S_f = \frac{LH}{T_{sat}}}$$

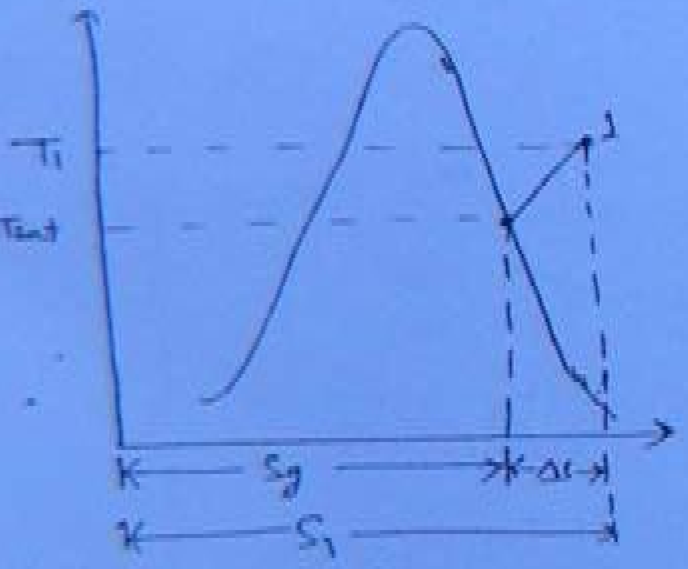
Case-II: When the point is in wet region.



$$S_1 = S_f + x(S_g - S_f)$$

$$\boxed{S_1 = S_f + \frac{x\,(LH)}{T_{sat}}}$$

Case-III: When the point is in super heated region.



$$S_1 = S_g + \Delta S$$

$$S_2 - S_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

∵ It is a constant pressure process.

∴ $P_1 = P_2$

∴ $\ln P_2/P_1 = \ln 1 = 0$.

∴ $S_2 - S_1 = \Delta S = C_p \ln\frac{T_2}{T_1}$

Here, $\Delta S = C_{p\,vap} \ln\frac{T_1}{T_{sat}}$

$$\boxed{S_1 = S_g + C_{p\,vap}.\ln\frac{T_1}{T_{sat}}}$$

⟹ ****** VVI CLAUSIUS CLAPEYRON EQUATION :→

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial s}{\partial v}\right)_T$$

During phase change P & T are independent of volume.

$$\therefore \frac{dP}{dT} = \frac{s_g - s_f}{v_g - v_f}$$

But $s_g - s_f = \frac{LH}{T_{sat}}$.

$$\therefore \boxed{\frac{dP}{dT} = \frac{LH}{T_{sat}(v_g - v_f)}} \longrightarrow \text{Clapeyron Equation}$$

05/08/2011

$$\frac{dP}{dT} = \frac{LH}{T_{sat}(v_g - v_f)}$$

$\because v_g >> v_f$

$\therefore v_f$ can be neglected.

Thus, $$\frac{dP}{dT} = \frac{LH}{T v_g} \quad \text{——} \ ①$$

Applying ideal gas equation,

$$PV = mRT$$

$$\Rightarrow \frac{V}{m} = \frac{RT}{P}$$

$$\text{or } v = \frac{RT}{P} \quad \text{——} \ ②$$

Putting the value of ② in eqⁿ ①, we have.

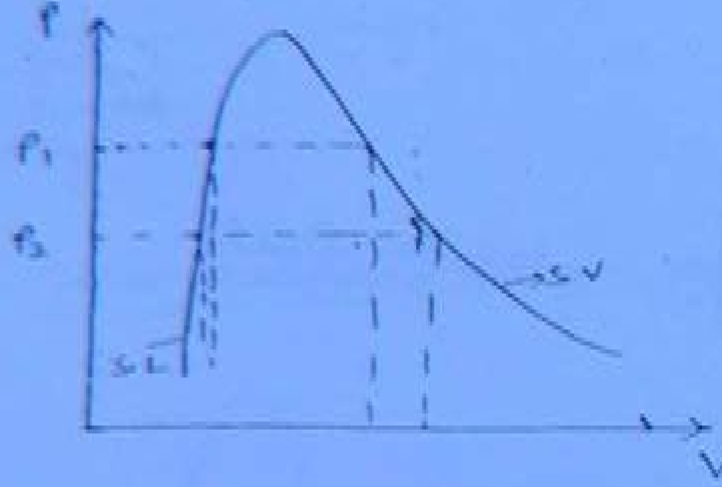
$$\frac{dP}{dT} = \frac{LH}{T \times \frac{RT}{P}}$$

$$\therefore \boxed{\frac{dP}{dT} = \frac{P(LH)}{RT^2}}$$

***
V.V.I

— This is known as Clausius - Clapeyron Equation.

⇒ P-V DIAGRAM:→



~~MOLLIER DIAGRAM:→~~

ote:

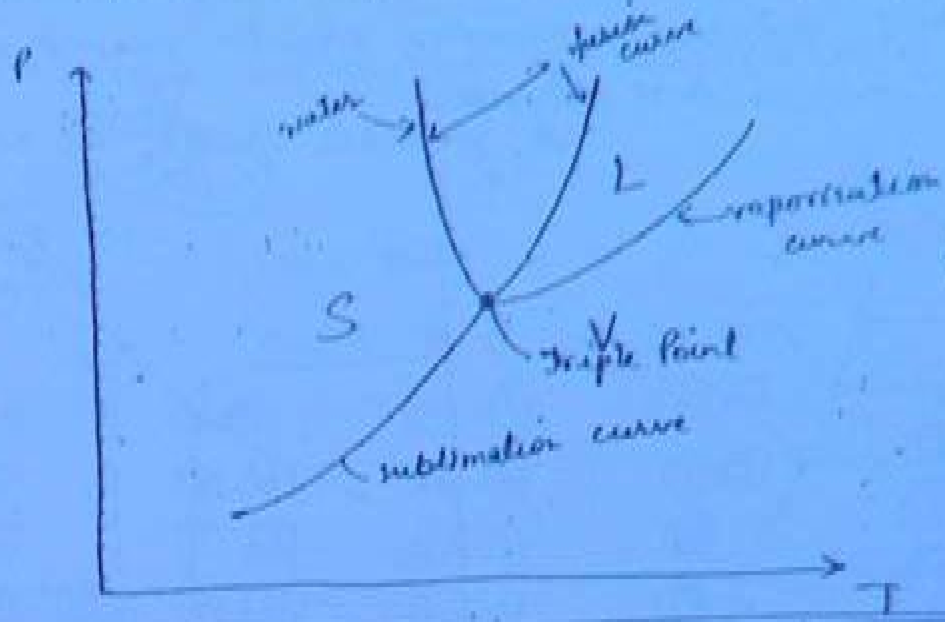
In subcooled region and superheated region, both temperature and pressure can be varied independently and hence, the degree of freedom in these regions is equal to two (2).

But in wet region, constant pressure and constant temperature lines are same and hence, the degree of freedom in wet region is equal to one (1).

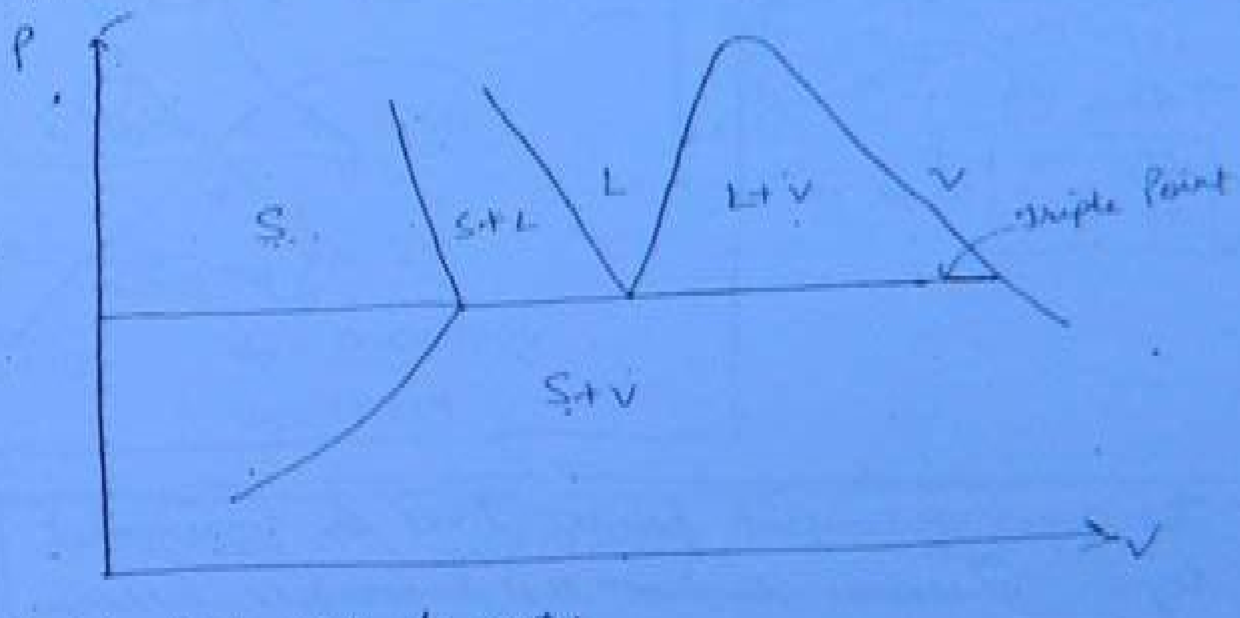
→ L → Melting
→ S → freezing
→V → Vaporization
V→L → condensation
S → V → Sublimation

General Substances	water
S-L→Melts → Expands	S-L→Melts→Contract
L-S→freeze → Contracts	L-S→freeze→Expands
	because of element of hydrogen bonding and cage like structure.

⇒ Solid - Liquid and Vapour Curve with triple point :-



Note:- The slope of fusion curve on P-T diagram is positive for general substances and negative for water.

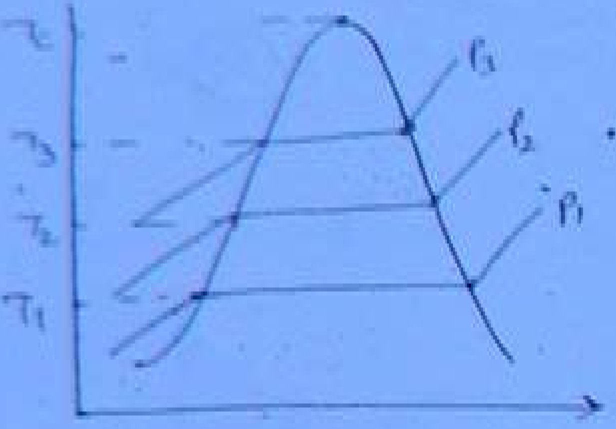


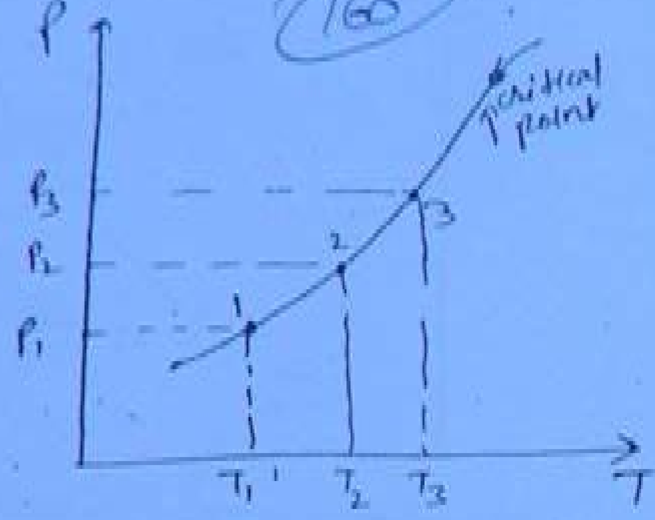
Triple Point data for water

$P_{tp} = 0.006113$ bar

$t_{tp} = 0.01°C$

According to Gibb's Phase Rule, the degree of freedom at triple point is 0 i.e. no intensive parameters can be varied at triple point. Therefore, it is a point on P-T diagram, but extensive properties can be varied and hence it is a line on P-V diagram.





⇒ * MOLLIER DIAGRAM :→

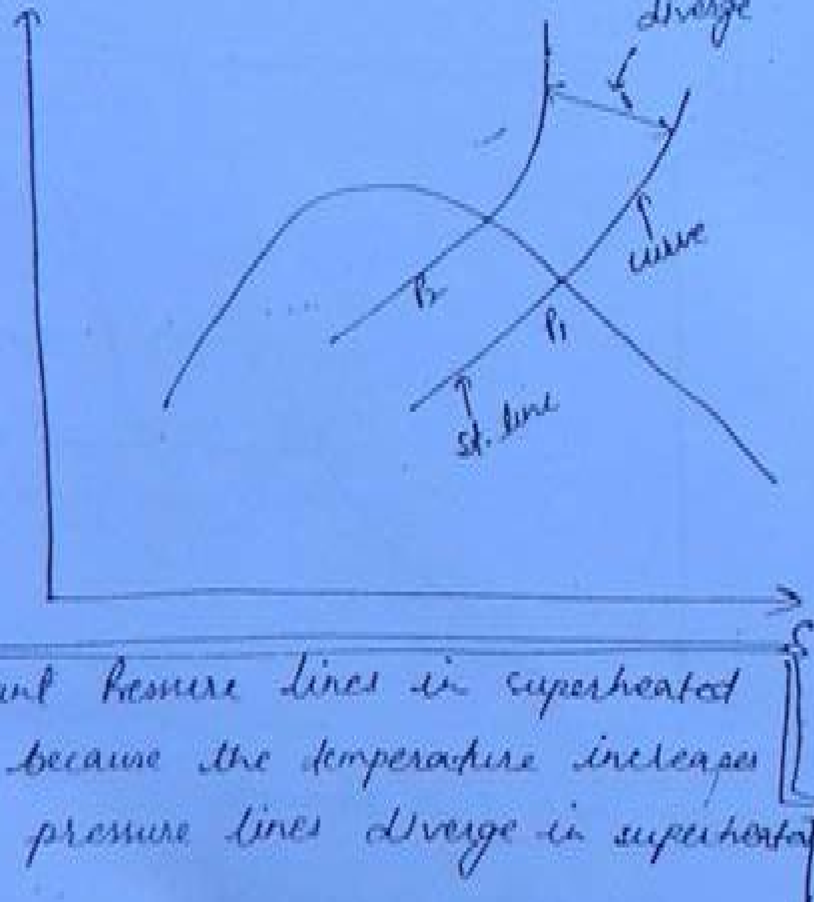
$Tds = dh - vdP$

$P = \text{const}$

$dP = 0$

$Tds = dh$

or, $\boxed{\dfrac{dh}{ds} = T}$



Note:- The slope of constant Pressure lines in superheated region increases because the temperature increases and hence constant pressure lines diverge in superheated region.

⇒ * REFERENCE STATE IN STEAM TABLE :→

Internal energy and entropy of saturated water at triple point is arbitrarily taken as zero.

Work Book

Chapter - 6

① ⓓ
② ⓐ
③ ⓒ
④ ⓐ & ⓑ
small capital
⑤ ⓒ
⑥ ⓒ
⑦ ⓑ
⑧ ⓓ
⑨ ⓑ
⑩ ⓒ
⑪ ⓐ
⑫ ⓒ
⑬ ⓑ
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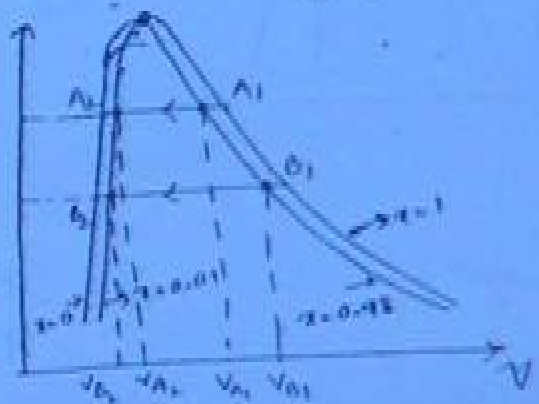
①

B₁
200 kPa
x = 0.01

A₁
300 kPa
x = 0.98



ⓓ $\left(v_{B_1} > v_{A_1} \ \& \ v_{A_2} > v_{B_2} \right)$

③ ~~$x = \frac{m_v}{m_v + m_l}$~~ $V_v = 8V_l$

$x = \frac{m_v}{m_v + m_l}$ $v_f = 0.001044$

$v_g = 1.6729$

$$x = \frac{m_v}{m_v + m_l} = \frac{m_v}{m_v\left(1 + \frac{m_l}{m_v}\right)} = \frac{1}{1 + \frac{m_l}{m_v}}$$

$$v = V/m \quad \text{or} \quad m = V/v$$

$$\therefore m_v = \frac{V_v}{v_v} \quad \& \quad \frac{8V_l}{v_v}$$

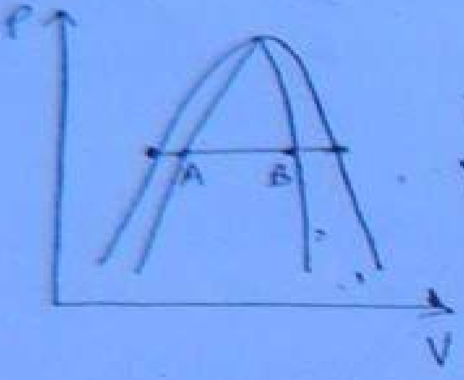
$$m_l = \frac{V_l}{v_l}$$

$$x = \frac{1}{1+\frac{\frac{V_l}{v_l}}{\frac{8V_l}{v_v}}} \Rightarrow x = \frac{1}{1+\frac{V_l}{v_l}\times\frac{v_v}{8V_l}}$$

$$= \frac{1}{1+\frac{1}{8}\left(\frac{1.6729}{0.001044}\right)}$$

$$\therefore \boxed{x = 0.0049 \simeq 0.005} \quad ⓐ$$

③



$$U_A = U_B$$

$$m_A u_A = m_B u_B$$

$$\frac{m_A}{m_B} = \frac{u_B}{u_A} = \frac{u_f + x_B(u_g - u_f)}{u_f + x_A(u_g - u_f)}$$

$$= \frac{\cancel{u_f} + u_g - \cancel{u_f}}{u_f}$$

$$= \frac{u_g}{u_f} = \frac{2583.6}{761.7} = \underline{\underline{3.4}} \quad ⓒ$$

⑤

⑦ $\rho = 36.12\ kg/m^3$

$T = -30°C = 243\ K.$

$\rho_f = 1399\ kg/m^3, \quad \rho_g = 7.379\ kg/m^3.$

$v = v_f + x(v_g - v_f)$

$v = V/m = \frac{1}{m/V} = 1/\rho.$

$\rho = m/V$

$$\therefore \frac{1}{\rho} = \frac{1}{\rho_f} + x\left(\frac{1}{\rho_g} - \frac{1}{\rho_f}\right)$$

$$\Rightarrow \frac{1}{36.12} = \frac{1}{1397} + x\left(\frac{1}{7.379} - \frac{1}{1377}\right)$$

$\therefore \boxed{x = 0.2}$ ⓑ

⑧ $Q = h_2 - h_1$

Given: $x = 0.4$, $V = 1\ m^3$, $P = 200\ kPa$, $h_2 = 125.7\ kJ/kg.$

$h_1 = h_f + x(h_g - h_f)$

$\quad = 504.7 + 0.4(2706.7 - 504.7)$

$\therefore h_1 = 1385.5\ kJ/kg.$

$\therefore Q = h_2 - h_1 = (125.7 - 1385.5) = -1259.8\ kJ/kg$

(−)ve sign means heat rejection.

Now, $v = V/m \Rightarrow m = V/v$

$v = v_f + x(v_g - v_f)$

$\quad = 0.001061 + 0.4(0.8857 - 0.001061)$

$\therefore v = 0.355$

$\therefore m = V/v = \frac{1}{0.355} = 2.8$ kg.

$\therefore Q = -1259.8 \times 2.8$

$= -3550$ kJ   ⓓ   (164)

⑨ $T = 99.63°C$

~~$s = s_f + x(s_g - s_f)$~~

$(s_g - s_f) = \frac{L_H}{T_{sat}}$

$\Rightarrow (s_g - s_f) = \frac{(h_g - h_f)}{T_{sat}} = \frac{2675.5 - 417.5}{372.63}$

$\therefore (s_g - s_f) = 6.0596$

$\therefore s_g = s_f + 6.0596 = 1.3026 + 6.0596$

$\therefore \boxed{s_g = 7.362 \text{ kJ/kg K}}$ ⓑ

⑩ $V = 20 m^3$

$x = 1$, $P_1 = 1$ MPa. $P_2 = 0.1$ MPa

$T = 300$ K

$Q = u_2 - u_1$

The system undergoes constant volume process and hence heat transfer is equal to change in internal energy.

$u_1 = u_f - x(u_g - u_f)$

$=$

$u_2 =$

(14) Adiabatic ($dQ = 0$)

Rev.

$ds = \frac{dQ}{T}$

$dS = 0$

$S = \text{Const.}$

Irrev.

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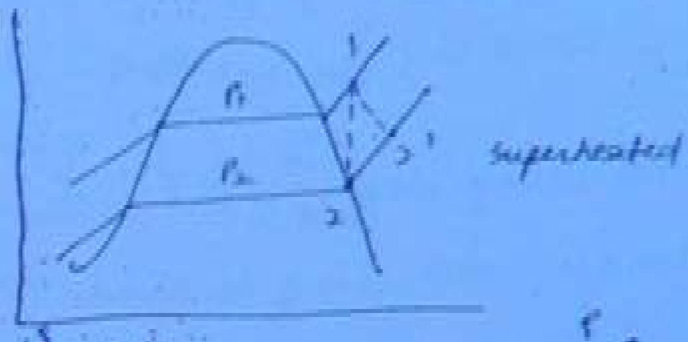
$dS = \left(\frac{dQ}{T}\right)_{\text{irr}} + (\delta s)_{\text{gen}}$

$dS = (\delta S)_{\text{gen}}$; $(\delta s) > 0$

$\boxed{dS > 0}$

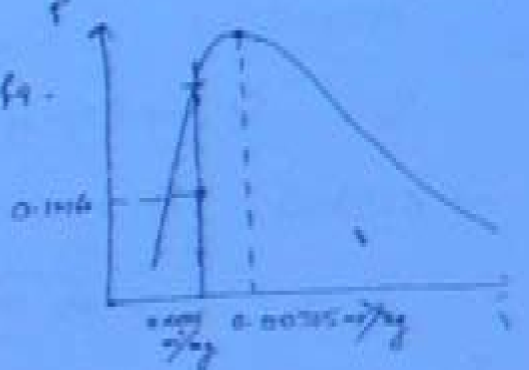


(18) $v_c = 0.003155\ m^3/kg$

$V = 0.025\ m^3$, $P = 0.1\ MPa = 100\ kPa$

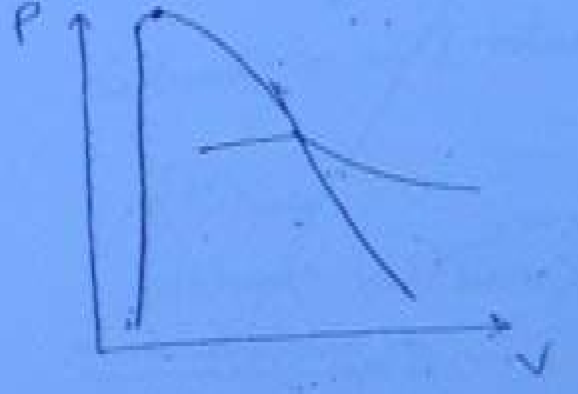
$m = 10\ kg$

$v = V/m = 0.0025\ m^3/kg$



As the in constant volume heating, the line is moving towards liquid line. Hence, liquid level will rise.

(19)

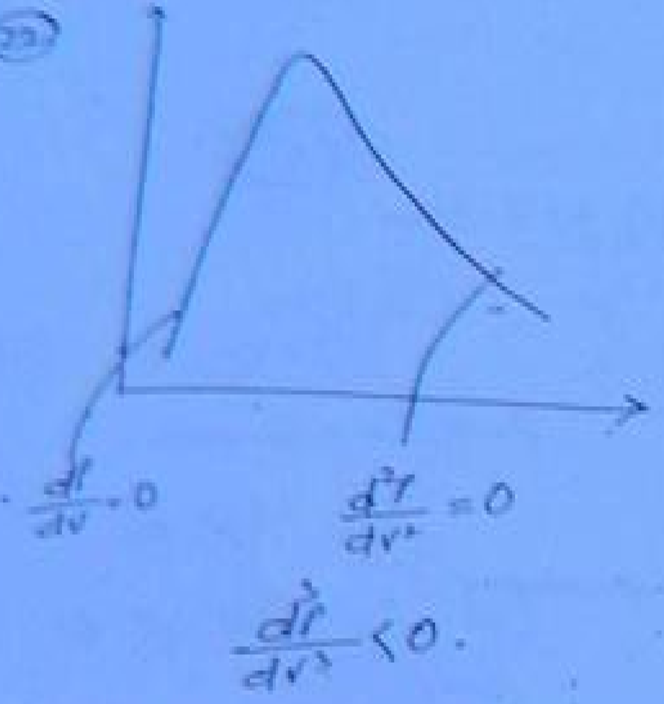


$PV = mRT \rightarrow C$

$\boxed{PV = C}$

(21)

(22)



from Mathematics.

for max^m:

$\left(\frac{dy}{dx}\right)=0$ & $\frac{d^2y}{dx^2}<0$.

for min^m:

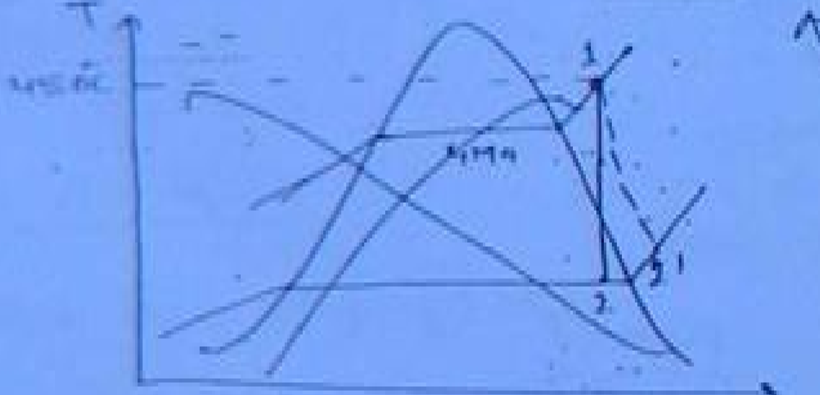
$\left(\frac{dy}{dx}\right)=0$ & $\frac{d^2y}{dx^2}>0$.

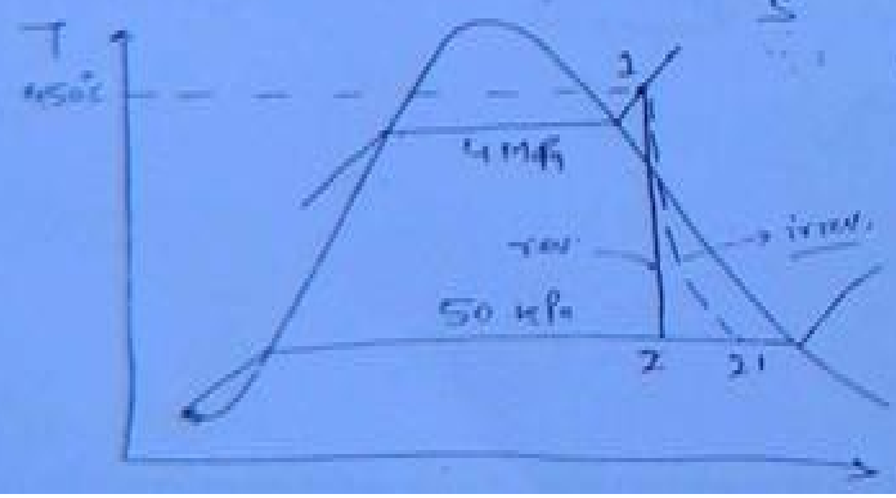
(23)

$S_2 = S_f + x(s_g - s_f)$

$\Rightarrow 6.9362 = 1.091 + x(7.5939 - 1.091)$

$\therefore x = 0.8988 \simeq \underline{0.9}$.





for adiabatic process,
only two conditions
are possible
S- constant or
S increasing.
x is minimum for 2.

㉕

1	2	3	4	5	6	7	8
t°C	P(kPa)	u_f	s_f	h_f	s_g	u_g	h_g
-20	190	88.76	0.3657	89.05	5.6155	1299.5	1418
40	1554.9	368.74	1.3574	371.43	4.8662	1341	1470.2

$h = u + Pv$

$h_f = u_f + Pv_f$

v_f is small, ∴ Pv_f is small.

$h_f \simeq u_f$

But h_f will be slightly greater than u_f.

u_g & h_g should be greater than u_f and h_f.

∴ Column ④ and ⑥ are not u_g & h_g. So they are s_f and s_g.

Also, s_g is greater than s_f

∴ Column ⑥ is s_g and column ④ is s_f.

Again $h_g = u_g + P_g v_g$

∴ $h_g > u_g$.

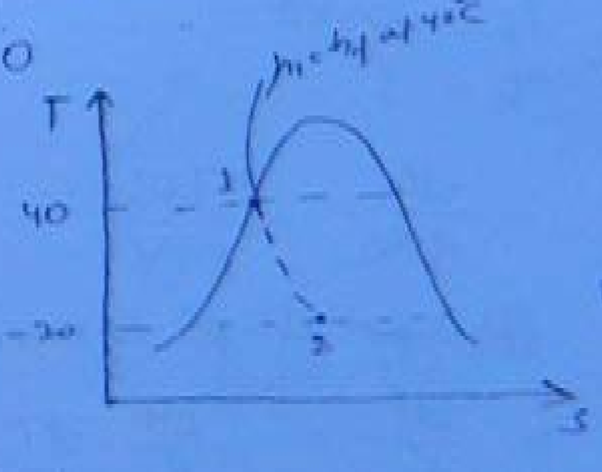
Column ⑦ is u_g & column ⑧ is h_g.

㉖

$(dS)_{univ} > 0$ ∵ (∵ throttling adiabatic)

$(dS)_{sys} + \overset{\to 0}{(dS)_{surr}} > 0$

$(dS)_{sys} > 0$



$h_1 = h_2 = 371.43$

$h_2 = h_f + x(h_g - h_f)$

$371.43 = 89.05 + x(1418 - 89.05)$

∴ $x = 0.212$

Work Book

Chapter - 7

① - ⓓ
② - ⓐ&ⓑ
③ - ⓑ
④ - ⓑ
⑤ - ⓒ

⑥ - ⓓ
⑦ - ⓑ
⑧ - ⓐ
⑨ - ~~ⓐ~~ ⓑ
⑩ - ⓒ

⑪ - ⓑ
⑫ - ⓒ
⑬ - ⓓ
⑭ - ⓓ
⑮ ....

⑯ - ⓐ
⑰ - ⓑ
⑱ - ⓐ
⑰ - ⓐ

① $\left(\frac{dP}{dT}\right) = 17.69 \text{ kPa/K}$.

$v_f = 0.00081\ 57 \text{ m}^3/\text{kg}$, $v_g = 0.0358 \text{ m}^3/\text{kg}$.

from Clapeyron eqn :-

$$\frac{dP}{dT} = \frac{LH}{T(v_g - v_f)}$$

$$LH = 17.69 \times 293\,(0.0358 - 0.00081\,57)$$

$$\ldots = \underline{\underline{181.4}}\ ⓓ$$

② $T_1 = 273 + 76 = 349 \text{ K}$.

$P_1 = 101 \text{ kPa}$.

$LH = 195 \text{ kJ/kg}$.

$R = 0.055 \text{ kJ/kg K}$.

$P_2 = 202 \text{ kPa}$

$T_2 = ?$

$$\frac{dP}{dT} = \frac{P(LH)}{RT^2}$$

Assuming LH to be constant,

$$\frac{dP}{P} = \frac{LH}{R} \cdot \frac{dT}{T^2}$$

Integrating above equation, we get.

$$\int_{P_1}^{P_2} \frac{dP}{P} = \int_{T_1}^{T_2} \frac{\Delta H}{R} \cdot \frac{dT}{T^2}$$

$$\ln \frac{P_2}{P_1} = \frac{\Delta H}{R}\left[-\frac{1}{T}\right]_{T_1}^{T_2}$$

$$\ln \frac{P_2}{P_1} = \frac{-\Delta H}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

$$\Rightarrow \ln \frac{202}{101} = -\frac{13.5}{0.055}\left[\frac{1}{T_2} - \frac{1}{349}\right].$$

$$\therefore \boxed{T_2 = 374.5k}$$ 9(a)

④ $G - F$

$(H - TS) - (U - TS)$

$H - \cancel{TS} - U + \cancel{TS}$

$\therefore H - U = PV$

But, for ideal gas, $PV = nRT$

$H - U = nRT$

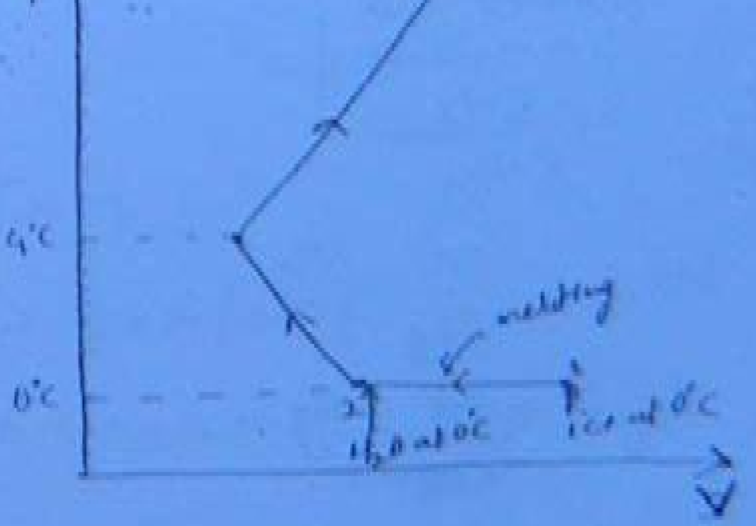
$\therefore H - U$ depends on T. (b)

⑥ $\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{-\partial V}{\partial T}\right)_P$

$\left(\frac{\partial V}{\partial T}\right)_P$ from 0°C to 4°C is $\ominus$ve.

But $\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{-\partial V}{\partial T}\right)_P$

$\therefore \left(\frac{\partial S}{\partial P}\right)_T$ for 0°C to 4°C is positive.



$\left(\frac{\partial V}{\partial T}\right)_P$ above 4°C is +ve. 1.70

and $\left(\frac{\partial S}{\partial P}\right)_T = \left(-\frac{\partial V}{\partial T}\right)_P$

$\therefore \left(\frac{\partial S}{\partial P}\right)_T$ is -ve for temp above 4°C

$\therefore$ Answer is (d)

$\left(\frac{\partial S}{\partial P}\right)_T > 0$ at 3°C and < 0 at 5°C.

⑦

$$\frac{dP}{dT} = \frac{LH}{T(v_g - v_f)} = \frac{LH}{T \cdot v_{fg}} \quad \text{—①}$$

$\ln P = A - \frac{B}{T}$

differentiating:

$$\frac{1}{P}\frac{dP}{dT} = 0 - B\left(-\frac{1}{T^2}\right)$$

$$\frac{1}{P}\frac{dP}{dT} = \frac{B}{T^2}$$

$$\frac{dP}{dT} = \frac{BP}{T^2} \quad \text{—②}$$

from ① & ②

$$\frac{BP}{T^2} = \frac{LH}{T\, v_{fg}}$$

$$\Rightarrow LH = \frac{BP\, v_{fg}}{v_{fg}\, T}$$

$$\Rightarrow h_{fg} = \frac{BP\, v_{fg}}{v_{fg}\, T}$$

$$\therefore \boxed{h_{fg} = \frac{P_{sat}\, v_{fg}\, B}{T_{sat}}} \quad ③$$

(b)

(8)

Temp^n decrease & irreversible (a).

from Maxwell's eq^n:

(9) $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

for ideal gas $PV = mRT$

$PV = RT$ (Take $m = 1$).

$P = \frac{R}{V} \cdot T$

This is of the form $y = cx$.
$dy/dx = c$

$\therefore \left(\frac{\partial P}{\partial T}\right)_V = R/V$ (b).

$\frac{C_p}{C_v} = 1.044$

$C_p - C_v$) $C_p - \frac{C_p}{1.044} = 1.87$

$\left(\frac{\partial T}{\partial P}\right)_h$ [illegible]

(12) — (c)
(13) — (a)
(14) — (d)
(15) — (b)
(16)

(10) — (c)

(11) $(\Delta S) = -m_1 R_1 \ln \frac{P_1}{P} - m_2 R_2 \ln \frac{P_2}{P}$

$(\Delta S) = -m_1 R_1 \ln x_1 - m_2 R_2 \ln x_2$

$x_1 = \frac{n_1}{n_1 + n_2} = \frac{1}{1+4} = 0.2$

$x_2 = \frac{n_2}{n_2 + n_1} = 0.8$

$PV = mRT = n\bar{R}T$
$mR\cancel{T} = n\bar{R}\cancel{T}$
$mR = n\bar{R}$
$m_1 R_1 = n_1 \bar{R}$
$m_2 R_2 = n_2 \bar{R}$

$(\Delta S) = -n_1 \bar{R} \ln x_1 - n_2 \bar{R} \ln x_2$

$= -\bar{R}\left[1 \times \ln(0.2) + 4 \times \ln(0.8)\right]$

$= -8.314 \times \left[1 \times \ln 0.2 + 4 \ln 0.8\right] = 20.8$

⑰ $C_p - C_v = \frac{TV\beta^2}{K_T}$

$$C_p - C_v = \frac{(273+35)\times 0.000114 \times (5\times10^{-5})^2}{8.6\times10^{-12}}$$

$\therefore C_p - C_v = 9.8$

Again $\frac{C_p}{C_v} = 1.024$

$\Rightarrow C_v = \frac{C_p}{1.024}$

$\therefore C_p - \frac{C_p}{1.024} = 9.87$

$\therefore C_p = \frac{9.87}{0.024} = \underline{421.3}$

Chapter - 3
Work Book

① — ⓓ
② — ⓐ
③ — ⓐ
④ — ⓓ
⑤ — ⓒ
⑥ — ⓐ
⑦ — ⓐ
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㉜ —

② $h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$

$C_p = 1.005\ kJ/kg\,K$

$P_1 = 100\ kPa,\ T_1 = 500K,\ C_1 = 150\ m/s,\ T_2 = 510K.$

$$h_1 + \frac{C_1^2}{2} + \cancel{Z_1 g}^{0} + \cancel{q}^{0} = h_2 + \frac{C_2^2}{2} + \cancel{Z_2 g}^{0} + \cancel{W}^{0}$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}.$$

$$h_1 + \frac{C_1^2}{2000} = h_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_pT_1 + \frac{C_1^2}{2000} = C_pT_2 + \frac{C_2^2}{2000}$$

$$= 1.0005 \times 500 + \frac{(150)^2}{2000} = 1.005 \times 510 + \frac{C_2^2}{2000}$$

$\Rightarrow C_2 = 48.98 \simeq 49\ m/s$ ⓐ diffuser ∵ final velocity is less than initial velocity.

③ $m = 1\ kg/s,\ T_1 = 800K,\ P_1 = 200\ kPa,\ C_1 = 50\ m/s$

$T_2 = 650K,\ P_2 = 100\ kPa,\ v = -25\ kW$

$C_p = 1.005\ kJ/kg\,K.$

$$h_1 + \frac{C_1^2}{2} + \cancel{Z_1 g}^{0} + v = h_2 + \frac{C_2^2}{2} + \cancel{Z_2 g}^{0} + \cancel{W}^{0}$$

$$\Rightarrow C_pT_1 + \frac{C_1^2}{2000} + v = C_pT_2 + \frac{C_2^2}{2000}$$

1 kw

$v = \frac{Q}{m} = \frac{25\ kW}{1\ kg/sec}$

$= -25\ kJ/kg$

$$\Rightarrow 1.005 \times 800 + \frac{(50)^2}{2000} - 25 = 1.005 \times 650 + \frac{(C_2)^2}{2000}$$

$\Rightarrow C_2 = 503.98 \simeq \underline{504\ m/s}$. ⓑ

⑨ $\delta Q = dE + \delta W$

$dE = \delta Q - \delta W$

(Properties are exact differentials).

⇒ * OPEN SYSTEM WORK * ⇐

Assumptions:

① Steady flow
② Neglect K.E and P.E changes
③ Reversible

$$h_1 + \cancel{\frac{c_1^2}{2}}^{0} + \cancel{z_1 g}^{0} + q = h_2 + \cancel{\frac{c_2^2}{2}}^{0} + \cancel{z_2 g}^{0} + w$$

$h_1 + q = h_2 + w$

or, $q = (h_2 - h_1) + w$

or, $q = dh + w$ — ①

Also

$\frac{dQ}{T} = ds$

$dQ = Tds$

$dq = Tds$

$q = Tds$ — ②

from ① & ②

$Tds = dh + w$ — ③

we also know $Tds = dh - vdP$ — ④

from ③ & ④

$\boxed{w = -vdP}$ open system work.

③ $P_1 = 400\,kPa$, $T_1 = 900\,K$, $C_1 = 0$.

$P_2 = 100\,kPa$.

~~$PV = mRT$~~ flow is ideal means reversible.

$$h_1 + \frac{C_1^2}{2} + \cancel{z_1 g}^{0} + \cancel{q}^{0} = h_2 + \frac{C_2^2}{2} + \cancel{z_2 g}^{0} + \cancel{w}^{0}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

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$$\Rightarrow T_2 = 900 \times \left(\frac{100}{400}\right)^{\frac{0.4}{1.4}} = 605.6\,K$$

$$h_1 + \cancel{\frac{C_1^2}{2000}}^{0} = h_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow C_p T_1 + \cancel{\frac{C_1^2}{2000}} = C_p T_2 + \frac{C_2^2}{2000}$$

$$\Rightarrow 1.005 \times 900 = 1.005 \times 605.6 + \frac{C_2^2}{2000}$$

$$\Rightarrow \underline{C_2 = 769\,m/s}$$ (a)

④ $m = 1\,kg/s$

$P_1 = 0.1\,MPa = 100\,kPa$ $P_2 = 1000\,kPa$

$T_1 = 300\,K$.

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300 \times \left(\frac{1000}{100}\right)^{\frac{0.4}{1.4}}$$

$$\therefore \underline{T_2 = 579.2\,K}.$$

$$W = h_2 - h_1$$

$$= C_p (T_2 - T_1)$$

$$= 1.005(579.2 - 300) = 280.6\,kJ/kg$$

$$\therefore P = (280.6 \times 1) = \underline{280.6\,kW} \text{ or } \underline{280\,kW}$$

⑨ ⑩ $m = 1 kg$

$dQ = dU + dW$

$0 = dU + dW$

$0 = mC_V(T_2 - T_1) + dW$

$dW = -mC_V(T_2 - T_1)$

$dW = mC_V(T_1 - T_2)$

$= 1 \times C_V(T_1 - T_2) = C_V\,\underline{(T_1 - T_2)}$. ⓒ

⑨ - ⓒ , ⑩ ⓓ

$\delta Q = dU + dW$

[illegible] $= 30 + 150$

$50 = 30 - 80$

176

⑪

1-2	2-3	3-1
$V = C$; $Q_{12} = 50$ $\because V = C$ $\therefore W_{12} = 0$	$T = C$ $W_{23} = 500\ kJ$ for $T = C$ $W = Q$ $\therefore Q_{23} = 500\ kJ$	$P = C$. $W_{31} = -200\ kJ$ $Q_{31} = ?$ $dU_{31} = ?$

for a cycle,

$\Sigma Q = \Sigma W$

$\Rightarrow Q_{12} + Q_{23} + Q_{31} = W_{12} + W_{23} + W_{31}$

$\Rightarrow 50 + 500 + Q_{31} = 0 + 500 - 200$

$\Rightarrow \boxed{Q_{31} = -250\ kJ}$

$\delta Q = \delta U + \delta W$

$\Rightarrow -250 = dU + 200$

$\Rightarrow \boxed{\delta U_{31} = -50\ kJ}$

ⓓ

(14) $E = (25 + 0.25t)$ kJ

$dQ = dE + dW$

$$\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$

$$\frac{dQ}{dt} = 0.25 + 0.75$$

$$= \underline{\underline{1.}}$$ (d)

(17)

$dQ = dE + dW$

$-50 = 30 - 80$

$E = 25 + 0.25t$

$$\frac{dE}{dt} = 0.25$$

$$\frac{dW}{dt} = 0.75$$

(15)

1-2

$V = C,$

$Q_{12} = 170$ kJ

$W_{12} = 0$

$U_{1} = 100$ kJ

$dQ_{12} = du_{12} + dW_{12}$

$\Rightarrow 170 = U_2 - U_1 + 0$

$\Rightarrow U_2 = 170 + 100 = \underline{270 \text{ kJ}}$

2-3

$P = C$

$Q_{23} = -180$ kJ

$W_{23} = -40$ kJ

$dQ_{23} = du_{23} + dW_{23}$

$\Rightarrow -180 = (U_3 - U_2) + 40$

$\Rightarrow -140 = (U_3 - 270)$

$\Rightarrow \underline{U_3 = 130 \text{ kJ}}$

3-1

$Q_{31} = 0.$

$\therefore U_1 = 100$ kJ

$U_2 = 270$ kJ

$U_3 = 130$ kJ

$\therefore$ (a)

(16) $P_1 = 10$ bar, $P_2 = 1$ bar.

In throttling $h_1 = h_2$

or $u_1 + P_1 v_1 = u_2 + P_2 v_2$

$\Rightarrow (u_2 - u_1) = P_1 v_1 - P_2 v_2$

$= (1000 \times 0.5) - (100 \times 2)$

$= \underline{300 \text{ kJ/kg}}$ (d)

(17) $dQ = dU + dW$

Question : $\frac{dU}{Q_p} = ?$ $\frac{dU}{Q_p} = \frac{dU}{dh} = \frac{mc_v dT}{mc_p dT}$ ($\because$ ideal gas)

$$= \frac{c_v}{c_p} = \frac{1}{\gamma} = \underline{\underline{\left(\frac{5}{7}\right)}}$$ (d)

⑱

A

$T_1 = 300K$

$P = C$

$Q_A = Q_B$

$T_2 = 330K.$

~~$Q_A \; dh = mc_p dT$~~

$(Q_A)_P = \overset{\neq}{dh} = mc_p dT$

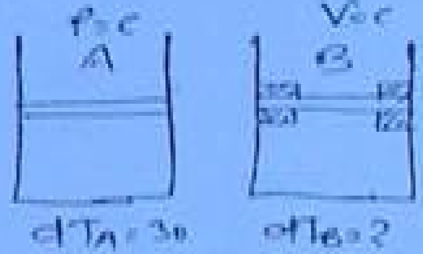
B

$T_1 = 300K.$

$V = C$

$Q_B = Q_A$

$T_2 = ?$



$(Q_A = Q_B).$

$(Q_B)_V = mc_v dT$ (178)

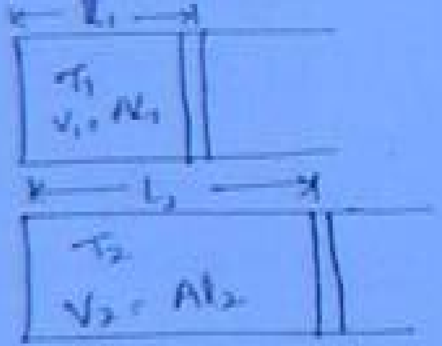
Now, $\cancel{m_A} c_p dT_A = \cancel{m_B} c_v dT_B$

$\Rightarrow dT_B = \frac{c_p}{c_v} \cdot dT_A$

$= \gamma \times 30$

$\therefore \boxed{dT_B = 1.4 \times 30 = 42\,K.}$ ⓓ.

⑲



for adiabatic process:

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$= \left(\frac{\cancel{A} L_2}{\cancel{A} L_1}\right)^{\frac{5}{3}-1}$

$= \left(\frac{L_2}{L_1}\right)^{2/3}$ ⓓ.

(20) $\delta Q = dU + \delta W$

$\Rightarrow \delta Q = dU \quad (\because \delta W = 0)$.

for ideal gas $U \rightarrow f(T)$.

$\cancel{\phi}$ $\delta Q < 0$ mean $dU < 0$.

∴ Temp^{r} will decrease.

(21) $\Sigma Q = 5\,kj$

$\Sigma Q = \Sigma W = 5\,kj$

∴ $W_{AB} + W_{BC} + W_{CA} = \cancel{Q}\,5$

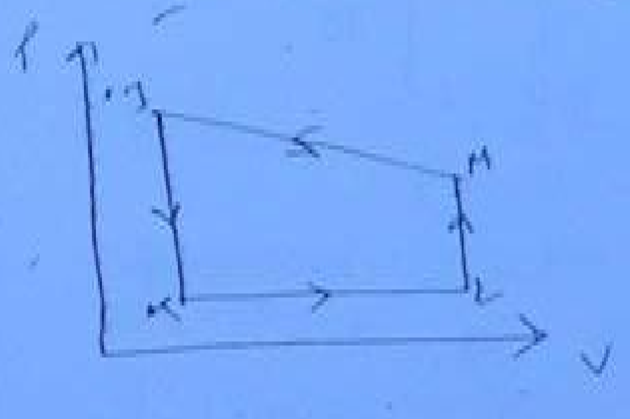
$\Rightarrow W_{AB} + 0 + W_{CA} = 5$

Now $W_{AB} =$ Area under curve

$= (10 \times 1) = 10\,kj$

∴ $W_{CA} = 5 - 10 = -5\,kj$ (a)

(22)



$J \rightarrow K \Rightarrow W = 0$

& $Q < 0$ ($\because$ heat rej.)

$P\cancel{V} = \cancel{n}RT$

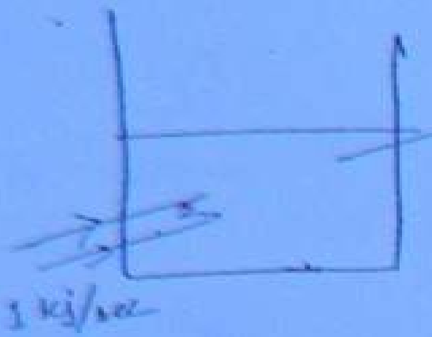
$P \propto T$

$P\downarrow \rightarrow T\downarrow$

$L - M \Rightarrow W = 0$.

(a)

(23)



Net accumulation = 1 − 0.16

= 0.84 kJ/sec

(180)

for 't' seconds

$Q = 0.84\,t$ kJ

$Q = mc(\Delta T)$

$\therefore\ 0.84t = mc(\Delta T)$

$\Rightarrow 0.84t = 2 \times 4.2(77-27)$

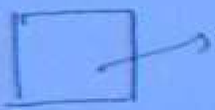
$\therefore\ t = 500$ seconds

$\therefore\ t = \underline{8 \text{ min. } 40 \text{ sec.}}$ (a).

(25) → (a)

(26) → (c)

(24)



1-2 discharging

$Q_{12} = -250$ kcal.

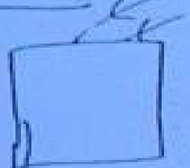
$W_{12} = 0$.

1 cal = 4.18 J

1 kcal = 4.18 kJ

$\therefore\ Q_{12} = -250 \times 4.18$ kJ

$= -1045$ kJ



2-1 charging

$W_{21} = -0.53$ kW hr.

$Q_{21} = ?$

$= -0.53 \frac{\text{kJ}}{\text{sec}} \times 3600$ sec

$W_{21} = -1908$ kJ

$\Sigma Q = \Sigma W$

$Q_{12} + Q_{21} = W_{12} + W_{21}$

$-1045 + Q_{21} = 0 + (-1908)$

$\therefore\ \boxed{Q_{21} = -863 \text{ kJ}}$ — (c)

(27) $dQ = dU + dW$

$$\frac{dQ}{dt} = \frac{dU}{dt} + \frac{dW}{dt}$$

$$\frac{dW}{dt} = -260\ \text{J/sec}$$

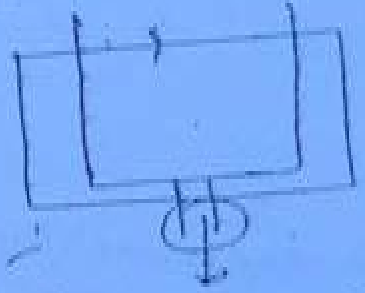
$$\frac{dQ}{dt} = -10\ \text{J/sec}$$

$$-10 = -260 + \frac{dU}{dt}$$

$$\therefore \frac{dU}{dt} = 250\ \text{J/sec}$$

$$\frac{250\ \text{J}}{\text{sec}} \times 3600 = 900 \times 10^3\ \text{J} = \underline{900\ \text{kJ}}.$$ ⓑ

(28)

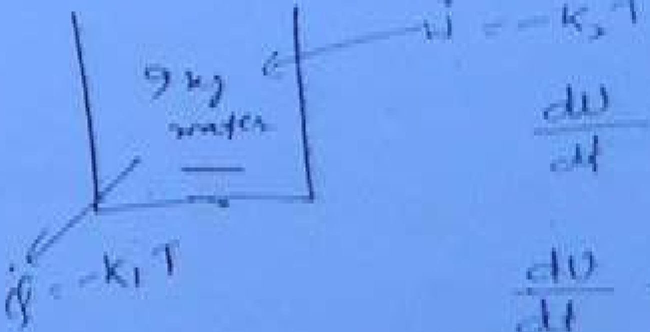


$$\left(\frac{dU}{dt}\right)_{cv} = \cancelto{0}{\underline{\dot{m}_i h_i}} + \cancelto{0}{\dot{Q}} - \dot{m}_e h_e - \cancelto{0}{\dot{W}}$$

$$\left(\frac{dU}{dt}\right)_{cv} = -\dot{m}_e h_e$$

$$\frac{d}{dt}(mu) = -\dot{m}_e h_e$$ —— ⓐ.

(29)



$$\frac{dU}{dt} = \cancelto{0}{\dot{m}_i h_i} + \dot{Q} - \cancelto{0}{\dot{m}_e h_e} - \dot{W}$$

(∵ no mass entering no mass leaving)

$$\frac{dU}{dt} = \dot{Q} - \dot{W} = -K_1T - (-K_2T)$$

$$\frac{dU}{dt} = (K_2 - K_1)T$$

$$\frac{dU}{dt} \cdot \frac{dT}{dT} = (K_2 - K_1)T$$

$$\frac{dU}{dT} \cdot \frac{dT}{dt} = (K_2 - K_1)T$$

from ideal gas eqⁿ, $dU = c\,dT$ or $\frac{dU}{dT} = c$

$$\therefore\ c \cdot \frac{dT}{dt} = (K_2 - K_1)T.$$

$$\text{or}\quad \frac{dT}{T} = \frac{(K_2 - K_1)}{c} \cdot dt$$

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Integrating :-

$$\ln T = \frac{(K_2 - K_1)}{c} \cdot t$$

$$T = e^{\left[\frac{(K_2 - K_1)}{c} \cdot t\right]}$$

If $K_2 > K_1$, T will increase exponentially

If $K_2 < K_1$, T will decrease exponentially.

(5) Use steam table and find enthalpy at 1 MPa & 400°C and then calculate.

Ans is (c) - 0.612

㉛ $P_1 = 0.7 \text{ bar} = 70 \text{ kPa}.$

$t_1 = 57^\circ C = 330K.$

$C_1 = 200$

$A_1 = 0.4$

$P_2 = 1 \text{ bar}.$

$A_2 = 0.6$

(183)

$\dot m = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$

$\dot m = \rho_1 A_1 C_1.$

∵ Ideal gas, ... $PV = mRT$

$$\frac{m}{V} = \frac{P}{RT}$$

$$\therefore \rho = P/RT.$$

$$\therefore \rho_1 = \frac{P_1}{R_1T_1} = \frac{70}{0.287 \times 330} = 0.739.$$

$$\therefore \dot m = 0.739 \times 0.4 \times 200$$

$$\therefore \boxed{\dot m = 59.1 \text{ kg/sec}}$$ Ans ⓐ

㉜ from steady flow equation :-

$$h_1 + \frac{C_1^2}{2000} + \cancel{Zg}^{0} + \cancel{q}^{0} = h_2 + \frac{C_2^2}{2000} + \cancel{Zg}^{0} + \cancel{W}^{0}$$

$$\therefore h_1 + \frac{C_1^2}{2000} = h_2 + \frac{C_2^2}{2000}$$

$$\text{or } C_pT_1 + \frac{C_1^2}{2000} = C_pT_2 + \frac{C_2^2}{2000} \quad \text{— ①}$$

Also $59.\overline{10} = \frac{P}{RT_2} A_2 C_2 \quad (\rho_2 A_2 C_2).$

$$59.1 = \frac{100}{0.287 T_2} \times 0.6 \times C_2$$

$$\Rightarrow C_2 = 0.283\, T_2 \quad \text{— ②}$$

Putting ② in ①, we can get answer

UNIT-V

:- I.C. ENGINE :-

An I.C. Engine compraises of a piston cylinder arrangement where the combustion takes place internaly for attaining the useful work output. The different important cycles that are involved for attaining useful work output are :-

1> The otto cycle or the constant volume cycle. 185

2> The Diesel cycle or the constant pressure cycle.

3) The Dual combustion cycle

The common processes that are involved for the above cycles are :-

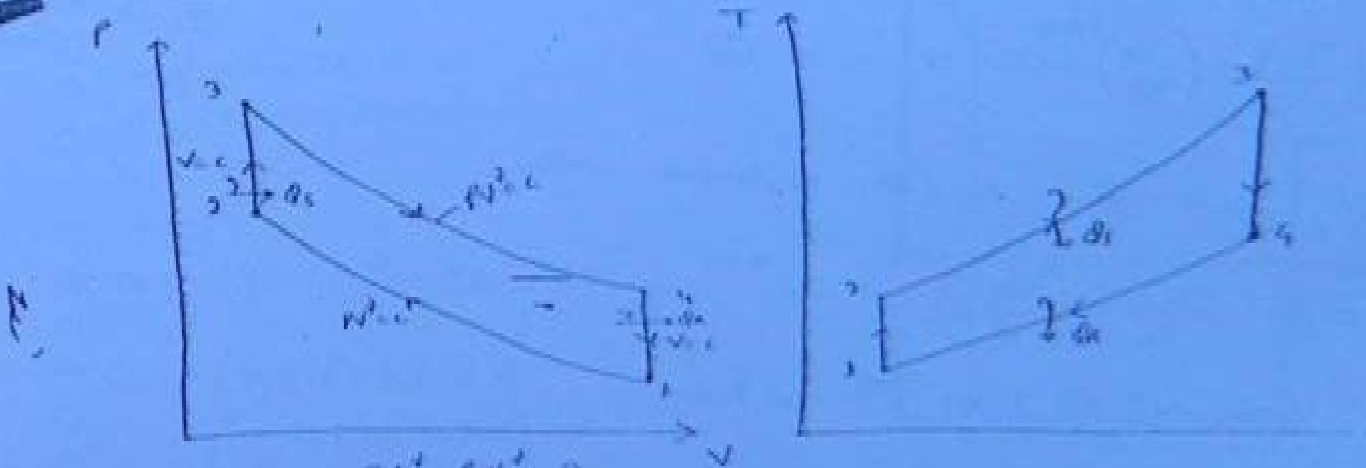
a> Isentropic expansion.

b> Heat rejection at constant volume;

c) Isentropic compression.

The different cycles are different from one another because of heat addition taking place differently.

=> THE OTTO CYCLE OR THE CONSTANT VOLUME CYCLE :-



$$\left.\begin{array}{l} P_1V_1^{\gamma} = P_2V_2^{\gamma} \\ \dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2} \end{array}\right\} \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

$$ds = mc_v \ln \frac{T_2}{T_1}$$ (Change in entropy when volume is constant)

If T_2 is greater, ds will greater than 0.

→ Derivation of efficiency of otto cycle :-

~~$\eta = \frac{WD}{HS} = \frac{HS - HR}{HS} = 1 - \frac{HR}{HS}$~~

~~$\eta = \frac{WD}{HA} = \frac{HA - HR}{HA} = 1 - \frac{HR}{HA}$~~

~~HS=~~

We know that the efficiency of the cycle

$$\eta_v = \frac{WD}{HA} = \frac{HA - HR}{HA} = 1 - \frac{HR}{HA}$$

$$HA = mc_v (T_3 - T_2)$$

$$HR = mc_v (T_4 - T_1)$$

$$\eta_v = 1 - \frac{mc_v (T_4 - T_1)}{mc_v (T_3 - T_2)} \text{ ——— ⓐ}$$

Let $\frac{V_1}{V_2} = r$, (the compression ratio)

We know that for an adiabatic process,

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} \text{ —— ⓑ}$$

But, $\frac{V_4}{V_3} = \frac{V_1}{V_2} = r$, (the expansion ratio)

$$\& \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1} \text{ —— ⓒ}$$

from eq^n ⓑ & eq^n ⓒ, we have

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\therefore \quad \frac{T_4}{T_3} = \frac{T_1}{T_2}$$

from componendo and dividendo, we have :-

$$\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$$

$$\therefore \quad \frac{T_4}{T_3} = \frac{T_1}{T_2} = \frac{T_4 - T_1}{T_3 - T_2} \quad \text{(from algebra)} \text{ —— ⓐ}$$

Thus we have:-

$$\eta_v = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2} \quad \text{(from eq}^n \text{ (A))}$$

$$\text{or,} \quad \boxed{\eta_v = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}} \text{ ——— ①} \quad \text{(from equation (B))}.$$

Problem

ESE-2010

Q:- The bore and stroke of the cylinder of a [illegible] 6-cylinder engine working on an otto cycle are 17 cm and 30 cm respectively. Total clearance volume is 9225 cm³. find the compression ratio.

Solⁿ:- Given:

No. of cylinders $n = 6$

$d = 17$ cm, $L = 30$ cm

$V_c = 922.5$ cm³

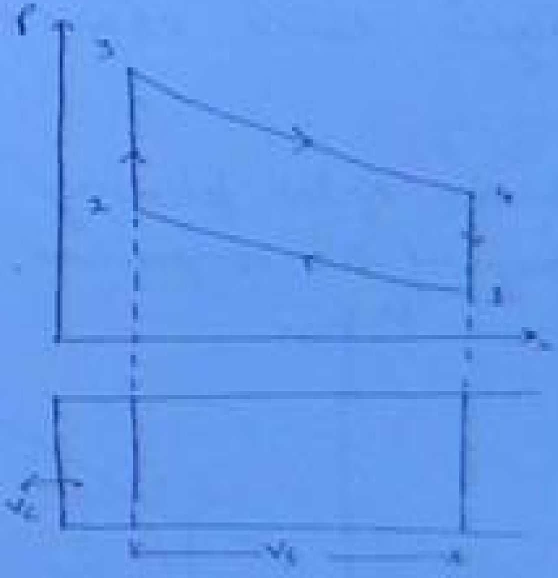
$r = ?$

$$r = \frac{V_1}{V_2} = \frac{V_s + V_c}{V_c} \text{ ——}$$

$$\therefore V_s = \left(\frac{\pi}{4} d^2 \times L\right) \times 6$$

$$= \left(\frac{\pi}{4} \times (17)^2 \times 30\right) \times 6$$

$$\therefore V_s = 40856.4 \text{ cm}^3$$



Now,

$$r = \frac{V_s + V_c}{V_c}$$

$$= \frac{40856.4 + 9225}{9225}$$

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$\therefore \boxed{r = 5.43}$ (Ans).

-2004

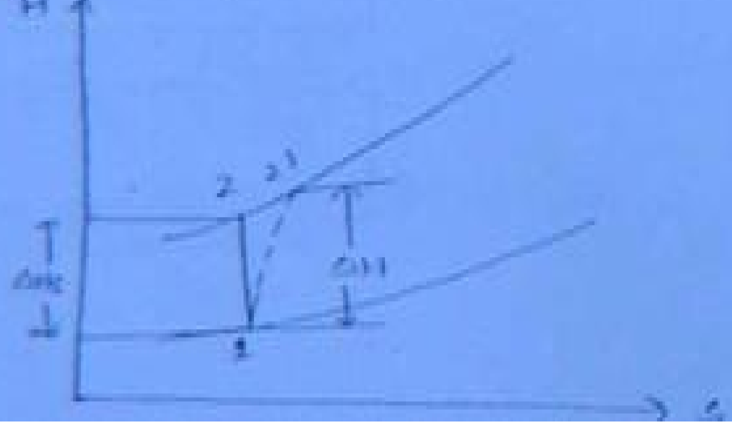
Q. Which gas power cycle consists of four processes & during which work done alone is transferred during two processes and heat added alone during two ot processes?

- (a) Atkinson cycle
- (b) Carnot cycle
- (c) Diesel cycle
- ✓ (d) Otto cycle (Ans).

During compression and expansion process, there is no heat transfer for an otto cycle, there is existence of work done. The heat addition and heat rejection process is at constant volume. So work done during constant volume process is zero (0). As, there is only one type of transfer during each process of a otto cycle, hence, choice (d) is correct.

-2005

Which one of the following expressions, the isentropic efficiency of the compression process is indicated in the figure



[✓ (a)] $\eta = \frac{\Delta H_s}{\Delta H}$

(b) $\eta = \frac{\Delta H}{\Delta H_s}$

(c) $\eta = \frac{\Delta H - \Delta H_s}{\Delta H}$

(d) $\eta = \frac{\Delta H - \Delta H_s}{\Delta H_s}$

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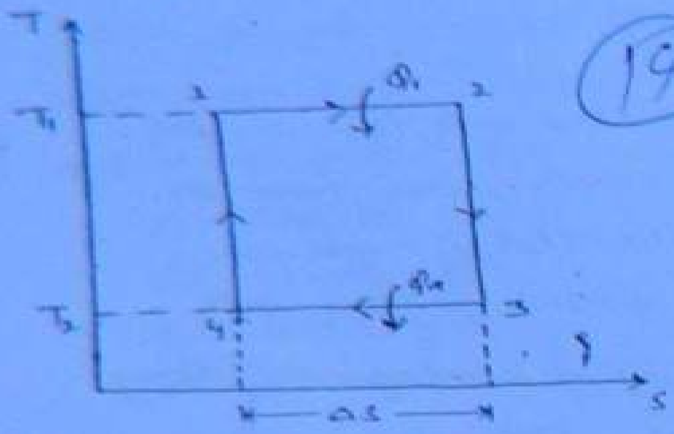
When the compression is without friction, the work done is 'ΔH_s' (see figure). When friction is taken in account, then from the figure, the work done will be 'ΔH'. The ratio of work done without friction to the work done ~~with~~ when friction is taken into account is defined as compression efficiency or isentropic efficiency. This efficiency for the given figure is $\eta = \frac{\Delta H_s}{\Delta H}$ (choice (a) is correct).

*- AVAILABLE ENERGY, AVAILABILITY & IRREVERSIBILITY :-

* Available Energy (AE):-

→ The maximum possible amount of work that can be obtained in a cycle is known as available energy.



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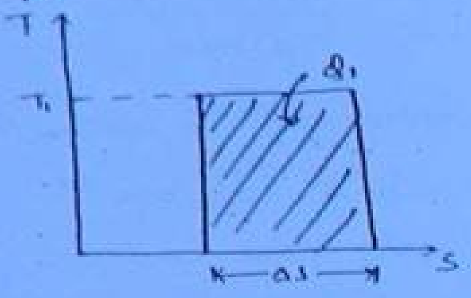
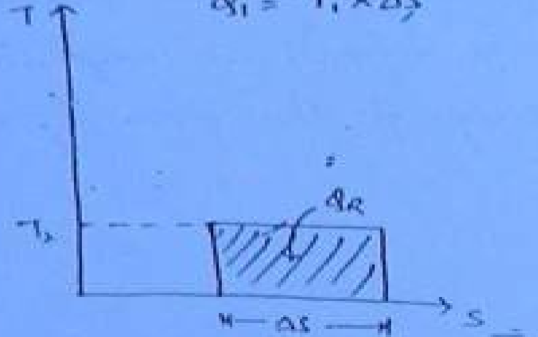


$Q_1 = T_1 \times \Delta S$



$Q_R = T_2 (\Delta S)$

$$\eta = W/Q_1$$

$$\eta_{max} = \frac{W_{max}}{Q_1}$$

also, $\eta_{max} = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \frac{W_{max}}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \boxed{W_{max} = Q_1 (1 - T_2/T_1)}$$

For a given source T_1 and for a given input Q_1 this work is further maximum when T_2 is minimum. The lowest possible temp^r of heat rejection is that of surroundings ($T_2 = T_0$). Therefore,

$$\boxed{AE = W_{max} = Q_1 \left(1 - \frac{T_0}{T_1}\right)}$$

Now, by process graph:-

$$Q_1 = T_1 \Delta S$$

$$\Rightarrow \frac{Q_1}{T_1} = \Delta S$$

Now, $AE = Q_1\left(1 - \frac{T_0}{T_1}\right)$

$$= Q_1 - \frac{Q_1}{T_1} T_0$$

$$= Q_1 - T_0 \Delta S$$

$$\Rightarrow Q_1 = AE + T_0 \Delta S$$

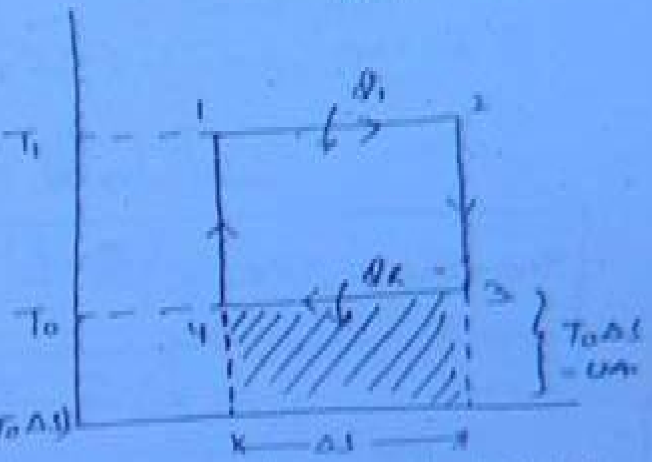
$$\boxed{Q_1 = AE + UAE}$$

Heat Rejection:-

$$Q_R = \underbrace{T_0 \Delta S}_{\text{min. heat rejection}}$$

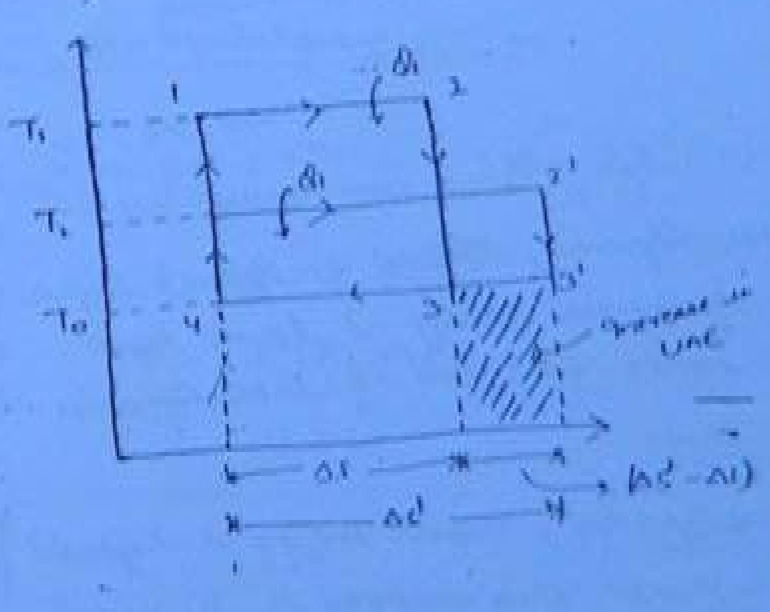
↓

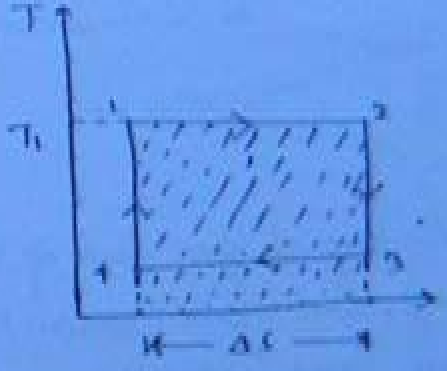
UAE



The minimum possible heat rejection ($T_0\Delta S$) is known as unavailable energy (UAE) i.e. area below atmospheric temp^n T_0 always represents unavailable energy.

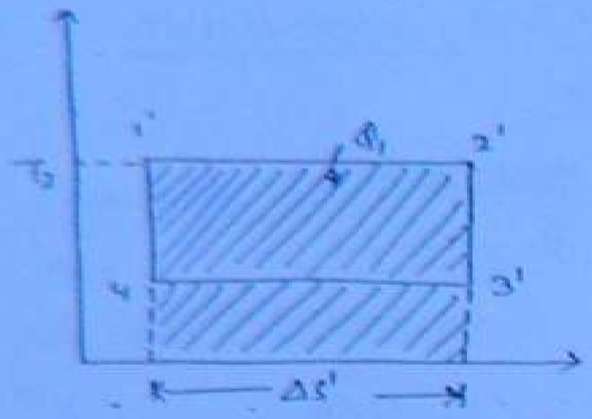
* Loss of Available Energy (increase in unavailable energy) when heat is transferred through a finite temp^n.





$$Q_1 = T_1 \Delta S$$

$$\Rightarrow \Delta S = \frac{Q_1}{T_1}$$



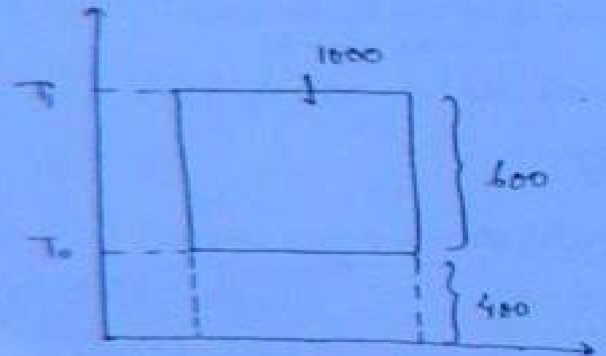
$$Q_1 = T_2 \times \Delta s'$$

$$\Rightarrow \Delta s' = \frac{Q_1}{T_2}$$

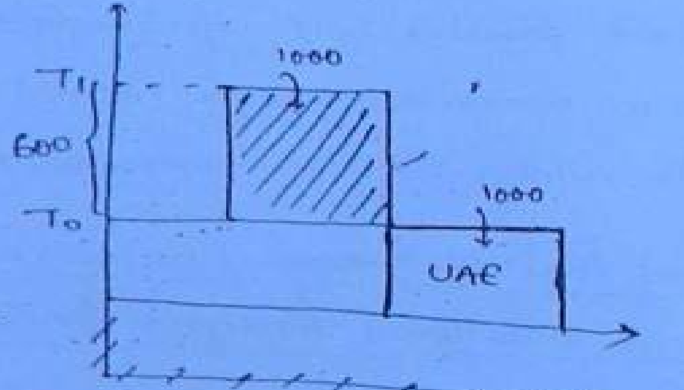
Now. $\uparrow$ in UAE $= T_0(\Delta s' - \Delta s)$

$$= T_0\left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1}\right)$$

$$\boxed{\text{Increase in UAE} = Q_1 T_0 \left(\frac{T_1 - T_2}{T_1 T_2}\right)}$$



$$Q_1 = 1000 \begin{cases} AE = W_{max} = 600 \\ UAE = 400 \end{cases}$$



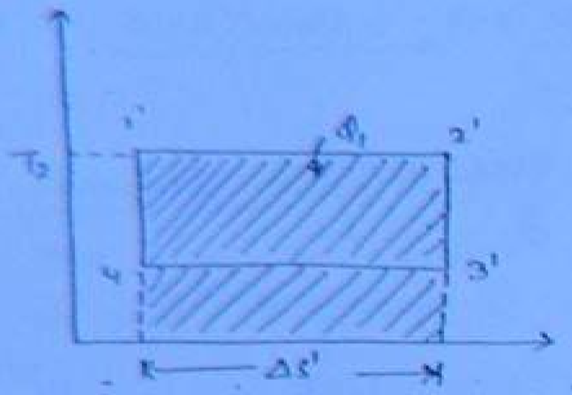
$T_1 \rightarrow 1000 \rightarrow 600$ AE = W

$\quad\quad\quad\quad \rightarrow 400$ UAE

$T_0 \rightarrow 1000 \rightarrow$ UAE but $W = 0$.

** According to first law of thermodynamics, thermal energy at higher temp and equal amount of thermal energy at lower temp have same meaning and hence <u>first law</u> of thermodynamics is known as <u>quantitative law</u>.

=> According to 2nd law of thermodynamics, thermal energy at higher temperature has greater significance when compared to the same amount of thermal energy at lower temp? because thermal energy at higher temp? is capable of producing more power and



$$Q_1 = T_2 \times \Delta s'$$

$$\Rightarrow \Delta s' = \frac{Q_1}{T_2}$$

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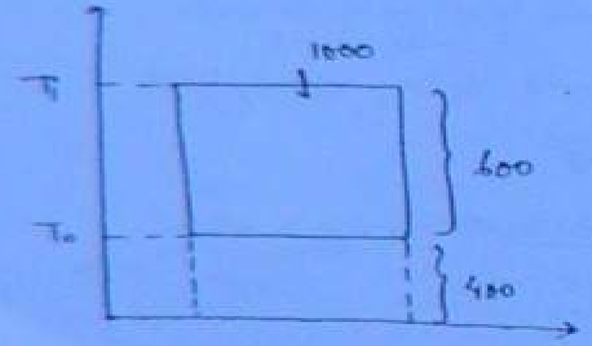
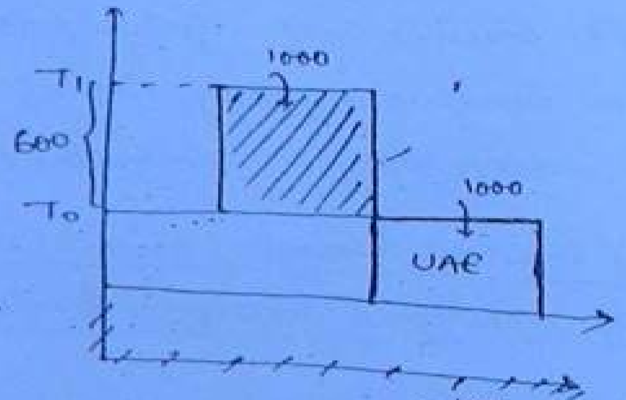
Now. $\uparrow$ in UAE $= T_0(\Delta s' - \Delta s)$

$$= T_0\left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1}\right)$$

$$\boxed{\text{Increase in UAE} = Q_1 T_0 \left(\frac{T_1 - T_2}{T_1 T_2}\right)}$$





$Q_1 = 1000$ — AE = Wmax = 600

— UAE = 400

$T_1 \rightarrow 1000 \rightarrow 600$ AE = W

$\rightarrow 400$ UAE

$T_0 \rightarrow 1000 \rightarrow$ UAE but $W = 0$.

*⇒ According to first law of thermodynamics, thermal energy at higher temp and equal amount of thermal energy at lower temp have same meaning and hence first law of thermodynamics is known as quantitative law.

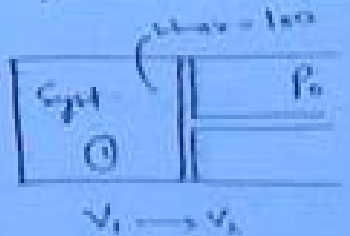
⇒ According to 2nd law of thermodynamics, thermal energy at higher temperature has greater significance when compared to the same amount of thermal energy at lower temp. because thermal energy at higher temp. is capable of producing more power and

$$\Rightarrow dW_{sys} = T_0 (\Delta S)_{sys} - dU_{syst} \quad \text{—— (from ①)}$$

$$\Rightarrow W_{max} = T_0 (S_2 - S_1) - (U_2 - U_1)$$

$$= T_0 (S_2 - S_1) - U_2 + U_1$$

$$\Rightarrow \boxed{W_{max} = (U_1 - U_2) - T_0 (S_1 - S_2)}$$ — for closed system only.



$$W_{max\ useful} = W_{max} - W_{atm}$$

$$W_{atm} = P_0 (V_2 - V_1)$$

So, $$\boxed{W_{max\ useful} = (U_1 - U_2) - T_0 (S_1 - S_2) - P_0 (V_2 - V_1)}$$

$$= (U_1 - T_0 S_1 + P_0 V_1) - [U_2 - T_0 S_2 + P_0 V_2]$$

$W_{max\ useful} = \phi_1 - \phi_2$ where, $\phi_1 = U_1 - T_0 S_1 + P_0 V_1$

$$\Rightarrow \boxed{\phi = U - T_0 S + P_0 V}$$ — availability function for closed system.

<u>Max</u>

Max^m work for open system :-

$$h_1 + \cancel{\frac{C_1^2}{2}} + \cancel{Z_1 g} + Q = h_2 + \cancel{\frac{C_2^2}{2}} + \cancel{Z_2 g} + W$$

$$\Rightarrow h_1 + Q = h_2 + W$$

$$Q = h_2 - h_1 + W$$

$$Q = dh + W$$

So, $$\boxed{W_{max} = h_1 - h_2 - T_0 (S_1 - S_2)}$$ ~~no change of availability~~

$$= (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$$

$$\boxed{W_{max} = \phi_1 - \phi_2}$$ → change of availability.

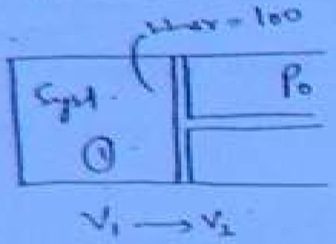
$\Rightarrow \quad dW_{sys} = T_0(\Delta S)_{sys} - dU_{syst}$ —— (from ①)

$\Rightarrow \quad W_{max} = T_0(S_2 - S_1) - (U_2 - U_1)$

$= T_0(S_2 - S_1) - U_2 + U_1$

$\Rightarrow \quad \boxed{W_{max} = (U_1 - U_2) - T_0(S_1 - S_2)}$ — for closed system only.

Now,



$W_{max\ useful} = W_{max} - W_{atm}$

$W_{atm} = P_0(V_2 - V_1)$

So, $\boxed{W_{max\ useful} = (U_1 - U_2) - T_0(S_1 - S_2) - P_0(V_2 - V_1)}$

$= (U_1 - T_0 S_1 + P_0 V_1) - [U_2 - T_0 S_2 + P_0 V_2]$

$W_{max\ useful} = \phi_1 - \phi_2$ where, $\phi_1 = U_1 - T_0 S_1 + P_0 V_1$

$\Rightarrow \quad \boxed{\phi = U - T_0 S + P_0 V}$ — availability function for closed system.

Max^m work for open System :-

$h_1 + \cancel{\frac{C_1^2}{2}} + \cancel{Z_1 g} + Q = h_2 + \cancel{\frac{C_2^2}{2}} + \cancel{Z_2 g} + W$

$\Rightarrow h_1 + Q = h_2 + W$

$Q = h_2 - h_1 + W$

$Q = dh + W$

So, $\boxed{W_{max} = h_1 - h_2 - T_0(S_1 - S_2)}$ — ~~change of availability~~

$= (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$

$\boxed{W_{max} = \phi_1 - \phi_2}$ ⟶ change of availability.

$$B_1 = h_1 - T_0 s_1$$

generalisation $\boxed{B = h_g - T_0 s}$

↓ availability function for open system

* IRREVERSIBILITY :— (I) :-

$$I = W_{max} - W_{actual}$$

$$I = T_0 \left[(\Delta S)_{sys} + (\Delta S)_{surr} \right]$$

$\boxed{I = T_0 (\Delta S)_{univ}}$ ⟶ valid for open and closed system.

* Guoy Stodala Theorem :-

A/C to it, rate of increase of irreversibility is proportional to rate of increase of entropy of the universe.

$$\boxed{\dot{I} \propto (\Delta \dot{S})_{univ}}$$

* Gibb's function (G) :-

$\boxed{G = H - TS}$ — gives sy open system work.

* Helmholtz Function (F) :-

$\boxed{F = U - TS}$ — gives closed system work.

** Equations to be remembered :-

1> $AE = Q_1 \left(1 - \frac{T_0}{T_1} \right)$

2> Increase in UAE due to heat transfer = $Q T_0 \left(\frac{T_1 - T_2}{T_1 T_2} \right)$.

3). $W_{max} = U_1 - U_2 - T_0 (S_1 - S_2)$ for closed system.

4) $W_{max} = h_1 - h_2 - T_0 (S_1 - S_2)$ for open system.

5). $W_{max\ useful} = U_1 - U_2 - T_0 (S_1 - S_2) - P_0 (V_2 - V_1)$ closed system

6). $I = W_{max} - W_{act} = T_0 (\Delta S)_{univ}$

Q. find the loss of available energy associated with in the transfer of 1000 kj of heat from a const. temp^n system at 600 K to another at 400 K and ambient temp^n is 300 K.

Sol^n: ↑ in UAE $= Q_1 T_0 \left(\frac{T_1 - T_2}{T_1 T_2}\right)$ 196

$$= 1000 \times 300 \left(\frac{600-400}{600 \times 400}\right)$$

$$= 250 \text{ kj}$$

→ loss of available energy = increase in UAE.

Q. for a steady flow process from state 1 to state 2 enthalpy changes from $h_1 = 400$ kj/kg to $h_2 = 100$ kj/kg and entropy changes from $S_1 = 1.1$ kj/kg K to $S_2 = 0.7$ kj/kg K and $T_0 = 300$K. find the change in availability.

Sol^n:

$$W_{max} = Q_1 - Q_2$$

$$= (h_1 - h_2) - T_0 (S_1 - S_2)$$

$$= (400 - 100) - 300\,(1.1 - 0.7)$$

$$= 180 \text{ kj/kg}$$ (Ans)