

**LECTURE NOTES ON**  
**MICROWAVE ENGINEERING**

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# **MICROWAVE ENGINEERING**

## **LECTURE NOTES**

UNIT- I  
**MICROWAVE TRANSMISSION LINES-I**

INTRODUCITON

Microwaves are electromagnetic waves with frequencies between 300MHz (0.3GHz) and 300GHz in the electromagnetic spectrum. Radio waves are electromagnetic waves within the frequencies 30KHz - 300GHz, and include microwaves.

Microwaves are at the higher frequency end of the radio wave band and low frequency radio waves are at the lower frequency end. Mobile phones, phone mast antennas (base stations), DECT cordless phones, Wi-Fi, WLAN, Wi MAX and Bluetooth have carrier wave frequencies within the microwave band of the electromagnetic spectrum, and are pulsed/modulated. Most Wi-Fi computers in schools use 2.45GHz (carrier wave), the same frequency as microwave ovens. Information about the frequencies can be found in Wi-Fi exposures and guidelines.

It is worth noting that the electromagnetic spectrum is divided into different bands based on frequency. But the biological effects of electromagnetic radiation do not necessarily fit into these artificial divisions.

The electromagnetic spectrum is the range of frequencies (the spectrum) of electromagnetic radiation and their respective wavelengths and photon energies.

The electromagnetic spectrum covers electromagnetic waves with frequencies ranging from below one hertz to above  $10^{25}$  hertz, corresponding to wavelengths from thousands of kilometers down to a fraction of the size of an atomic nucleus. This frequency range is divided into separate bands, and the electromagnetic waves within each frequency band are called by different names; beginning at the low frequency (long wavelength) end of the spectrum these are: radio waves, microwaves, infrared, visible light, ultraviolet, X-rays, and gamma rays at the high-frequency (short wavelength) end. The electromagnetic waves in each of these bands have different characteristics, such as how they are produced, how they interact with matter, and their practical applications. The limit for long wavelengths is the size of the universe itself, while it is thought that

the short wavelength limit is in the vicinity of the Planck length.<sup>[4]</sup> Gamma rays, X-rays, and high ultraviolet are classified as *ionizing radiation* as their photons have enough energy to ionize atoms, causing chemical reactions. Exposure to these rays can be a health hazard, causing radiation sickness, DNA damage and cancer. Radiation of visible light wavelengths and lower are called *nonionizing radiation* as they cannot cause these effects.

Class			Freq- uency	Wave- length	Energy
Ionizing radiation	$\gamma$	Gamma rays	300 EHz	1 pm	1.24 MeV
			30 EHz	10 pm	124 keV
	HX	Hard X-rays	3 EHz	100 pm	12.4 keV

	SX	Soft X-rays	300 PHz	1 nm	1.24 keV
			30 PHz	10 nm	124 eV
	EUV	Extreme ultraviolet	3 PHz	100 nm	12.4 eV
	NUV	Near ultraviolet			
Visible			300 THz	1 $\mu\text{m}$	1.24 eV
	NIR	Near infrared			
			30 THz	10 $\mu\text{m}$	124 meV
	MIR	Mid infrared			
			3 THz	100 $\mu\text{m}$	12.4 meV

	FIR	Far infrared			
			300 GHz	1 mm	1.24 meV
Micro- waves  and radio waves	EHF	Extremely high frequency	30 GHz	1 cm	124 $\mu$ eV
	SHF	Super high frequency	3 GHz	1 dm	12.4 $\mu$ eV
	UHF	Ultra high frequency	300 MHz	1 m	1.24 $\mu$ eV
	VHF	Very high frequency	30 MHz	10 m	124 neV
HF	High frequency	3 MHz	100 m	12.4 neV	

	MF	Medium frequency			
			300 kHz	1 km	1.24 neV
	LF	Low frequency			
			30 kHz	10 km	124 peV
	VLF	Very low frequency			
			3 kHz	100 km	12.4 peV
	ULF	Ultra low frequency			
			300 Hz	1000 km	1.24 peV
	SLF	Super low frequency			
			30 Hz	10000 km	124 feV
	ELF	Extremely low frequency			
			3 Hz	100000 km	12.4 feV

$\gamma$ = Gamma rays	MIR = Mid infrared	HF = High freq.			
HX = Hard X-rays	FIR = Far infrared	MF = Medium freq.			
SX = Soft X-rays	Radio waves	LF = Low freq.			
EUV = Extreme ultraviolet	EHF = Extremely high freq.	VLF = Very low freq.			
NUV = Near ultraviolet	SHF = Super high freq.	VF/ULF = Voice freq.			
Visible light	UHF = Ultra high freq.	SLF = Super low freq.			
NIR = Near Infrared	VHF = Very high freq.	ELF = Extremely low freq.			
		Freq = Frequency			

### Microwave Frequency Bands

The microwave spectrum is usually defined as a range of frequencies ranging from 1 GHz to over 100 GHz. This range has been divided into a number of frequency bands, each represented by a letter. There are a number of organizations that assign these letter bands. The most common being the IEEE Radar Bands followed by NATO Radio Bands and ITU Bands. Below you can see tables with details on each letter band. Click on the letter band to learn more about it and find products on everything RF that can be used for in this band.

#### Frequency Bands

Letter Designation	Frequency Range	Wavelength Range
L band	1 to 2 GHz	15 cm to 30 cm
S band	2 to 4 GHz	7.5 cm to 15 cm



Letter Designation	Frequency Range	Wavelength Range
C band	4 to 8 GHz	3.75 cm to 7.5 cm
X band	8 to 12 GHz	25 mm to 37.5 mm
Ku band	12 to 18 GHz	16.7 mm to 25 mm
K band	18 to 26.5 GHz	11.3 mm to 16.7 mm
Ka band	26.5 to 40 GHz	5.0 mm to 11.3 mm
Q band	33 to 50 GHz	6.0 mm to 9.0 mm
U band	40 to 60 GHz	5.0 mm to 7.5 mm
V band	50 to 75 GHz	4.0 mm to 6.0 mm
W band	75 to 110 GHz	2.7 mm to 4.0 mm
F band	90 to 110 GHz	2.1 mm to 3.3 mm
D band	110 to 170 GHz	1.8 mm to 2.7 mm

At microwave frequencies (above 1GHz to 100 GHz) the losses in the two line transmission system will be very high and hence it cannot be used at those frequencies. Hence microwave signals are propagated through the waveguides in order to minimize the losses.

Microwaves are radio waves radio waves with wave lengths ranging from as long as one meter to as short as one millimeter, or equivalently, with frequencies between 300 MHz (0.3 GHz) and 300 GHz.

This broad definition includes both UHF and EHF (millimeter waves), and various sources use different boundaries. In all cases, microwave includes the entire SHF band (3 to 30 GHz, or 10 to 1 cm) at minimum, with RF engineering often putting the lower boundary at 1 GHz (30 cm), and the upper around 100 GHz (3 mm). The prefix "micro-" in "microwave" is not meant to suggest a wavelength in the micrometer range. It indicates that microwaves are "small" compared to waves used in typical radio broadcasting, in that they have shorter wavelengths. The boundaries between far infrared light, terahertz radiation, microwaves, and ultra-high-frequency radio waves are fairly arbitrary and are used variously between different fields of study. Microwave technology has wide range of application areas. Traditionally it has been used for telecommunication/communication purposes but it is also used for different kinds of sensing and imaging applications. Heating of different substance such as food is another area. The application areas are many can be categories in different ways.

- Telecom
- Point-to-point communication, Satellite, Cellular access technologies
- Space
- Sensing/Spectroscopy, Communication, Radio astronomy
- MedTech
- Diagnostics, imaging, and treatment applications.
- Defense
- Radar, Communication
- Security
- Car avoidance radar, Traffic surveillance, Air traffic security “cameras”
- Navigation, Positioning & Measurement
- GPS
- Food
- Heating & detection of foreign bodies in food New and novel application areas are constantly being added.

Waveguide:

A waveguide consists of a hollow metallic tube of a rectangular or circular shape used to guide an electromagnetic wave. Waveguides are used principally at frequencies in the microwave range.

In waveguide the electric and magnetic fields are confined the space with in the guides. Thus no power is lost through radiation and even the dielectric loss is negligible since the guides are normally air-filled. However, there is some power loss as heat in the walls of the guide, but the loss is very small.

It is possible to propagate several modes of EM waves with in a waveguide. These modes correspond to solutions of Maxwell's Equations for particular waveguide.

If the frequency of the impressed signal is above the cut-off frequency for a given mode, the EM energy can be transmitted through the guide for that particular mode without attenuation.

The mode which is having the lowest cut-off frequency is called the 'Dominant Mode'

**Properties and characteristics of waveguide:**

The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave through multiple reflections.

When the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall .the process results in a component of either electric or magnetic fields in the direction of propagation of the resultant wave.

TEM waves cannot propagate through the waveguide since it requires an axial conductor for axial current flow.when the wavelength inside the waveguide differs from that outside the guide, the velocity of wave

propagation inside the waveguide must also be different from that through free space.

if one end of the waveguide is closed using a shorting plate and allowed a wave to propagate from other end, then there will be complete reflection of the waves resulting in standing waves.

### **Wave guides and its types:**

There are five types of waveguides. They are:

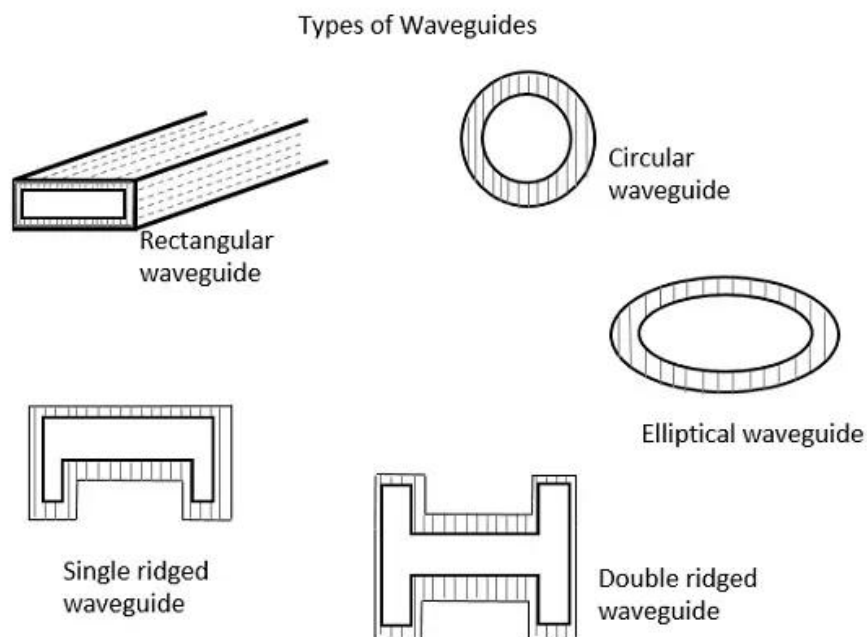
Rectangular waveguide

Circular waveguide

Elliptical waveguide

Single ridged waveguide

Double ridged waveguide



### **Advantages of Waveguides:**

Waveguides are easy to manufacture.

They can handle very large power (in kilowatts)

Power loss is very negligible in waveguides

They offer very low loss (low value of alpha-attenuation)

The microwave energy when travels through the waveguide, experiences lower losses than a coaxial cable.

### **Rectangular Waveguide:**

A Rectangular waveguide is a hollow metallic tube with a rectangular cross section.

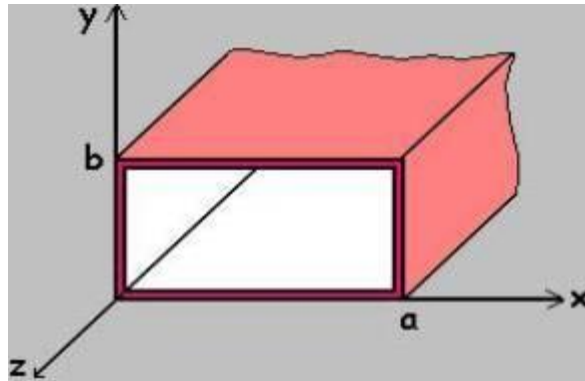


Fig. Rectangular wave guide

When the waves travel longitudinally down the guide because of conducting walls plane waves are reflected from wall. This process results in a component of either electric or magnetic field in the direction of propagation of the resultant wave. Therefore the wave is no longer a transverse electromagnetic wave. Any uniform plane wave in a lossless guide may be resolved into TE and TM waves. In rectangular guide the modes are designed  $TE_{mn}$  or  $TM_{mn}$ .

### **Propagation of waves in Rectangular waveguides**

Consider a rectangular waveguide situated in the rectangular coordinate system with its breadth along x-axis, width along y-axis and the wave is assume to propagate along the z-direction. Waveguide is filled with air. In a waveguide no TEM wave is exists.

**TEM(Transverse Electromagnetic wave):** in TEM both electric and magnetic fields are purely transverse to the direction of propagation and consequence have no 'z' directed E & H components.

**TE(Transverse Electric Wave)** In TE wave only the E field is purely transverse to the direction of

propagation and the magnetic field is not purely transverse

i.e.  $E_z=0, H_z \neq 0$

**TM(Transverse Magnetic Wave)** In TE wave only the H field is purely transverse to the direction of propagation and the Electric field is not purely transverse

i.e.  $E_z \neq 0, H_z = 0$

**HE(Hybrid wave)** In this neither electric nor magnetic fields are purely transverse to the direction of propagation.

i.e.  $E_z \neq 0, H_z \neq 0$

**WAVE EQUATIONS**

Since we assumed that the wave direction is along z-direction then the wave equation are

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z \quad \text{for TM wave - (1)}$$

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \text{-----for TE wave (2)}$$

$$\text{Where } E_z = E_0 e^{-\gamma z}, H_z = H_0 e^{-\gamma z} \text{-----(3)}$$

The condition for wave propagation is that  $\gamma$  must be imaginary. Differentiating eqn(3) w.r.t 'z' we get

$$\partial E_z / \partial z = E_0 e^{-\gamma(-\gamma)} = -\gamma E_z \text{----- (4)}$$

Hence we can define operator  $\partial / \partial z = -\gamma \text{----- (5)}$

By differentiating eqn(4) w.r.t 'z' we get  $\partial^2 E_z / \partial z^2 = \gamma^2 E_z$

We can define the operator

$$\partial^2 / \partial z^2 = \gamma^2 \text{-- (6)}$$

From eqn(1) we can write

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z$$

By solving above two partial differential equations we get solutions for  $E_z$  and  $H_z$ . Using Maxwell's equations. it is possible to find the various components along x and y-directions.

From Maxwell's first equation, we have

$$\nabla \times H = j\omega \epsilon E$$

$$= j\omega \epsilon [E_x a_x + E_y a_y + E_z a_z]$$

$$a_x \rightarrow \gamma H_y + \partial H_z / \partial y = j\omega \epsilon E_x \quad \text{---(9)}$$

$$a_y \rightarrow \gamma H_x + \partial H_z / \partial x = -j\omega \epsilon E_y \quad \text{---10}$$

$$a_z \rightarrow \partial H_y / \partial x + \partial H_x / \partial y = j\omega \epsilon E_z \quad \text{---(11)}$$

similarly from Maxwell's 2<sup>nd</sup> equation we have

$$\nabla \times E = -j\omega \mu H \quad \text{By expanding } \partial / \partial z = -j\omega [H_x a_x + H_y a_y + H_z a_z]$$

By comparing  $a_x, a_y, a_z$  components

$$a_x \rightarrow \gamma E_y + \partial E_z / \partial y = -j\omega \mu H_x \quad \text{---(12)}$$

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega \mu H_y \quad \text{---(13)}$$

$\partial x$

$$\partial E_y / \partial x - \partial E_x / \partial y = -j\omega \epsilon H_z \quad \text{---(14)}$$

From eqn(13)

$$H_y = \gamma E_x + \frac{\partial E_z}{\partial x} / j\omega \mu \quad \text{---(15)}$$

$\partial x$

By substituting eqn(15) in eqn(9) we get

$$\gamma^2 / j\omega \mu E_x + \gamma / j\omega \epsilon \frac{\partial E_z}{\partial x} + \partial H_z / \partial y = j\omega \epsilon E_x$$

$\partial x$

$$\text{since } \gamma^2 + \omega^2 \mu \epsilon = h^2$$

by dividing the above equation with  $h^2$  we get

$$E_x = -\gamma/h^2 \frac{\partial E_z}{\partial x} - j\omega\mu/h^2 \frac{\partial H_z}{\partial y} \text{---- (15)}$$

Similarly

$$E_y = -\gamma/h^2 \frac{\partial E_z}{\partial y} + j\omega\epsilon/h^2 \frac{\partial E_z}{\partial x} \text{---- (16)}$$

And

$$H_x = -\gamma/h^2 \frac{\partial H_z}{\partial x} + j\omega\mu/h^2 \frac{\partial E_z}{\partial y} \text{---- (17)}$$

$$H_y = -\gamma/h^2 \frac{\partial H_z}{\partial y} - j\omega\mu/h^2 \frac{\partial E_z}{\partial x} \text{---- (18)}$$

These equations give a general relationship for field components with in a waveguide. Propagation of TEM Waves:

For TEM wave  $E_z=0$  and  $H_z=0$

Substituting these values in equns (15) to (18) all the field components along x and y directions  $E_x, E_y, H_x, H_y$  vanish and have a TEM wave cannot exist inside a waveguide.

### **Modes:**

The electromagnetic wave inside a waveguide can have an infinite number of patterns which are called modes. The electric field cannot have a component parallel to the surface i.e. the electric field must always be perpendicular to the surface at the conductor. The magnetic fields on the other hand always parallel to the surface of the conductor and cannot have a component perpendicular to it at the surface.



## TE Mode Analysis

The  $TE_{mn}$  modes in a rectangular waveguide are characterized by  $E_z=0$ . The z component of the magnetic field,  $H_z$  must exist in order to have energy transmission in the guide.

The wave equation for TE wave is given by

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \quad (1)$$

This is a partial differential equation whose solution can be assumed. Assume a solution

$$H_z = XY$$

Where  $X$  = pure function of  $x$  only  $Y$  = pure function of  $y$  only

$$-B^2 - A^2 + h^2 = 0$$

$$\text{i.e. } h^2 = A^2 + B^2 \quad (4)$$

$$X = c_1 \cos Bx + c_2 \sin Bx \quad Y = c_3 \cos Ay + c_4 \sin Ay$$

i.e. the complete solution for  $H_z = XY$  is

$$H_z = (c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay + c_4 \sin Ay) \quad (5)$$

Where  $c_1, c_2, c_3$  and  $c_4$  are constants which can be evaluated by applying boundary conditions.

### Boundary Conditions

Since we consider a TE wave propagating along z direction. So  $E_z=0$  but we have components along

x and y direction.  $E_x=0$  waves along bottom and top walls of the waveguide

$E_y=0$  waves along left and right

walls of the waveguide 1<sup>st</sup>

Boundary condition:

$E_x=0$  at  $y=0 \forall x \rightarrow 0$  to  $a$  (bottom wall) 2<sup>nd</sup> Boundary condition

$E_x=0$  at  $y=b$

$\forall x \rightarrow 0$  to

a (top wall) 3<sup>rd</sup> Boundary condition

$E_y=0$  at  $x=0 \forall y \rightarrow 0$  to  $b$  (left side wall) 4<sup>th</sup> Boundary condition

$E_y=0$  at  $x=a \forall y \rightarrow 0$  to  $b$  (right side wall)

i) Substituting 1<sup>st</sup> Boundary condition in eqn(5) Since we have

$$E_x = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu/h^2 \partial H_z / \partial y \quad (6)$$

Since  $E_z=0 \rightarrow$

$$E_x = -j\omega\mu/h^2 [(c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay + c_4 \sin Ay) / \partial y] \quad E_x = -j\omega\mu/h^2 \partial [(c_1 \cos Bx + c_2 \sin Bx)(-A c_3 \sin Ay + A c_4 \cos Ay) / \partial y]$$

From the first boundary condition we get

$$0 = -j\omega\mu/h^2 [(c_1 \cos Bx + c_2 \sin Bx) \neq 0, A \neq 0 \quad c_4 = 0]$$

Substituting the value of  $c_4$  in eqn (5), the solution reduces to

$$H_z = (c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay) \quad (7)$$

ii) from third boundary condition  $E_y=0$  at  $x=0 \forall y \rightarrow 0$  to  $b$

Since we have

$$E_y = -\gamma/h^2 \partial E_z / \partial y + j\omega\mu/h^2 \partial H_z / \partial x \quad \text{--- (8)}$$

Since  $E_z=0$  and substituting the value of  $H_z$  in eqn(7), we get  $E_y = j\omega\mu/h^2 [(c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay)] / \partial x$

$$E_y = j\omega\mu/h^2 [(-Bc_1 \sin Bx + Bc_2 \sin Bx)(c_3 \cos Ay)]$$

From third condition,  $0 = j\omega\mu/h^2 (0 + Bc_2) c_3 \cos Ay$  Since  $\cos Ay \neq 0, B \neq 0, c_3 \neq 0 \quad c_2 = 0$

from eq (7)  $H_z = c_1 c_3 \cos Bx \cos Ay$  (9)

iii) From boundary condition 2<sup>nd</sup> Boundary condition

since we have

$$E_x = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu/h^2 \partial H_z / \partial y$$

$$= -j\omega\mu/h^2 \partial / \partial [c_1 c_3 \cos Bx \cos Ay] \quad [E_z=0] \quad E_x = j\omega\mu/h^2 c_1 c_3 \cos Bx \sin Ay$$

From the second boundary condition,  $E_x=0$  at  $y=b \forall x \rightarrow 0$  to  $a$

$$0 = j\omega\mu/h^2 c_1 c_3 \cos Bx \sin Ab$$

$$\cos Bx \neq 0, c_1 c_3 \neq 0$$

$$\sin Ab = 0 \text{ or } Ab = n\pi \text{ where } n=0,1,2,\dots$$

$$A = n\pi/b \quad (10)$$

iv) 4<sup>th</sup> Boundary condition since

$$E_y = -\gamma/h^2 \partial E_z / \partial y + j\omega\mu/h^2 \partial H_z / \partial x$$

$$E_y = -j\omega\mu/h^2 \partial / \partial [c_1 c_3 \cos Bx \cos Ay]$$

$$E_y = -j\omega\mu/h^2 c_1 c_3 \sin Bx \cdot B \cos Ay$$

From the 4<sup>th</sup> Boundary condition  $E_y=0$  at  $x=a \forall y \rightarrow 0$  to  $b$

$$0 = -$$

$$j\omega\mu/h^2 B c_1 c_3 \sin Bx \cdot \cos A$$

$$y \quad \forall \quad y \rightarrow 0 \quad \text{to} \quad b$$

$$\cos Ay \neq 0, c_1 c_3 \neq 0$$

$$\sin Ba = 0$$

$$B = m\pi/a \quad (11)$$

From eq(9)

$$H_z = c_1 c_3 \cos(m\pi/a)x \cos(n\pi/b)y$$

Let  $c_1 c_3 = c$

$$H_z = c \cos(m\pi/a)x \cos(n\pi/b)y e^{(j\omega t - \gamma z)} \quad \dots \quad (12)$$

### Field Components

$$E_x = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu/h^2 \partial H_z / \partial y \quad \text{Since } E_z=0 \text{ for TE wave}$$

$$E_x = j\omega\mu/h^2 c(n\pi/b) \cos(m\pi/a)x \sin(n\pi/b)y e^{j\omega t - \gamma z} \quad (13)$$

$$E_y = \gamma/h^2 \partial E_z /$$

$$\partial y + j\omega\mu/h^2 \partial$$

$H_z / \partial x$  Since

$E_z = 0$  for TE

wave

$$E_y = j\omega\mu/h^2 \partial H_z / \partial x$$

$$E_y = -j\omega\mu/h^2 c[m\pi/a] \sin(m\pi/a)x \cos(n\pi/b)y e^{j\omega t - \gamma z} \quad (14)$$

Similarly

$$H_x = -\gamma/h^2 \partial H_z / \partial x - j\omega\epsilon/h^2 \partial E_z / \partial y$$

$$H_x = \gamma/h^2 c(m\pi/a) \sin(m\pi/a)x \cos(n\pi/b)y e^{j\omega t - \gamma z} \quad (15)$$

$$H_y = -\gamma/h^2 \partial H_z / \partial y - j\omega\epsilon/h^2 \partial E_z / \partial x$$

$$H_y = -\gamma/h^2 c(n\pi/b)^2 \cos(m\pi/a)x \sin(n\pi/b)y e^{j\omega t - \gamma z} \quad (16)$$

### TM Mode Analysis

For TM wave  $H_z = 0$   $E_z \neq 0$

$$\partial^2 E_z / \partial x^2 + \partial^2 E_z / \partial y^2 + h^2 E_z = 0 \quad (1)$$

This is a partial differential equation which can be solved to get the different field components

$E_x$ ,  $E_y$ ,  $H_x$  and  $H_y$  by variable separable method.

$$\text{Let us assume a solution } E_z = XY \quad (2)$$

Using these two equations from eqn(1) we get

$$-B^2 - A^2 + h^2 = 0$$

$$\text{i.e. } h^2 = A^2 + B^2 \quad (5)$$

the solution of eqn(3) and(4) are

$$X=c_1\cos Bx+c_2\sin Bx$$

$$Y=c_3\cos Ay+c_4\sin Ay$$

Where  $c_1, c_2, c_3$  and  $c_4$  are constants which can be evaluated by applying boundary conditions From eqn(1)

$$EZ= XY$$

$$EZ= (c_1\cos Bx+c_2\sin Bx)(c_3\cos Ay+c_4\sin Ay)$$

$$\text{----- (6)}$$

### Boundary Conditions

Since we consider a TE wave propagating along z direction. So  $Ez=0$  but we have components along x and y direction.  $Ex=0$  waves along bottom and top walls of the waveguide

$Ey=0$  waves along left and right walls of the waveguide 1<sup>st</sup> Boundary condition:

$Ex=0$  at  $y=0 \forall x \rightarrow 0$  to  $a$  (bottom wall) 2<sup>nd</sup> Boundary condition

$Ex=0$  at  $y=b \forall x \rightarrow 0$  to  $a$  (top wall) 3<sup>rd</sup> Boundary condition

$Ey=0$  at  $x=0 \forall y \rightarrow 0$  to  $b$  (left side wall) 4<sup>th</sup> Boundary condition

$Ey=0$  at  $x=a \forall y \rightarrow 0$  to  $b$  (right side wall)

i)

Substituting 1<sup>st</sup> Boundary condition in eqn(10) Since we have

$$0=Ez= [c_1\cos Bx+c_2\sin Bx][c_3\cos A0+c_4\sin A0] [c_1\cos Bx+c_2\sin Bx]c_3=0 c_1\cos Bx+c_2\sin Bx \neq 0$$

$$c_3=0$$

$$\text{i.e. } Ez=[c_1\cos Bx+c_2\sin Bx]c_4\sin Ay \quad (11)$$

Substituting 2<sup>nd</sup> Boundary condition in eqn(11), we get  $Ez=c_2c_4\sin Bx\sin Ay$  (12)

ii) Substituting 3<sup>rd</sup> Boundary condition in eqn(12), we get

$$\sin Ab=0$$

$$A = n\pi/b \quad (13)$$

iii)

Substituting 4<sup>th</sup> Boundary condition

in eqn(12), we get  $\sin Ba = 0$

$$B = m\pi/a \quad (14)$$

From (12),(13),(14)

$$E_z = c \sin(m\pi/a) x \sin(n\pi/b) y e^{j(\omega t - \gamma z)} \quad (15)$$

$$E_x = -\gamma/h^2 \partial E_z / \partial x$$

$$E_x = -\gamma/h^2 c (m\pi/a) \cos(m\pi/a) x \sin(n\pi/b) y e^{j(\omega t - \gamma z)} \quad (16)$$

$$E_y = -\gamma/h^2 c (n\pi/b) \sin(m\pi/a) x \cos(n\pi/b) y e^{j(\omega t - \gamma z)} \quad (17)$$

$$H_x = j\omega\epsilon/h^2 c (n\pi/b) \sin(m\pi/a) x \cos(n\pi/b) y e^{j(\omega t - \gamma z)} \quad (18)$$

$$H_y = j\omega\epsilon/h^2 c [m\pi/a] \cos(m\pi/a) x \sin(n\pi/b) y e^{j(\omega t - \gamma z)} \quad (19)$$

### Cut-off Frequency of a Waveguide

$$\text{Since we have } \gamma^2 + \omega^2 \mu\epsilon = h^2 = A^2 + B^2$$

$\gamma$  then becomes real and positive and equal to the attenuation constant  $\alpha$  i.e. the wave is completely attenuated and there is no phase change. Hence the wave cannot propagate.

However at higher frequencies,  $\gamma < 0$

$\gamma$  becomes imaginary there will be phase change  $\beta$  and hence the wave propagates.

At the transition  $\gamma$  becomes zero and the propagation starts. The frequency at which  $\gamma$  just becomes zero is defined as the cut-off frequency  $f_c$

At  $f = f_c$ ,

$$\gamma = 0$$

$$0 = (m\pi/a)^2 + (n\pi/b)^2 - \omega^2 \mu \epsilon \text{ or}$$

$$f_c = 1/2\pi \sqrt{\mu \epsilon} [(m\pi/a)^2 + (n\pi/b)^2]^{1/2}$$

$$f_c = c/2 [(m\pi/a)^2 + (n\pi/b)^2]^{1/2}$$

The cut-off wavelength ( $\lambda_c$ ) is

$$\lambda_c = c/f_c =$$

$$c/2 [(m\pi/a)^2 + (n\pi/b)^2]^{1/2}$$

$$\lambda_{cm,n} = 2ab / [m^2 b^2 + n^2 a^2]^{1/2}$$

All wavelengths greater than  $\lambda_c$  are attenuated and those less than  $\lambda_c$  are allowed to propagate inside the waveguide.

### **Guided Wavelength ( $\lambda_g$ ):**

It is defined as the distance travelled by the wave in order to undergo a phase shift of  $2\pi$  radians.

It is related to phase constant by the relation  $\lambda_g = 2\pi/\beta$  the wavelength in the waveguide is different from the wavelength in free space. Guide wavelength is related

to free space wavelength  $\lambda_0$  and cut-off wavelength  $\lambda_c$  by

$$1/\lambda_g^2 = 1/\lambda_0^2 - 1/\lambda_c^2$$

The above equation is true for any mode in a waveguide of any cross section

### **Phase Velocity ( $v_p$ )**

Wave propagates in the waveguide when guide wavelength  $\lambda_g$  is greater than the free space wavelength  $\lambda_0$ .

In a waveguide,  $v_p = \lambda_g f$  where  $v_p$  is the phase velocity. But the speed of light is equal to product of  $\lambda_0$

and  $f$ .

This  $v_p$  is greater than the speed of light since  $\lambda_g > \lambda_0$ .

The wavelength in the guide is the length of the cycle and  $v_p$  represents the velocity of the phase. It is defined as the rate at which the wave changes its phase in terms of the guide wavelength.

$$V_p = \omega / \beta$$

$$V_p = c / [1 - (\lambda_0 / \lambda_c)^2]^{1/2}$$

### **Group Velocity ( $v_g$ ):**

The rate at which the wave propagates through the waveguide and is given by  $V_g = d\omega / d\beta$

$$\text{Since } \beta = [\mu\epsilon(\omega^2 - \omega_c^2)]^{1/2}$$

Now differentiating  $\beta$  w.r.t  $\omega$  we get  $V_g = c [1 - (\lambda_0 / \lambda_c)^2]^{1/2}$

Consider the product of  $V_p$  and  $V_g$

$$V_p \cdot V_g = c^2$$

### **Dominant Mode**

The mode for which the cut-off wavelength assumes a maximum value.

$$\lambda_{c\,mn} = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$$

**Dominant mode in TE** For TE<sub>01</sub> mode  $\lambda_{c\,01} = 2b$  TE<sub>10</sub> mode  $\lambda_{c\,10} = 2a$

Among all  $\lambda_{c\,10}$  has the maximum value since 'a' is the larger dimension than 'b'.

Hence TE<sub>10</sub> mode is the dominant mode in rectangular waveguide.

### **Dominant Mode in TM**

Minimum possible mode is TM<sub>11</sub>. Higher modes than this also exist.

### **Degenerate Modes**

Two or more modes having the same cut-off frequency are called 'Degenerate modes'

For a rectangular waveguide TE<sub>mn</sub>/TM<sub>mn</sub> modes for which both  $m \neq 0, n \neq 0$  will always be degenerate modes.



## Wavelengths and Impedance Relations[TE &TM WAVES]

### Guide Wavelength( $\lambda_g$ )

It is defined as the distance travelled by the wave in order to under go a phase shift of  $2\pi$  radians.

$$1/\lambda_g^2 = 1/\lambda_0^2 - 1/\lambda_c^2$$

Wave impedance is defined as the ratio of the strength of electric field in one transverse direction to the strength of the magnetic field along the other transverse direction.

$$Z = E_x/H_y$$

Wave impedance for a TM wave in rectangular waveguide

$$Z = -\gamma/h^2 \partial E_z / \partial x - j\omega\mu \partial H_z / \partial y / -\gamma/h^2 \partial H_z / \partial y - j\omega\epsilon/h^2 \partial E_z / \partial x$$

For a TM wave  $H_z=0$   $Z_{TM} = \gamma/j\omega\epsilon$

$$= \beta/\omega\epsilon$$

Since we have  $\beta = [\omega^2\mu\epsilon - \gamma^2]^{1/2}$

$$Z_{TM} = \eta [1 - (\lambda_0/\lambda_c)^2]^{1/2}$$

Since ' $\lambda_0$  is always less than  $\lambda_c$  for wave propagation  $Z_{TM} < \eta$

1) Wave impedance of TE waves in rectangular waveguide  $Z_{TE} = \eta/[1 - (\lambda_0/\lambda_c)^2]^{1/2}$

Therefore  $Z_{TE} > \eta$

For TEM waves between parallel planes the cut-off frequency is zero and wave impedance for TEM wave is the free space impedance itself

$$Z_{TEM} = \eta$$

### Cavity Resonators:

A cavity resonator is a metallic enclosure that confines the electromagnetic energy i.e. when one end of the waveguide is terminated in a shorting plate there will be reflections and hence standing waves. When another shorting plate is kept at a distance of a multiple of  $\lambda_g/2$  than the hollow space so formed can support a signal which bounces back and forth

between the two shorting plates. This results in resonance and hence the hollow space is called “cavity” and the resonator as the ‘cavity resonator’.

The waveguide section can be rectangular or circular.

The microwave cavity resonator is similar to a tuned circuit at low frequencies having a resonant

frequency  $f_0 = \frac{1}{2\pi\sqrt{LC}}$

The cavity resonator can resonate at only one particular frequency like a parallel resonant circuit.

From the figure

$$d = 3\lambda_g/2$$

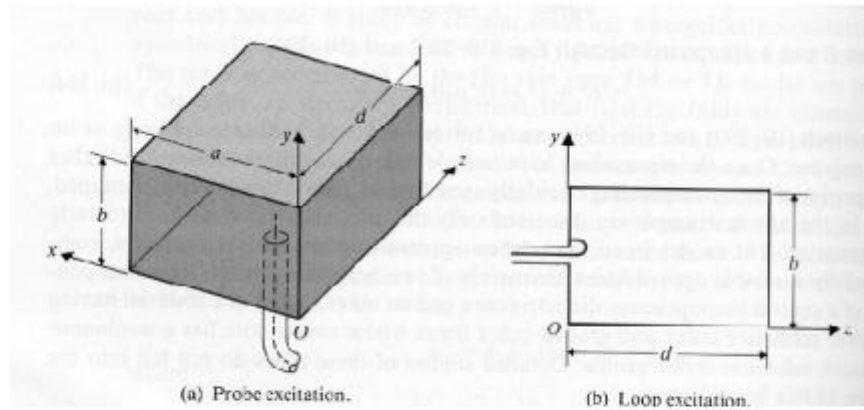
The stored electric and magnetic energies inside the cavity determine its equivalent inductance and capacitance. The energy dissipated by the finite conductivity of the cavity walls determines its equivalent resistance.

A given resonator has an infinite number of resonant modes and each mode corresponds to a definite resonant frequency.

When the frequency of an impressed signal is equal to a resonant frequency a maximum amplitude of the standing wave occurs and the peak energies stored in the electric and magnetic fields are equal.

The mode having the lowest resonant frequency is called as the ‘Dominant mode’.

### **Rectangular cavity Resonator**



**Case 1  $TM_{mnp}$  mode:  $H_z=0$ , neither  $m$  nor  $n=0$ ,  $p$  can be 0.**

The electromagnetic field inside the cavity should satisfy Maxwell's equations subject to the boundary conditions that the electric field tangential to and the magnetic field normal to the metal walls must vanish.

The wave equations in the rectangular resonator should satisfy the boundary condition of the zero tangential 'E' At four of the walls.

(1) TE waves

For a TE wave  $E_z=0, H_z \neq 0$  From Maxwell's equation

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \epsilon H_z$$

$$\text{Since } \frac{\partial^2 H_z}{\partial z^2} = -\gamma^2 H_z$$

$$\gamma^2$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \epsilon) H_z = 0$$

$$\text{Let } \gamma^2 + \omega^2 \mu \epsilon = h^2$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \text{ ----- (1)}$$

This is a partial differential

equation of 2<sup>nd</sup> order Let

$$H_z = XY \text{ -----(2)}$$

Where X is a function of 'x' alone, Y is a

function of 'y' alone  $Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$

$$1/X \frac{\partial^2 X}{\partial x^2} + 1/Y \frac{\partial^2 Y}{\partial y^2} + h^2 = 0 \text{ -----(3)}$$

Where  $h^2$  is a constant since  $\gamma^2$

and  $\omega^2 \mu \epsilon$  are constants.

So to satisfy the above equation sum of functions of 'X' and 'Y' must be equal to a constant. It is possible when individual one must be a constant.

$$\text{Let } 1/X \frac{\partial^2 X}{\partial x^2} = -B^2, 1/Y \frac{\partial^2 Y}{\partial y^2} = -A^2 \text{ -----(4)}$$

Where  $A^2$  and  $B^2$  are constants

$$-A^2 - B^2 + h^2 = 0$$

$$h^2 = A^2 + B^2 \text{ ----(5)}$$

solutions of

equation (4)

are

$$X = c_1 \cos Bx$$

$$+ c_2 \sin Bx \text{ ----- (6)}$$

$$Y = c_3 \cos Ay + c_4 \sin Ay \text{ ---- (7)}$$

Where  $c_1, c_2, c_3$  and  $c_4$  are constants

Which are determined by applying boundary conditions

i) Boundary condition (Bottom wall)

$E_x = 0$  for  $y = 0$  and all values of  $x$  varying

=  $-j\omega\mu/h^2[(c_2\sin Bx+c_1\cos Bx)(-Ac_3\sin Ay+Ac_4\cos Ay)]$  From the boundary condition (i)

$$0 = -j\omega\mu/h^2[c_1\cos Bx+c_2\sin Bx]Ac_4$$

$$[c_1\cos Bx+c_2\sin Bx]Ac_4=0$$

$A \neq 0$ ,

$$c_1\cos Bx+c_2\sin Bx \neq 0$$

$$c_4=0$$

$$H_z = (c_1\cos Bx+c_2\sin Bx)(c_3\cos Ay)$$

$$H_z = c_3(c_1\cos Bx+c_2\sin Bx)\cos Ay$$

### Resonant Frequency( $f_0$ )

Since we know that for a rectangular waveguide  $h^2 = \gamma^2 + \omega^2\mu\epsilon = A^2 + B^2 = (m\pi/a)^2 + (n\pi/b)^2$

$$\omega^2\mu\epsilon = (m\pi/a)^2 + (n\pi/b)^2 - \gamma^2$$

for a wave propagation  $\gamma = j\beta$  or  $\gamma^2 = -\beta^2$   $\omega^2\mu\epsilon = (m\pi/a)^2 + (n\pi/b)^2 +$

$$\beta^2$$

if a wave has to exist in a cavity resonator there must be a phase change corresponding to a given guide wavelength. The condition for the resonator to resonate is  $\beta = p\pi/d$ .

where  $p =$  a constant  $1, 2, 3, \dots$ , that indicates half wave variation of either electric or magnetic field along that z-direction

$d =$  length of the resonator when  $\beta = p\pi/d, f = f_c, \omega = \omega_0$

$$\omega_0^2\mu\epsilon = (m\pi/a)^2 + (n\pi/b)^2 + (p\pi/d)^2$$

$$(2\pi f_0)^2\mu\epsilon = (m\pi/a)^2 + (n\pi/b)^2 + (p\pi/d)^2$$

## UNIT -2

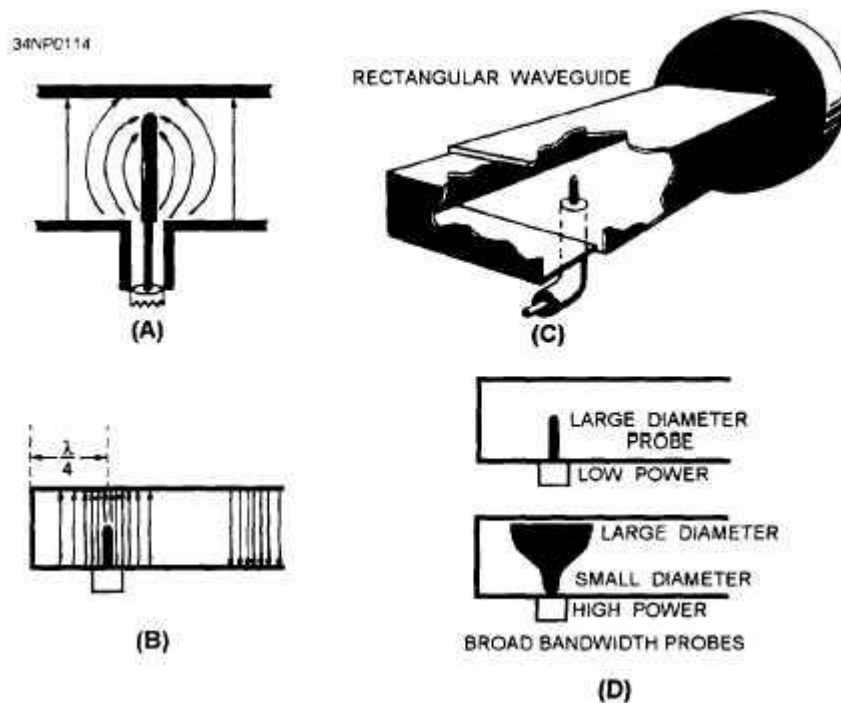
### Coupling mechanisms

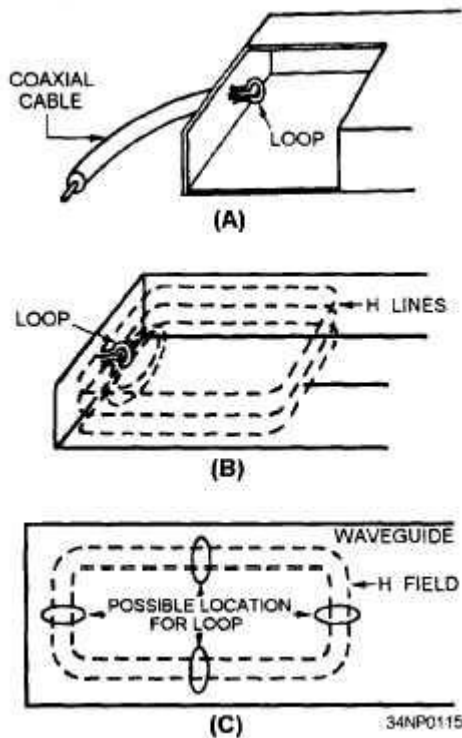
Probe coupling & Loop coupling: Probes & Loops are metallic wires used to couple coaxial line to a waveguide resonator to feed or extract microwave signal.

When a short antenna in the form of a probe or a loop inserted into a waveguide, it will radiate and if it is placed correctly, the wanted mode will be set up. The correct positioning of the coupling probes for launching dominant  $TE_{10}$  mode.

The probe is placed at a distance of  $\lambda_g/4$  from the shorted end of the waveguide and the centre of broader dimension of the waveguide because at that point electric field is maximum. This probe will now act as an antenna which is polarized in the plane parallel to that of electric field.

The coupling loop placed at the centre of shorted end plate of the waveguide can also be used to launch  $TE_{10}$  mode i.e., coupling is achieved by means of a loop antenna located in a plane perpendicular to the electric field and loops to a magnetic field but in each case both are set up because electric and magnetic fields are inseparable.





**Waveguide discontinuities:** Waveguide irises, tuning screws and posts.

**Waveguide irises:** In any waveguide system, when there is a mismatch there will be reflections. In transmission lines, in order to overcome this mismatch lumped impedances or stubs of required value are placed at precalculated points. In waveguides too, some discontinuities are made use for matching purposes.

Any susceptances appearing across the guide, causing mismatch ( production of standing waves) needs to be cancelled by introducing another susceptance of the same magnitude but of opposite nature.

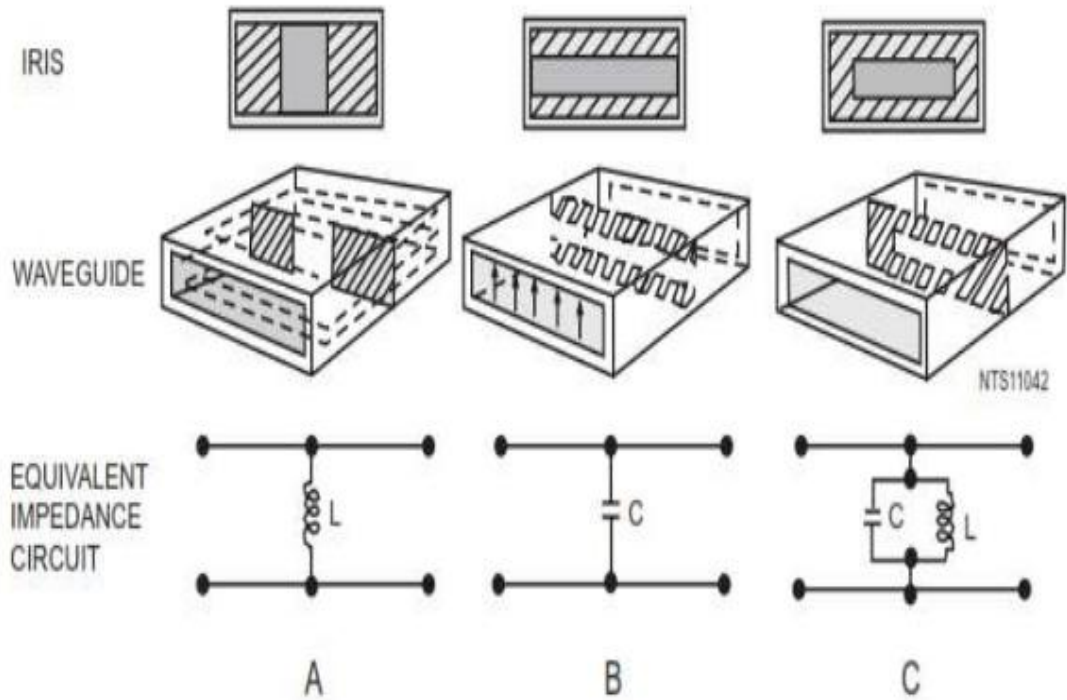
Irises (also called windows, apertures or diaphragms) are made use of for the purpose.

An inductive iris allows a current to flow where none flowed before. The iris is placed in a position where the magnetic field is strong (or where electric field is relatively weak). Since the plane of polarization of electric field is parallel to the plane of iris, the current flow due to iris causes a magnetic field to be set up.

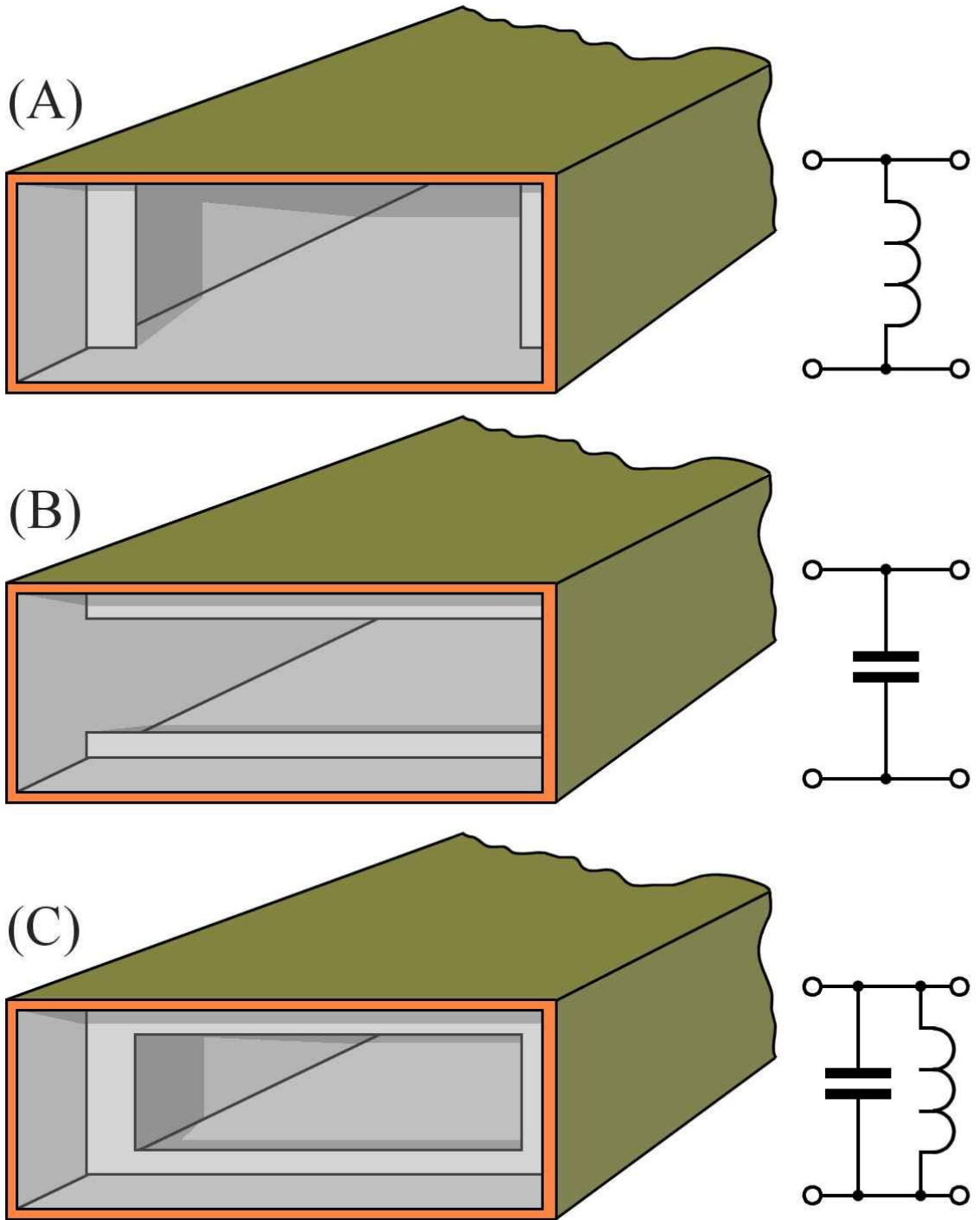
In capacitive iris, it is seen that the potential which existed between the top and bottom walls of the waveguide now exists between surfaces which are closer. The capacitive iris is placed in a position where the electric field is strong.

In parallel resonant iris, the inductive and capacitive irises are combined. For the dominant mode, the iris presents a high impedance and the shunting effect for this mode will be negligible. Parallel resonant iris acts as a band pass filter to suppress unwanted modes.

A series resonant iris which supported by a non metallic material and it is transparent to the flow of microwave energy.







**Posts & tuning screws:**

When a metallic cylindrical post is introduced into the broader side of waveguide, it produces the same effect as an iris in providing lumped reactance at that point.

If the post extends only a short distance ( $< \lambda_g/4$ ) into the waveguide, it behaves capacitively shown in figure 1. When the depth is equal to  $\lambda_g/4$ , the post acts as a series resonant circuit shown in figure 2. If it is greater than  $\lambda_g/4$ , the post behaves inductively.

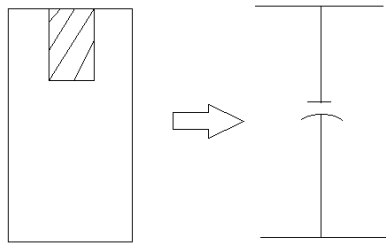


Figure: 1

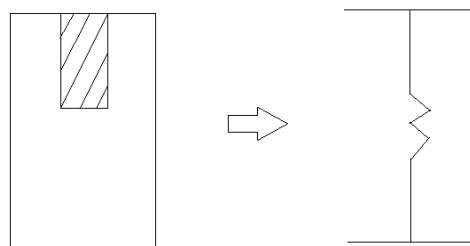


Figure: 2

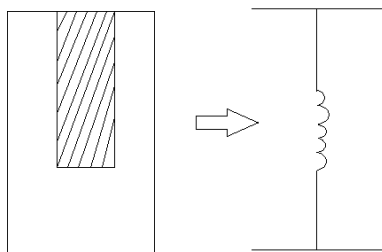


Figure: 3

When the post is extended completely across the waveguide the post becomes inductive. The big advantage of the post over an iris is that it is readily adjustable.

An adjustable post is known as a screw. As in case of posts depending up on the depth of penetration, the tuning screw may introduce inductive or capacitive susceptance.

**Matched load:** Matched Load is a device used to terminate a transmission line or waveguide so that all the energy from the signal source will be absorbed.



**CIRCUALTORS AND ISOLATORS:** Both microwave circulators and isolators are non reciprocal transmission devices that use the property of Faraday rotation in the ferrite material. A non reciprocal phase shifter

consists of thin slab of ferrite placed in a rectangular waveguide at a point where the dc magnetic field of the incident wave mode is circularly polarized. When a piece of ferrite is affected by a dc magnetic field the ferrite exhibits Faraday rotation. It does so because the ferrite is nonlinear material and its permeability is an asymmetric tensor.

**MICROWAVE CIRCULATORS:** A microwave circulator is a multiport waveguide junction in which the wave can flow only from the  $n$ th port to the  $(n + 1)$ th port in one direction. Although there is no restriction on the number of ports, the four-port microwave circulator is the most common. One type of four-port microwave circulator is a combination of two 3-dB side hole directional couplers and a rectangular waveguide with two non reciprocal phase shifters.

Each of the two 3- dB couplers in the circulator introduces a phase shift of  $90^\circ$ , and each of the two phase shifters produces a certain amount of phase change in a certain direction as indicated. When a wave is incident to port 1, the wave is split into two components by coupler 1. The wave in the primary guide arrives at port 2 with a relative phase change of  $180^\circ$ . The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of  $180^\circ$ . Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2. However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of  $270^\circ$ . The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of  $90^\circ$ . Since the two waves reaching port 4 are out of phase by  $180^\circ$ , the power transmission from port 1 to port 4 is zero. In general, the differential propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be where  $m$  and  $n$  are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on. As a result, the sequence of power flow is designated as  $1 \sim 2 \sim 3 \sim 4 \sim 1$ . Many types of microwave circulators are in use today. However, their principles of operation remain the same. A four-port circulator is constructed by the use of two magic tees and a phase shifter. The phase shifter produces a phase shift of  $180^\circ$ . A perfectly matched, lossless, and nonreciprocal four-port circulator has an S matrix of the form Using the properties of S parameters the S-matrix is

**MICROWAVE ISOLATORS:** An isolator is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus the isolator is usually called uniline.

**UNIT-3**  
**MICROWAVE LINEAR BEAM AND CROSS FIELD TUBES**  
**(O TYPE AND M TYPE)**

Limitations of conventional tubes at microwave frequencies: Conventional vacuum tube like triodes, tetrodes and pentodes are less useful signal source at the frequency above the 300 MHz. To see whether or not a conventional device works satisfactory at high frequencies or microwave frequencies, we consider a simple oscillator having LC tuned circuit and try to increase the operating frequency. For this purpose we reduce the tank circuit parameter, either L or C (since  $\omega = \frac{1}{\sqrt{LC}}$ ). But there is a certain limit by which we can reduce the value of L and C components of the tank circuits. For high frequency or microwave frequency the device parameters like the inter electrode capacitance and lead inductance takes the dominant part in the circuit and affect the operation of the oscillator. There are following reasons for that conventional tube cannot be used for microwave frequency or high frequency.

1. Inter electrode capacitance and lead inductance effect.
2. Transit time effect.
3. Gain-Bandwidth product limitation.
4. RF losses.
5. Radiation losses.

**1. Inter electrode Capacitance and Lead Inductance Effect:** The inter electrode capacitances and lead inductances are the order of 1 to 2 pF and 15 to 20 mH respectively. The shunt impedances due to inter electrode becomes very low and series impedances due to lead inductance become very high at the microwave or high frequency which makes these tube unstable. Refinements have been done in the design and fabrication of these tubes with the result that these tubes, like disk seal tube, are still used up to the lower end of microwave spectrum.

**2. Transit Time Effect:** In a conventional tube electrons emitted by the cathode take a finite (non-zero) time in reaching the anode. This interval, called the transit time, depends on the cathode anode spacing and the static voltage between the anode and the. Transit time ( $\tau$ ) =  $\frac{d}{v_0}$  where  $\tau$  is the transit time, d is the cathode anode spacing and  $v_0$  is the velocity of electrons.

**3. Gain-Bandwidth Product Limitation:** In ordinary vacuum tubes the maximum gain is generally achieved by resonating the output tunes circuit.

$$\text{Gain-bandwidth product} = A_{\text{max}} \text{ BW} = (g_m / G) (G/C)$$

Where  $g_m$  is the transconductance,

$$A_{\text{max}} \cdot \text{BW} = g_m / C .$$

It is important to note that the gain-bandwidth product is independent of frequency. As  $g_m$  and C are fixed for a particular tube or circuit, higher gain can be achieved only at the applicable to resonant circuit only. In

microwave device either re-entrant cavities or slow-wave structures are used to obtain a possible overall high gain over a broad bandwidth.

**4. RF Losses:** RF losses include the skin effect losses and dielectric losses.

**(a) Skin effect losses:** Due to skin effect, the conductor losses came into play at higher frequencies, at which the current has the tendency to confined itself to a smaller cross-section of the conductor towards its outer surface.

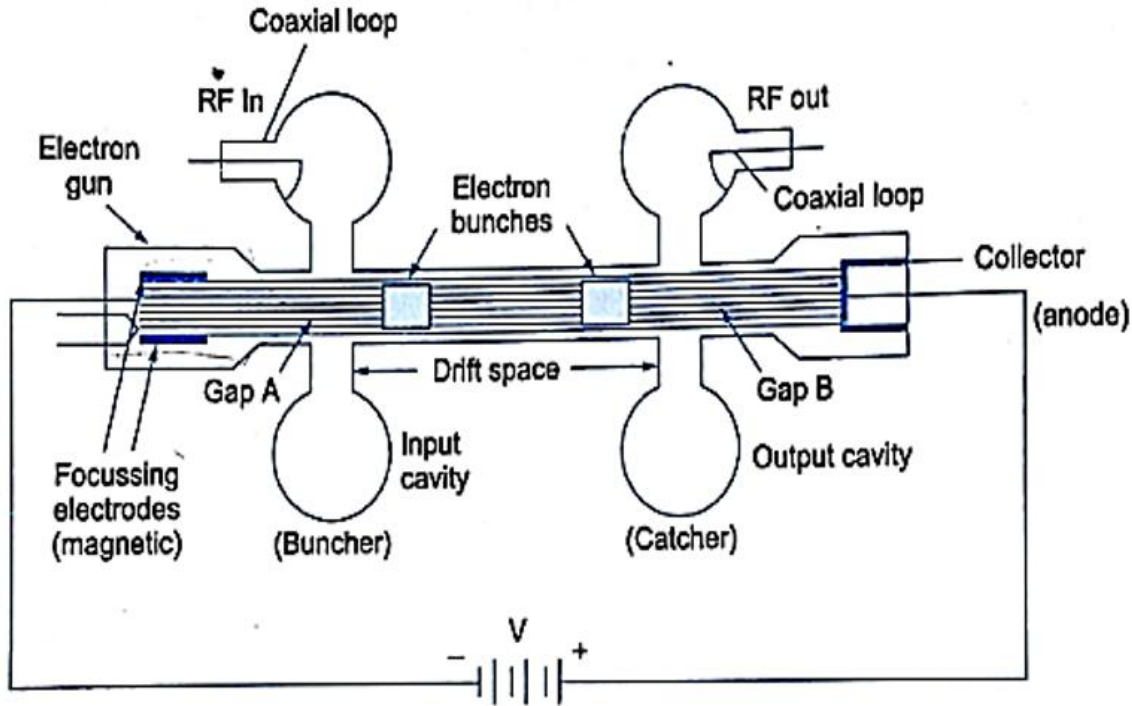
**(b) Dielectric losses:** At the microwave frequency or high frequency various insulating materials like glass envelope, silicon and plastic encapsulations are used. The losses occur due to dielectric materials is known as dielectric loss generally the relationship between the power loss in dielectric and frequency is given by  $PL \propto f$  So, if frequency increases then power loss will also increases. The effect of dielectric loss can reduced eliminating the tube base and reducing the surface area of the dielectric material.

**5. Radiation Losses:** At high frequency, when the dimensions of wire approaches near to the wavelength ( $\lambda = c/f$ ). It will emit radiation called radiation losses. Radiation losses are increases with the increase in frequency. Radiation loss can be reduced by proper shielding of the tube and its circuitry.

**Klystron:** Klystron is the simplest vacuum tube that can be used for amplification or generation (as an oscillator) of microwave signal. The operation of klystron depends upon velocity modulation which leads to density modulation of electrons. Klystron may be classified as gives below: 1. Two cavity klystron amplifier 2. Multi cavity klystron 3. Reflex klystron.

#### **TWO CAVITY KLYSTRON AMPLIFIER:**

One of the earlier form of velocity modulation device is the two cavity klystron amplifier, represented by the schematic of figure. It is seen that high velocity electron beam is formed, focused and sent down along a glass tube to a collector electrode, which is at a high positive potential with respect to the cathode. As it is clear from the figure, a two cavity klystron amplifier consists of a cathode, focussing electrodes, two buncher grids separated by a very small distance forming a gap A (Input cavity or buncher cavity), two catcher grids with a small gap B (output or catcher cavity) followed by a collector.



**Operation:** The input and output are taken from the tube via resonant cavity with the help of coupling loops. The region between buncher cavity and catcher cavity is called drift space. The first electrode (focussing grid) controls the number of electrons in the electron beam and serves to focus the beam. The velocity of electrons in the beam is determined by the beam accelerating potential. On leaving the region of focussing grid, the electrons pass through the grids of buncher cavity. The space between the grids is referred to as interaction space. When electrons travel through this space, they are subjected to RF potential at a frequency determined by the cavity resonant frequency which is nothing but the input frequency. The amplitude of this RF potential between the grids is determined by the amplitude of the input signal in case of an amplifier or by the amplitude of feedback signal from the second cavity if used as an oscillator. The working of two cavity klystron amplifier depends upon velocity modulation.

**Velocity Modulation** Consider a situation when there is no voltage across the gap. Electrons passing through gap A are unaffected and continue on to the collector with the same constant velocities they had before approaching the gap A. When RF signal to be amplified is used for exciting the buncher cavity thereby developing an alternating voltage of signal frequency across the gap A. The theory of velocity modulation can be explained by using the diagram known as Applegate diagram as shown in figure. At point X on the input RF cycle, the alternating voltage is zero and an electron which passes through gap A is unaffected by the RF signal. Let this electron be called reference electron  $e_R$  which travels with an unchanged velocity  $v_0 = \sqrt{\frac{2eV}{m}}$ , where V is the anode to cathode voltage. Consider another point Y of the RF cycle an electron passing the gap slightly later than the reference electron  $e_R$ , called the late electron  $e_L$  is subjected to positive RF voltage so late electron  $e_L$  is accelerated and hence travelling towards gap B

with an increased velocity and this late electron  $e_L$  tries to catch the reference electron  $e_R$ . Similarly, another point  $Z$  of RF cycle, an electron passing the gap slightly before than the reference electron  $e_R$ , called the early electron  $e_e$  and this early electron is subjected to negative RF voltage so early electron  $e_e$  is retarded and hence travelling towards gap B with reduced velocity and reference electron  $e_R$  catches up the early electron  $e_e$ . So, when the electron pass through the buncher gap their velocity will be change according to the input RF signal. This process is known as velocity modulation. However, as explained with reference to Applegate diagram, the electrons gradually bunch together as they travel in the drift space. When an electron catches up with another one, the electron will exchange energy with the slower electron, giving it some excess energy and they bunch together and move on with the average velocity of the beam. This phenomena is very vital to the operation of klystron tube as an amplifier. The pulsating stream of electrons passes through gap B and excited oscillation in the output cavity. The density of electron passing the gap B varies cyclically with time. This mean the electron beam contains an AC current and variation in current density (often called current modulation) enables the klystron to have a significant gain and hence drift space converts the velocity modulation into current modulation.

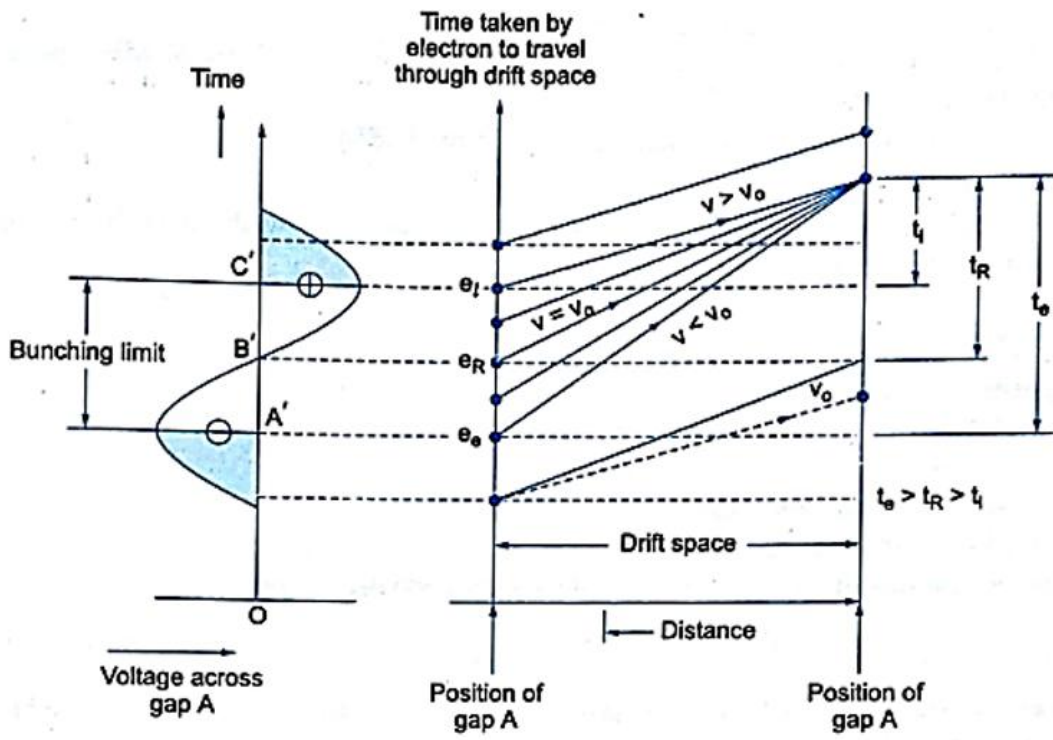


Fig. Apple gate diagram of a klystron amplifier



**MATHEMATICAL ANALYSIS OF TWO CAVITY KLYSTRON** The following assumptions are made in the mathematical treatment to follow:

1. Electron transit angle in buncher and catcher grids is very small.
2.  $V_1 \ll V_0$ , where  $V_1$  is the amplitude of RF signal at the buncher grid,  $V_0$  is the accelerating voltage.
3. Space charge effects are negligible.
4. Electron beam density is uniform throughout the length, i.e., no loss of electron takes place in buncher and catcher cavity.

**Equation of Velocity Modulation-**

Let the potential difference or dc voltage between cathode and anode be  $V_0$  (i.e. it is also called Beam Voltage) and  $v_0$  be the velocity of the high current density beam.  $L$  is the drift space length, electron charge  $e = 1.6 \times 10^{-19}$  coulombs and  $m = 9.1 \times 10^{-31}$  kg.

At equilibrium condition, Kinetic energy  $E = \frac{1}{2} m v_0^2 = e V_0$

$$\Rightarrow v_0 = \sqrt{2eV_0/m} = 0.593 \times 10^6 \sqrt{V_0}, \text{ m/sec ... (1)}$$

RF input signal is applied to the input terminal and the gap voltage between the buncher grids =  $V_s = V_1 \sin \omega t_1$

Where,  $V_1 =$  RF voltage in buncher cavity and is much-much less than  $V_0$

If the velocity of electron beam at the time of leaving buncher cavity, then at equilibrium condition, the energy of the electron at the time of leaving buncher cavity =  $\frac{1}{2} m v_1^2 = e (V_0 + V_s) = e (V_0 + V_1 \sin \omega t_1)$   
 .....(2)

$$\Rightarrow v_1 = \sqrt{(2e/m) (V_0 + V_1 \sin \omega t_1)} = \sqrt{(2eV_0/m) \cdot [1 + (V_1/V_0) \sin \omega t_1]}$$

Now put equation (1)

$$\Rightarrow v_1 = v_0 \cdot [1 + (V_1/V_0) \sin \omega t_1]^{1/2}$$

$$\Rightarrow v_1 = v_0 \cdot [1 + (V_1/2V_0) \sin \omega t_1] \text{ ..... (3)}$$

This is a equation of velocity modulation

If  $\theta_g =$  phase angle of the RF input voltage during which the electron is accelerated

= Transit Angle

$$\theta_g = \omega \tau = \omega (t_1 - t_0) = \omega d/v_0 \text{ ..... (4)}$$

$$\text{where, } \tau = t_1 - t_0 = d/v_0 \Rightarrow t_1 = t_0 + (d/v_0)$$

and  $d =$  buncher width  $t_1$

Now average microwave voltage at buncher gap is =  $V_{s(\text{avg})} = (1/\tau) \cdot \int_{t_0}^{t_1} V_1 \sin \omega t \cdot dt$

$$\begin{aligned} \Rightarrow V_{s(\text{avg})} &= - (V_1/\omega \tau) \cdot [\cos \omega t_1 - \cos \omega t_0] = (V_1/\omega \tau) \cdot [\cos \omega t_0 - \cos \omega t_1] \\ &= (V_1 \cdot v_0/\omega d) \cdot [\cos \omega t_0 - \cos \{\omega t_0 + (\omega d/v_0)\}], \end{aligned}$$

where,  $t_1 = t_0 + (d/v_0)$  and  $\tau = d/v_0$

$$= (V_1 \cdot v_0 / \omega d) \cdot 2 \cdot \sin \left\{ \omega t_0 + \omega \left[ t_0 + \frac{d}{v_0} \right] + \frac{\omega d}{2v_0} \right\} \cdot \sin \left( \frac{\omega d}{2v_0} \right)$$

$$= \left\{ V_1 / \left( \frac{\omega d}{2v_0} \right) \right\} \cdot \sin \left\{ \omega t_0 + \left( \frac{\omega d}{2v_0} \right) \right\} \cdot \sin \left( \frac{\omega d}{2v_0} \right)$$

Put equation (4)  $\Rightarrow$

$$V_{s(\text{avg})} = V_1 \cdot \sin \left\{ \omega t_0 + \left( \frac{\theta_g}{2} \right) \right\} \cdot \text{Sin} \left( \frac{\theta_g}{2} \right) \quad \text{, where } \theta_g = \omega d / v_0$$

Since  $\beta_i =$  Beam coupling coefficient of the input cavity

$$\Rightarrow \beta_i = \sin \left( \frac{\theta_g}{2} \right) \quad \text{ } \left( \frac{\theta_g}{2} \right)$$

When  $\theta_g$  increases,  $\beta_i$  decreases; thus the velocity modulation of beam decreases.

$$\Rightarrow \boxed{V_{s(\text{avg})} = V_1 \cdot \beta_i \text{Sin} \left\{ \omega t_0 + \left( \frac{\theta_g}{2} \right) \right\}} \quad \dots \dots \dots (5)$$

As electrons pass through the buncher gap, their velocities are increased, decreased or unchanged, depending upon the +ve, -ve or zero RF voltage across the grids when they pass through.. At time  $t_{av} = (t_0+t_1)/2$  the electrons are midway across the buncher gap with velocity  $v_{av}$ .

$$\text{From equation(2) } \frac{1}{2} m v_{av}^2 = e (V_0 + V_{s(\text{avg})})$$

$$\Rightarrow v_{av} = \sqrt{(2e/m) (V_0 + V_{s(\text{avg})})} = \sqrt{(2eV_0/m)} \sqrt{(1 + V_{s(\text{avg})}/V_0)}$$

Put equation (1) and (5)

$$\Rightarrow v_{av} = v_0 \cdot \sqrt{[1 + V_1 \beta_i \sin \{ \omega t_0 + (\theta_g/2) \}]}$$

$$\Rightarrow v_{av} = v_0 \cdot [1 + (V_1 \beta_i / V_0) \sin \{ \omega t_0 + (\theta_g/2) \}]^{1/2}$$

Applying Binomial theorem,

$$\Rightarrow \boxed{v_{av} = v_0 \cdot [1 + (V_1 \beta_i / 2V_0) \sin \{ \omega t_0 + (\theta_g/2) \}]}$$

and these electrons exit from the buncher gap at time  $(t_0+t_1)$  to the field free drift space between the two cavities with velocity  $v_1$

$$v_1 = v_0 \cdot [1 + (V_1 \beta_i / 2V_0) \sin \{ \omega t_1 - (\theta_g/2) \}]$$

The electrons in the beam are velocity modulated by the input RF signal with **depth of velocity modulation**

$$d_m = V_1 \beta_i / V_0.$$

$$\Rightarrow v_1 = v_0 \cdot [1 + (d_m/2) \sin \{ \omega t_1 - (\theta_g/2) \}] \quad \dots \dots \dots (6)$$

It is a standard equation of Velocity modulation.

**Transit time in the Drift Space:-** Let us assume that  $t_2$  is the time when the bunched electrons are at the catcher grid after travelling through the field free drift space  $L$ ,  $t_2 = t_1 + L/v_1$

Put equation (6)  $\Rightarrow t_2 = t_1 + L/v_0 \cdot [1 + (d_m/2) \sin\{\omega t_1 - (\theta_g/2)\}]$

$$\Rightarrow t_2 = t_1 + (L/v_0) \cdot [1 - (d_m/2) \sin\{\omega t_1 - (\theta_g/2)\}]$$

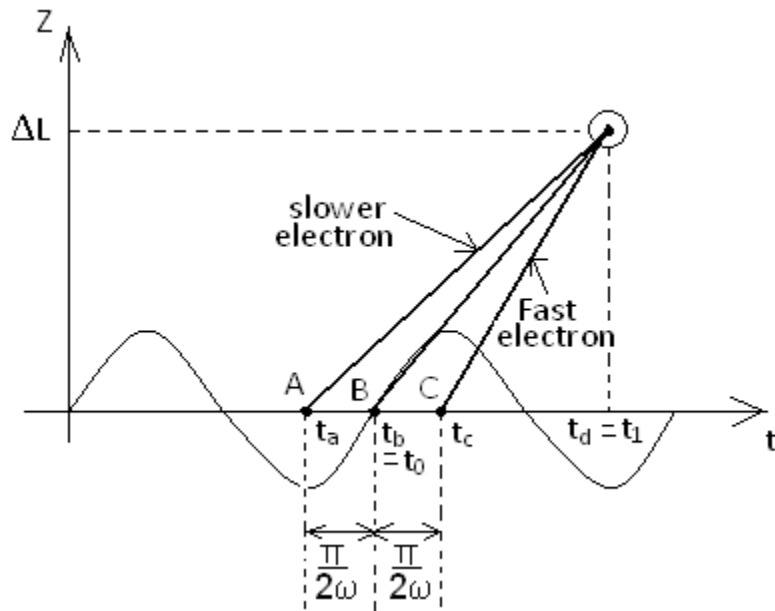
The transit time  $T_d$  in the drift space is given by

$$T_d = t_2 - t_1 = (L/v_0) \cdot [1 - (d_m/2) \sin\{\omega t_1 - (\theta_g/2)\}] \dots\dots\dots(7)$$

**Density Modulation and Bunching process:-** The electrons gradually bunch together due to the difference in velocities of the electrons, as they travel down the drift space. The variation in electron velocity in drift space is known as velocity modulation and the density of electrons in the bunches and catcher cavity gap varies cyclically with time, i.e. become density modulated.

According to fig. the distance from buncher grid to the buncher location is  $\Delta L$  and initially we consider for electron B,

$$\Delta L = v_0(t_d - t_b)$$



$$\begin{aligned} \text{For electron A, } \Delta L &= v_{1\min}(t_d - t_a) \\ &= v_{1\min} \{t_d - t_b + (\pi/2\omega)\} \dots\dots\dots(7) \end{aligned}$$

$$\text{Where, } t_a = t_b - (\pi/2\omega)$$

$$\begin{aligned} \text{For electron C, } \Delta L &= v_{1\max}(t_d - t_c) \\ &= v_{1\max} \{t_d - t_b - (\pi/2\omega)\} \dots\dots\dots(8) \end{aligned}$$

$$\text{where, } t_c = t_b + (\pi/2\omega)$$

From equation (6) the minimum value of  $v_1$  is found to put the minimum value of  $\sin \{\omega t_0 + (\theta_g/2)\} = -1$

$$\text{Thus equation (6)} \Rightarrow v_{1\min} = v_0 \cdot [1 - (d_m/2)]$$

Similarly, from equation (6) the maximum value of  $v_1$  is found to put the maximum value of  $\sin \{\omega t_0 + (\theta_g/2)\} = +1$

$$\text{Thus equation (6)} \Rightarrow v_{1\max} = v_0 \cdot [1 + (d_m/2)]$$

Now put the value of  $v_{1\min}$  and  $v_{1\max}$  in equation (7) and (8)

$$\text{Equation (7)} \Rightarrow \Delta L = v_0 \cdot [1 - (d_m/2)] \{t_d - t_b + (\pi/2\omega)\}$$

$$\text{Eq(8)} \Rightarrow \Delta L = v_0 \cdot [1 + (d_m/2)] \{t_d - t_b - (\pi/2\omega)\}$$

$$\Rightarrow \Delta L = v_0(t_d - t_b) - v_0(\pi/2\omega) + (v_0 d_m/2) (t_d - t_b) - (v_0 d_m/2) (\pi/2\omega) \dots\dots(9)$$

If the distance has to be the same for electrons A, B and C and let us consider the distance is

$$\Delta L = v_0(t_d - t_b)$$

$$\text{Eq(9)} \Rightarrow \cancel{\Delta L = v_0(t_d - t_b)} - v_0(\pi/2\omega) + (v_0 d_m/2) (t_d - t_b) - (v_0 d_m/2) (\pi/2\omega)$$

$$\Rightarrow - v_0(\pi/2\omega) + (v_0 d_m/2)(t_d - t_b) - (v_0 d_m/2) (\pi/2\omega) = 0$$

$$\Rightarrow (v_0 d_m/2) (t_d - t_b) = v_0(\pi/2\omega) + (v_0 d_m/2) (\pi/2\omega)$$

$$\Rightarrow (d_m/2) (t_d - t_b) = (\pi/2\omega) [1 + (d_m/2)]$$

$$\Rightarrow (t_d - t_b) = (2/ d_m)(\pi/2\omega) [1 + (d_m/2)]$$

$$\Rightarrow (t_d - t_b) = (\pi/\omega d_m) + (\pi/2\omega) \dots\dots\dots(10)$$

$$\text{Where, } d_m = V_1 \beta_i / V_0$$

Since,  $V_0/V_1 \gg (\pi/2\omega)$

Thus neglect  $(\pi/2\omega)$  in eq (10)  $\Rightarrow$

$$(t_d - t_b) \approx (\pi/\omega d_m) \dots\dots\dots(11)$$

Since,  $\Delta L = v_0(t_d - t_b)$

$$\text{Put eq (11)} \Rightarrow \Delta L = v_0(\pi/\omega d_m) = 3.14 (v_0/\omega d_m) \dots\dots\dots(12)$$

It is noted that

- (i) The mutual repulsion of space charge is neglected.
- (ii) The distance  $\Delta L$  is not the one for maximum degree of bunching.

**Output Power and Efficiency:-**

The electronic efficiency of the two cavity klystron amplifier is defined as the ratio of the output power to the input power

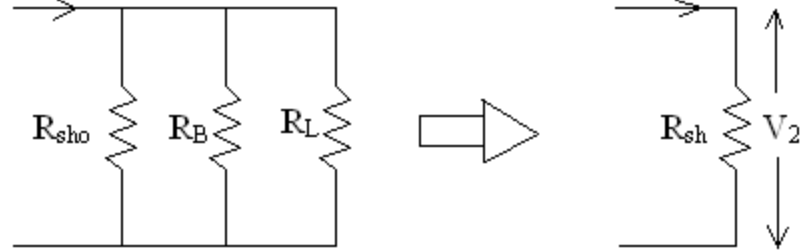
$$\text{Efficiency '}\eta\text{' = } P_o/P_{in} = P_{ac}/P_{dc}$$

$$\text{From previous equations, } \eta = (0.58) \cdot V_2 / V_0$$

and the voltage  $V_2$  is equal to  $V_0$ , then maximum efficiency  $\eta_{\max} = 58\%$   
 But in practice the efficiency is in the range of 15 to 40%.

The output equivalent circuit is

$$I_{2\text{indu}} = \beta_0 I_2 = \beta_0 2 I_0 J_1(x)$$



Where,  $I_{2\text{indu}}$  = Induced current,  $R_{\text{sho}}$  = Wall resistance of catcher cavity

$R_B$  = Beam loading resistance,  $R_L$  = External Load

$R_{\text{sh}}$  = Effective shunt resistance or total equivalent shunt resistance of the catcher circuit

The induced current by the electron beam in the walls of the catcher cavity is directly proportional to the amplitude of the microwave input voltage  $V_1$ . Magnitude of the induced current  $I_{2\text{indu}} = \beta_0 I_2$ ;

$$= (\beta_0) \cdot 2I_0 J_1(\chi);$$

where,  $I_2 = 2I_0 J_1(x)$ ;  $J_1(\chi) = 1^{\text{st}}$  order Bessel function

$$\chi = \text{Bunching Parameter} = (\beta_1 V_1 / 2V_0) \cdot \theta_0$$

$$\theta_0 = 2n\pi - \pi/2$$

The output power delivered to the catcher cavity and the load is

$$P_{\text{out}} = \frac{I_{2\text{indu}}^2 R_{\text{sh}}}{2} = \frac{\beta_0^2 I_2^2 R_{\text{sh}}}{2} \quad \text{where } I_{2\text{indu}} = \beta_0 I_2 / \sqrt{2}$$

$$P_{\text{out}} = \frac{\beta_0 I_2 V_2}{2}, \quad \text{Where } V_2 = \beta_0 \cdot I_2 R_{\text{sh}}$$

and power input  $P_{\text{in}} = I_0 V_0$

Since efficiency  $\eta = P_{\text{out}} / P_{\text{in}}$

$$\Rightarrow \eta = \beta_0 I_2 V_2 / 2 I_0 V_0$$

Now put  $I_2 = 2I_0 J_1(\chi)$

$$\Rightarrow \eta = \beta_0 2I_0 J_1(\chi) V_2 / 2 I_0 V_0$$

$$\Rightarrow \eta = \{\beta_0 J_1(\chi)\} \cdot V_2 / V_0$$

Since,  $\beta_i = \beta_0 = 1$  and  $J_1(\chi) = 0.58$

$$\text{Thus, } \eta = (0.58) \cdot V_2 / V_0$$

and the voltage  $V_2$  is equal to  $V_0$ , then maximum efficiency  $\eta_{\max} = 58\%$ . But in practice the efficiency is in the range of 15 to 40%.

**Applications of Two Cavity Klystron:**

1. In CW doppler radar.
2. For pumping parametric amplifiers.
3. As frequency modulated oscillators in high power microwave link.
4. In TV transmitters.
5. In satellite ground stations.

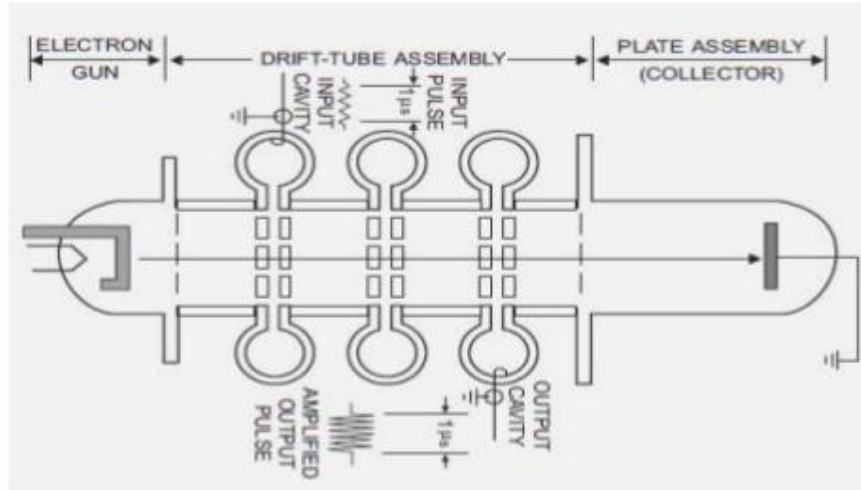
**MULTICAVITY KLYSTRON AMPLIFIER**

Klystron amplification, power output, and efficiency can be greatly improved by the addition of intermediate cavities between the input and output cavities of the basic klystron. Additional cavities serve to velocity-modulate the electron beam and produce an increase in the energy available at the output. Since all intermediate cavities in a multi cavity klystron operate in the same manner, a representative three-cavity klystron will be discussed.

**Construction:** A three-cavity klystron is illustrated in figure. The entire drift-tube assembly, the three cavities, and the collector plate of the three-cavity klystron are operated at ground potential for reasons of safety. The electron beam is formed and accelerated toward the drift tube by a large negative pulse applied to the cathode. Magnetic focus coils are placed around the drift tube to keep the electrons in a tight beam and away from the side walls of the tube. The focus of the beam is also aided by the concave shape of the cathode is high-powered klystrons.

# Multicavity Klystron

- In all modern klystrons, the number of cavities exceeds two.
- A larger number of cavities may be used to increase the gain of the klystron, or to increase the bandwidth.



**Three-cavity klystron**

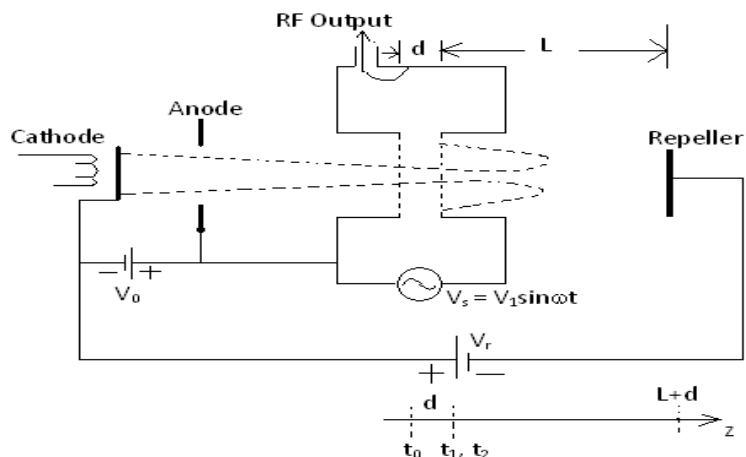
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Operation of Multi cavity Klystron The output of any klystron (regardless of the number of cavities used) is developed by velocity modulation of the electron beam. The electrons that are accelerated by the cathode pulse are acted upon by RF fields developed across the input and middle cavities. Some electrons are accelerated, some are decelerated, and some are unaffected. Electron reaction depends on the amplitude and polarity of the fields across the cavities when the electrons pass the cavity gaps. During the time the electrons are travelling through the drift space between the cavities, the accelerated electrons overtake the decelerated electrons to form bunches. As a result, bunches of electrons arrive at the output cavity at the proper instant during each cycle of the RF field and deliver energy to the output cavity. Only a small degree of bunching takes place within the electron beam during the interval of travel from the input cavity to the middle cavity. The amount of bunching is sufficient, however, to cause oscillations within the middle cavity and to maintain a large oscillating voltage across the middle cavity gap. Most of the velocity modulation produced in the three-cavity klystron is caused by the voltage across the input gap of the middle cavity. The high voltage across the gap causes the bunching process to proceed rapidly in the drift space between the middle cavity and the output cavity. The electron bunches cross the gap of the output cavity when the gap voltage is at maximum negative. Maximum energy transfer from the electron beam to the output cavity occurs under these conditions. The energy given up by the electrons is the kinetic energy that was originally absorbed from the cathode pulse. Klystron amplifiers have been built with as many as five intermediate cavities in addition to the input and output cavities. The effect of

the intermediate cavities is to improve the electron bunching process which improves amplifier gain. The overall efficiency of the tube is also improved to a lesser extent. Adding more cavities is roughly the same as adding more stages to a conventional amplifier. The overall amplifier gain is increased and the overall bandwidth is reduced if all the stages are tuned to the same frequency. The same effect occurs with multi cavity klystron tuning. A klystron amplifier tube will deliver high gain and a narrow bandwidth if all the cavities are tuned to the same frequency. This method of tuning is called synchronous tuning. If the cavities are tuned to slightly different frequencies, the gain of the amplifier will be reduced but the bandwidth will be appreciably increased. This method of tuning is called staggered tuning.

### REFLEX KLYSTRON

Reflex klystron is low power, low efficiency microwave oscillator. Reflex klystron is a single cavity variable frequency microwave generator. This is most widely used in application where variable frequency is desired like radar receiver and microwave receivers. Construction: Reflex klystron consists of an electron gun similar to that of multi cavity klystron, a filament surrounded by a cathode and a focussing electrode at the cathode as shown in figure. The reflex klystron contains a repeller which is at a high negative potential. The suitable formed electron beam is accelerated towards the cavity, where a high positive voltage applied to it. This acts as anode and known as anode cavity. After passing the gap in cavity electrons travel towards repeller which is at high negative potential. The electrons are repelled back from midway of the repeller space by the repeller electrode towards the anode. If conditions are properly adjusted, then the returning electrons give more energy to the gap than they took from it on forward journey, thus leads to sustained oscillations.



Where,  $t_0$  = time for electron entering cavity gap at  $z = 0$

$t_1$  = time for same electron leaving cavity gap at  $z = d$

$t_2$  = time for same electron returned by retarding field  $z=d$  and collected on walls of cavity

**Operation-** The electron beam injected from the cathode is first velocity modulated by the beam voltage.



Some electrons are accelerated and leave the resonator at an increased velocity than those with uncharged velocity. Some retarded electrons enter the repeller region with less velocity. Then the electrons, which are leaving the resonator, will need different time to return due to change in velocity. As a result returning electrons group together in bunches. It is seen that earlier electrons take more time to return to the gap than later electrons and so the conditions are right for bunching to take place.

On their return journey the bunched electrons pass through the gap during the retarding phase of the alternating field and give up their Kinetic energy to the e.m. energy of the field in the cavity.

Or as the electron bunches pass through resonator, they interact with voltage at resonator grids. If the bunches pass the grid at such time that the electrons are slowed down by the voltage, energy will be delivered to the resonator and electrons will oscillate. The electrons finally collected by the walls of the cavity or other grounded metal parts of the tube.

**Applegate Diagram-**

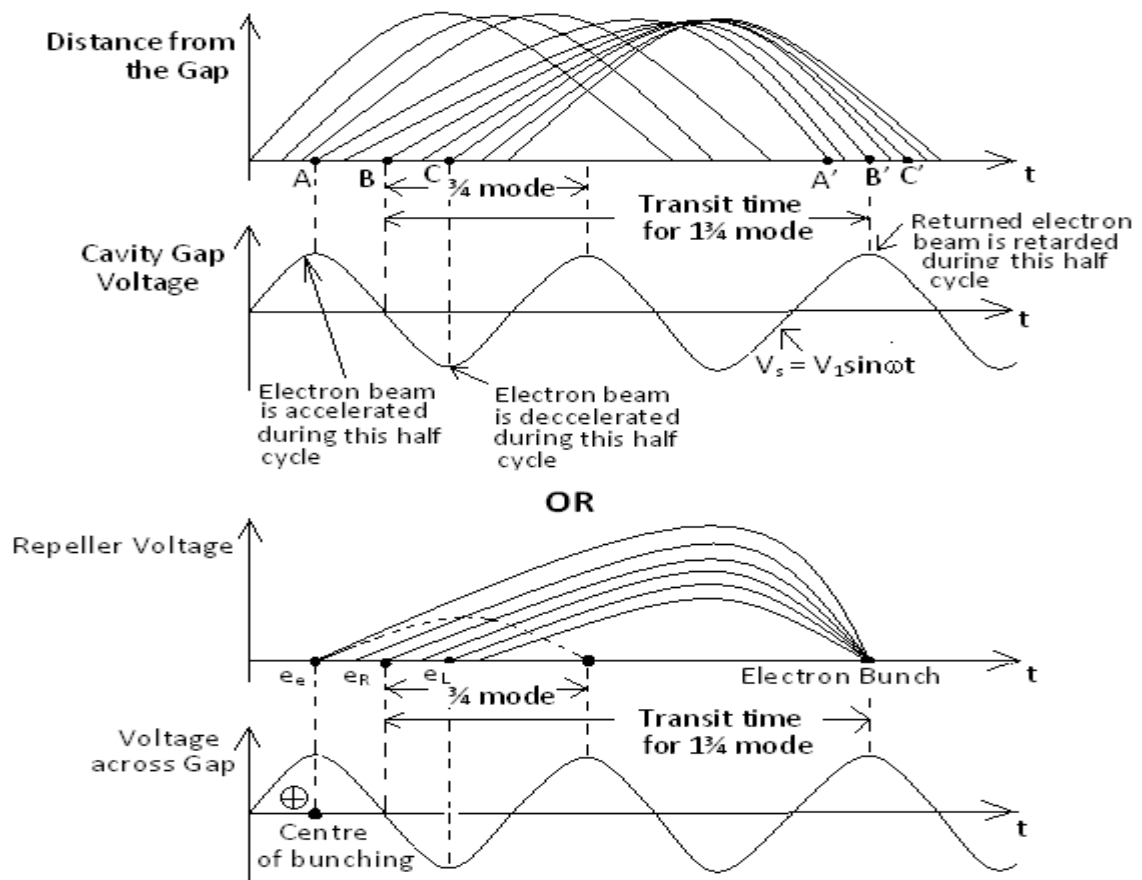


Fig.() Applegate diagram with gap voltage for a reflex klystron

**Operation through Applegate diagram-** The early electron  $e_e$  that passes through the gap, before the reference electron  $e_R$ , experiences a maximum +ve voltage across the gap and the electron is accelerated, it moves with greater velocity and penetrates deep into repeller space. The return time for electron  $e_e$  is greater as the depth of

penetration into the repeller space is more. Hence  $e_e$  and  $e_R$  appear at the gap for second time at the same instant.

The late electron  $e_L$  that passes the gap, later than reference electron  $e_R$ , experiences a maximum –ve voltage and moves with a retarding velocity. The return time is shorter as the penetration into repeller space is less and catches up with reference electron  $e_R$  and earlier electron  $e_e$  and forming a bunch.

Bunches return back and pass through the gap during the retarding phase of the alternating field and give up their maximum energy to the e.m. energy of the field in the cavity to sustained oscillations.

**Power Output and efficiency-**

$$\text{Efficiency } \eta = \frac{\text{ac power delivered to the load (output power, } P_o)}{\text{dc power supplied by beam voltage (input power, } P_{in})} = \frac{P_{ac}}{P_{dc}}$$

where,  $P_{ac} = V_1 I_2 / 2$  and  $P_{dc} = V_0 I_0$

$$\therefore \eta = V_1 I_2 / 2 V_0 I_0$$

where,  $I_2 = 2 I_0 \beta_0 J_1(\chi')$

$$\Rightarrow \eta = V_1 I_0 \beta_0 J_1(\chi') / V_0 I_0 \dots\dots\dots(1)$$

Now we have to find the value of  $V_1$  in terms of bunching parameter ( $\chi'$ )

Since Bunching parameter,  $\chi' = \beta_0 V_1 \theta_0' / 2 V_0$  where,  $\theta_0' = 2n\pi - \pi/2$

$$\therefore V_1 = 2 V_0 \chi' / \beta_0 (2n\pi - \pi/2) \dots\dots\dots(2)$$

$$\Rightarrow \eta = 2 V_0 \chi' I_0 \beta_0 J_1(\chi') / \beta_0 (2n\pi - \pi/2) V_0 I_0 = 2 \chi' J_1(\chi') / (2n\pi - \pi/2)$$

Since  $n=2$  or  $1\frac{3}{4}$  mode has the most power output and where,  $\chi' = 2.408$  &  $J_1(\chi') = 0.52$

$$\therefore \eta_{max} = 2(2.408)(0.52) / (2 \times 2 \times \pi - \pi/2) = 2.50432 / (12.56 - 1.57) = 0.227$$

$\eta_{max}$  in percentage is

$$\boxed{\eta_{max} = 22.7 \%}$$
 , but practically  $\eta = 10$  to  $20 \%$

**TRAVELLING WAVE TUBE**

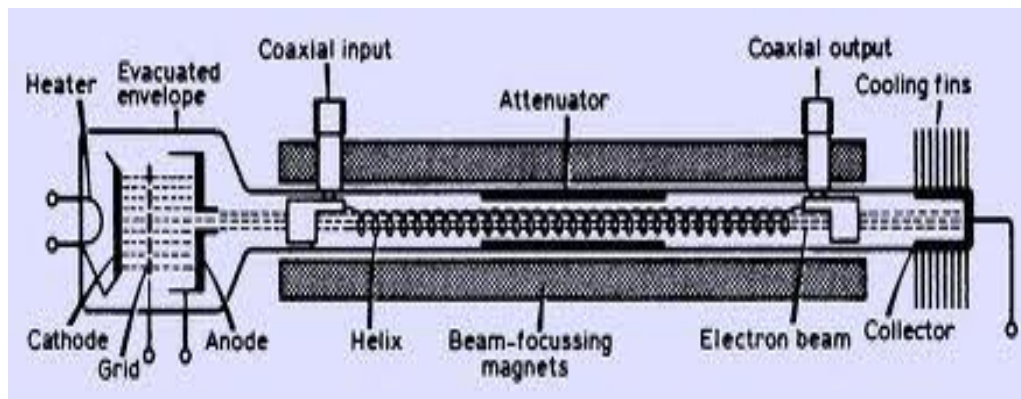
The travelling wave tube was invented by Rudolf Kompfner in 1944 during the Second WorldWar. The TWT is linear beam or O-type device like the klystron. The TWT is a high-gain, low noise, wide bandwidth microwave amplifier. It is capable to achieve gain greater the 40 dB with bandwidth exceeding an octave (1 octave in which the upper frequency is twice the lower frequency).

Travelling wave tube has been designed for frequencies as low as 300 MHz and high as 50 GHz. The wide bandwidth and low-noise characteristics makes the TWT ideal for used as an amplifier in microwave equipment. For broadband application, such as satellite, radar transmitter, the TWT are almost exclusively used. If we compare the basic operating principles of TWT and klystron, in TWT, the microwave circuit is non-resonant and the wave propagates with same speed as the electrons in the beam. The initial effect on the beam is a small amount of velocity

modulation caused by the weak electric field associated with the travelling wave. Just as in the klystron this velocity modulation later translates to current modulation, which then induces an RF current in the circuit, causing amplification.

TWTs are broad band devices in which there are no cavity resonators. The interaction space extends and the electron beam exchanges energy with the RF wave over the full length of the tube. But it is necessary to ensure that the electron beam and the RF wave both are travelling in the same direction with nearly the same velocity.

The electron beam travels with a velocity governed by the anode voltage. The RF field propagates with a velocity equal to velocity of light. The interaction between the RF field and electron beam will take place only when the RF field is retarded by slow wave structures, like helix.



Fig()

**Operation-** The applied RF signal propagates around the turns of the helix, and it produces an electric field at the centre of the helix. The axial electric field propagates with velocity of light multiplied by the ratio of the helix pitch to helix circumference. When the electrons enter the helix tube, an interaction takes place between the moving axial electric field and the moving electrons.

The interaction takes place between them in such a way that on an average the electron beam delivers or transfer energy to the RF wave on the helix. This interaction causes the signal wave grows amplified and becomes larger.

**Velocity Modulation-** When the axial field is zero, electron velocity is unaffected. This happens at the point of node of the axial electric field. Those electrons entering the helix, when the axial field is positive antinode, at the accelerating field are accelerated. At a later point where the axial RF field is -ve antinode, retarding field, the electrons are decelerated. The electrons get velocity modulated.

As the electrons travel further along the helix, bunching of electrons occur at the end which shifts the phase of  $\pi/2$ . Magnet produces axial magnetic field prevents spreading of electron beam as it travels down the tube.

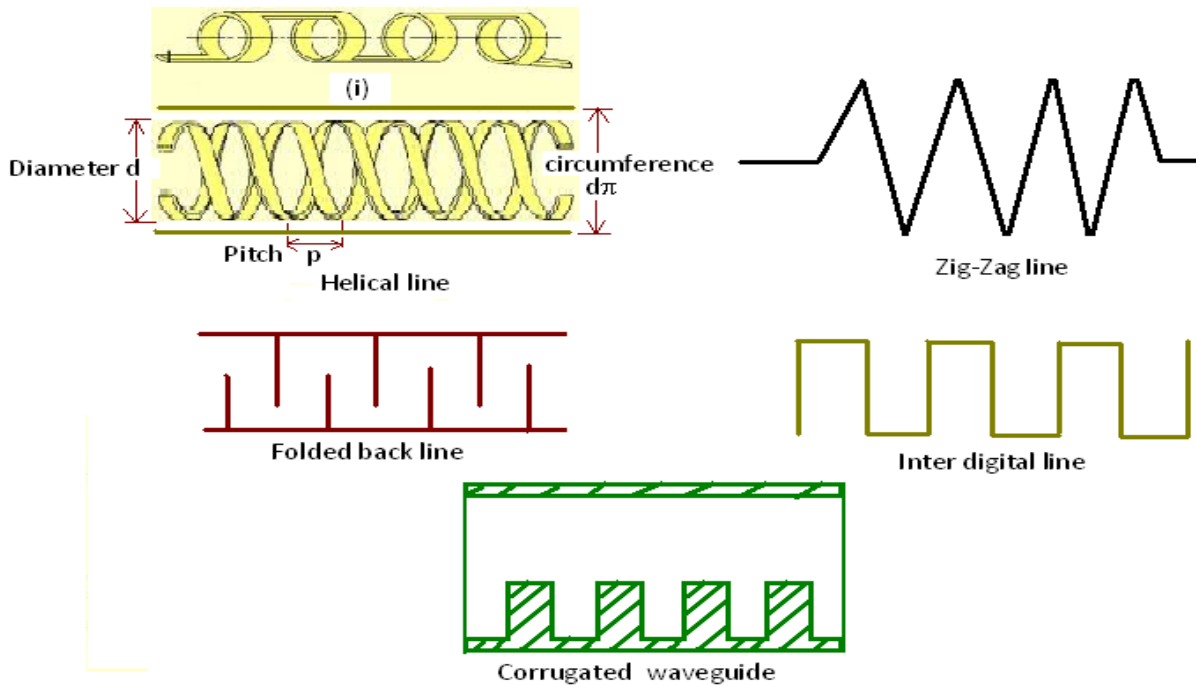
**Characteristics-** 1. Frequency range= 3 GHz and above

- 2. bandwidth = above 0.8 GHz
- 3. efficiency = 20 to 40 GHz
- 4. Power output = 10 kW (average)
- 5. Power gain = up to 60 dB
- 6. noise figure= 4 to 6 dB (low power TWTs; 0.5 to 16 GHz)  
25 dB (High power TWT at 40 GHz)

**Slow Wave Structures (SWS)-** SWSs are special circuits which are used in microwave tubes to reduce the velocity of wave in a certain direction so that the electron beam and the signal wave can interact.

The phase velocity of a wave in ordinary waveguide is greater than the velocity of light in a vacuum. Since the electron beam can be accelerated only to velocities that are about a fraction of the velocity of light, thus the electron beam must keep in step with the microwave signal and a slow wave structure must be incorporated in the microwave devices. By which electron beam and signal wave are travelling with nearly the same velocity and valuable interaction takes place.

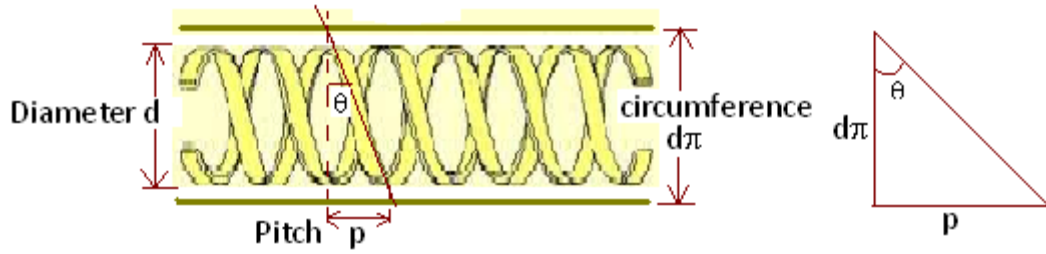
As the operating frequency is increased, both the inductance and capacitance of the resonant circuit must be decreased in order to maintain resonance at operating frequency. Because the gain bandwidth product is limited by the resonant circuit, the ordinary resonator cannot generate a large output. Several non-resonant periodic circuits or slow wave structures are designed for producing large gain over a wide bandwidth. Some slow wave structures are



Ratio of the phase velocity of electron beam  $v_p$  along the pitch to the phase velocity ( $c$ ) of RF field along the coil is equal to the sine of pitch angle  $\theta$ .

$$v_p/c = \sin\theta \dots\dots(i)$$

Now according to the below fig.(.),  $\sin\theta = p/\sqrt{[p^2+(\pi d)^2]}$  .....(ii)



Equating equation (i) and (ii)

$$v_p/c = p/\sqrt{[p^2+(\pi d)^2]}$$

$$\Rightarrow v_p = cp/\sqrt{[p^2+(\pi d)^2]} \dots\dots\dots(iii)$$

Since,  $p \ll \pi d$  and  $\theta$  is very small

Therefore, equation (iii) becomes

$$v_p = cp/\pi d = \omega/\beta \dots\dots\dots(iv)$$

Equation(iv) holds good for small pitch angle and the phase velocity along the coil in free space ( $\epsilon_r = 1$ ).

**SMALL SIGNAL ANALYSIS OF TWT:**

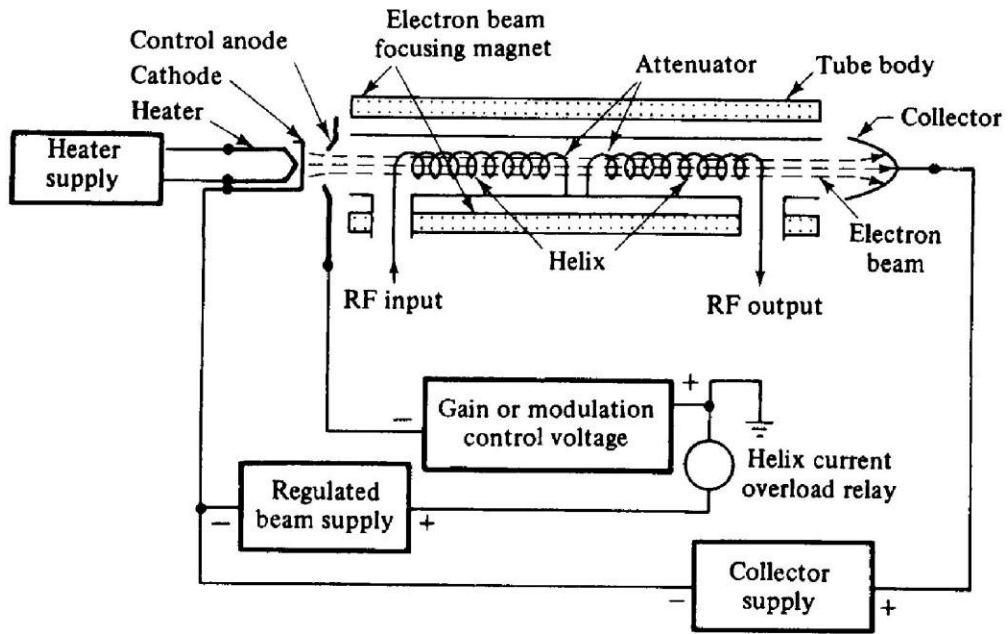
In addition to velocity modulation, the beam will also experience a fluctuation in charge density and current density. These are known as space charge waves on the electron beam. We shall carry out a small signal analysis of same. The purpose of the following analysis is to determine the propagation constants of the space charge waves that can exist on the slow wave structure in presence of an electron beam and to calculate the gain of the device from these constants.

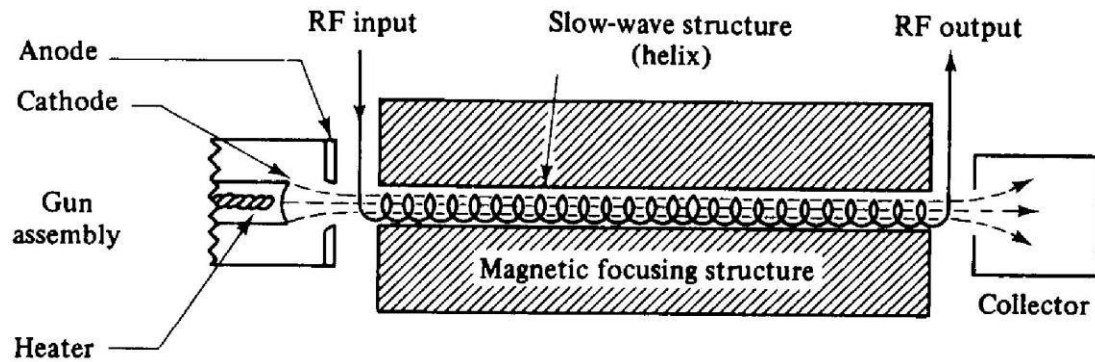
**Convection Current (Electronic Equation)**

**RF current produced by the circuit field:**

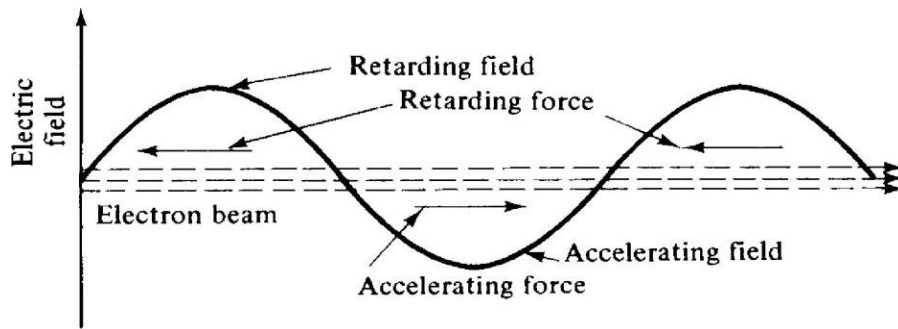
The convection current induced in the electron beam by the axial electronic field and the microwave axial field produced by the beam must first be developed? When the space charge effect is considered the electron velocity, charge density, current density and axial electric field will perturbate about their average DC values.

The schematic diagram and simplified circuit of helix TWT are shown below





The electrons entering the retarding field are decelerated and those in the accelerating field are accelerated. They begin forming a bunch centred about those electrons that enter the helix during the zero fields



Since the dc velocity of the electrons is slightly greater than the axial wave velocity, more electrons in the retarding field than in the accelerating field, and a great amount of energy is transferred from the beam to the electromagnetic field. The microwave signal voltage is amplified by the amplified field

The bunch continues to become more compact, and a larger amplification of the signal voltage occurs at the end of the helix. The magnet produces an axial magnetic field to prevent spreading the electron beam as it travels down the tube. An attenuator placed near the centre of the helix reduces all the waves travelling along the helix to nearly zero so that the reflected waves from the mismatched loads can be prevented from reaching the input and causing oscillation. The bunched electrons emerging from the attenuator induce a new electric field with the same frequency. This field in turn induces a new amplified microwave signal on the helix. The magnitude of the velocity fluctuation of the electron beam is directly proportional to the magnitude of the axial electric field

### Convection current

The relationship between the circuit and electron beam quantities can be determined by the convection current induced in the electron beam by the axial electric field and the microwave axial field produced by the beam. When the space-charge effect is considered, the electron velocity, charge density, the current density, and the axial field will perturbate about their average or dc values. These quantities can be expressed as

$$\begin{aligned}
 v &= v_0 + v_1 e^{j\omega t - \gamma z} \\
 \rho &= \rho_0 + \rho_1 e^{j\omega t - \gamma z} \\
 J &= -J_0 + J_1 e^{j\omega t - \gamma z} \\
 E_z &= E_1 e^{j\omega t - \gamma z} \quad \text{----- 1}
 \end{aligned}$$

For a small signal, the electron beam-current density can be written as

$$J = \rho v \approx -J_0 + J_1 e^{j\omega t - \gamma z} \quad \text{----- 2}$$

If an axial electric field exists in the structure, it will perturbate the electron velocity according to the force equation

$$\frac{dv}{dt} = -\frac{e}{m} E_1 e^{j\omega t - \gamma z} = \left( \frac{\partial}{\partial t} + \frac{dz}{dt} \frac{\partial}{\partial z} \right) v = (j\omega - \gamma v_0) v_1 e^{j\omega t - \gamma z} \quad \text{----- 3}$$

The convection current in the electron beam is given by

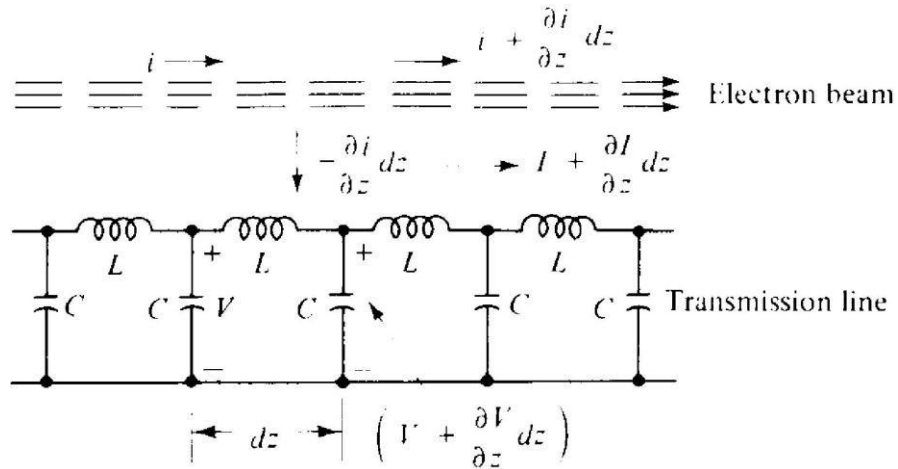
$$i = j \frac{\beta_e I_0}{2V_0 (j\beta_e - \gamma)^2} E_1 \quad \text{----- 4}$$

This equation is called the electronic equation, for it determines the convection current induced by the axial electric field

### Axial Electric Field

The convection current in the electron beam induces an electric field in the slow wave circuit. This induced field adds to the field already present in the circuit and causes the circuit power to increase the distance. The coupling relationship between the electron beam and the slow wave helix is shown in the figure:





The slow wave helix is represented by a distributed lossless transmission line.

The parameters are defined as follows:

$L$  = inductance per unit length

$C$  = capacitance per unit length

$I$  = alternating current in transmission line

$V$  = alternating voltage in transmission line

$i$  = convection current

application of transmission line theory and kirchoff's current law to the electron beam results in

$$\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t} - \frac{\partial i}{\partial z}$$

----- 5

----- 6

$$-\gamma I = -j\omega CV + \gamma i$$

From Kirchoff's voltage law the voltage equation

is

$$\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t} \quad \text{----- 7}$$

$$-\gamma V = -j\omega LI \quad \text{----- 8}$$

Elimination of the circuit current I yields,

$$\gamma^2 V = -V\omega^2 LC - \gamma j\omega L \quad \text{----- 9}$$

When the electron beam current is present, the above equation can be written as

$$V = \frac{\gamma\gamma_0 Z_0}{\gamma^2 - \gamma_0^2} i \quad \text{----- 10}$$

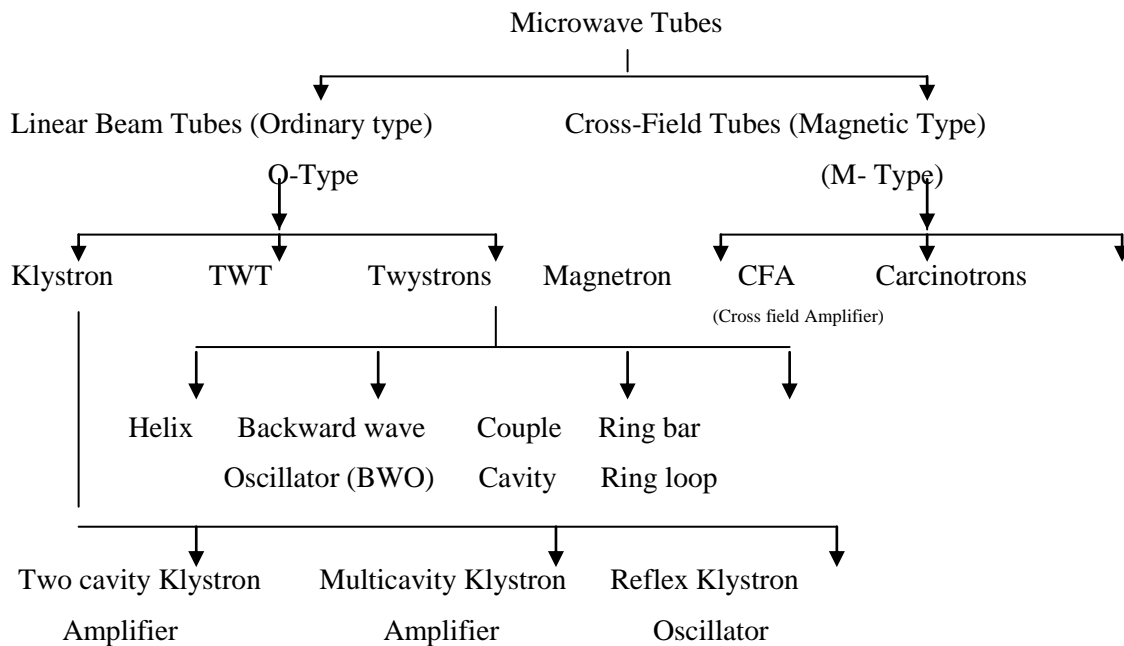
$$E_z = -\nabla V = -(\partial V / \partial z) = \gamma V$$

The axial electric field is given by

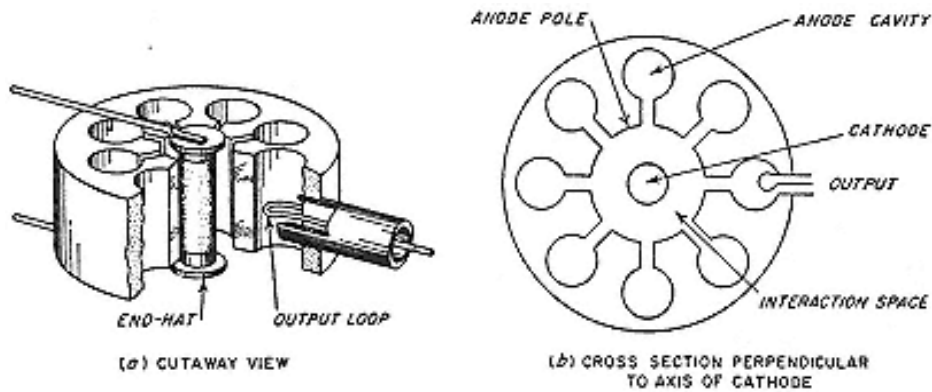
$$E_1 = -\frac{\gamma^2 \gamma_0 Z_0}{\gamma^2 - \gamma_0^2} i$$

This equation is called the circuit equation because it determines the how the axial electric field of the slow-wave helix is affected by the spatial ac electron beam current

## Cross-Coupled Tubes (Magnetron Oscillator)



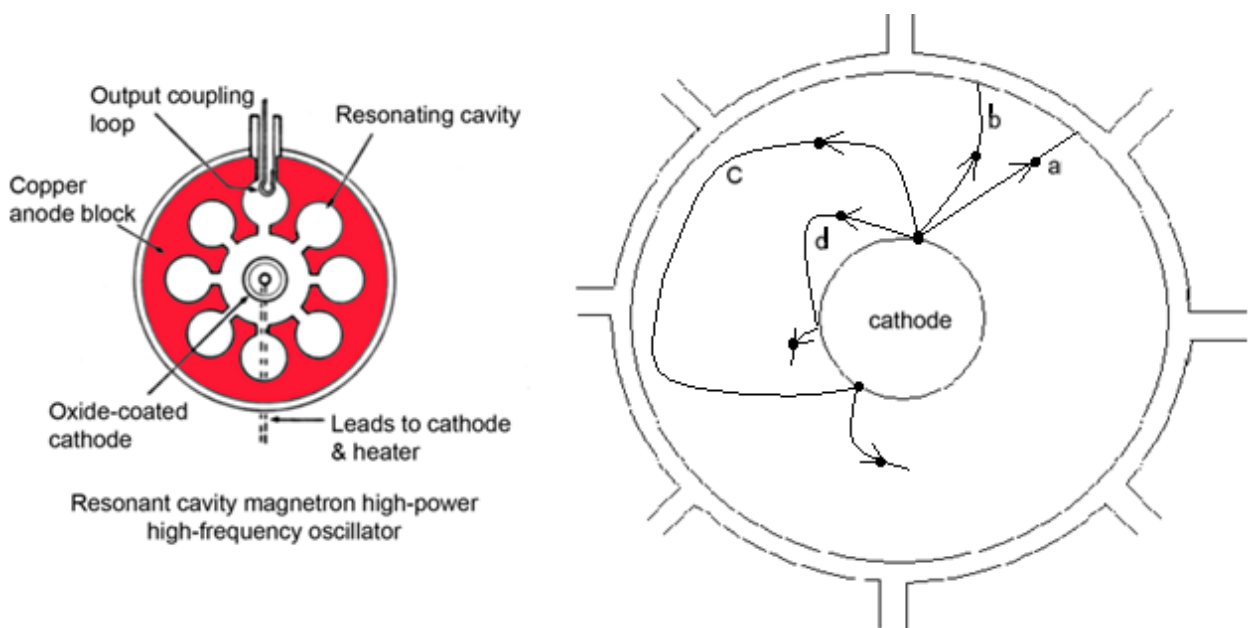
The Magnetron oscillator was the first device developed that was capable of generating large powers at microwave frequencies. This consists of a cylindrical cathode surrounded by anode structures that occupy cavities opening into the cathode-anode or interaction space.



Output power is with draw by means of a coupling loop or alternatively a tapered waveguide can be employed.

**Mechanism of oscillations in Magnetron-** The magnetron requires an external magnetic field with flux lines parallel to the axis of cathode. This field is provided by a permanent magnet or electromagnet. The dc magnetic field is normal to the dc electric field between the cathode and anode. Because of the cross-field between the cathode and anode, the electrons emitted from the cathode are affected by the cross-field to move in curved paths. If the dc magnetic field is strong enough, the electrons will not arrive in the anode but

return back to the cathode.



Fig(a). q

Fig(b).

Operation- From fig (b).

1. Path 'a'- If there is no magnetic field present; the electron would be drawn directly towards the anode in accordance with path 'a'.
2. Path 'b'- As the electron travels with a velocity the axial magnetic field exerts a force on it. When the magnetic field is weak the electron path is deflected as path 'b'.
3. Path 'c'- However, when the intensity of the magnetic field is sufficiently great, the electrons are turned back towards the cathode without ever reaching the anode accordance with path 'c'.
4. path 'd'- The magnetic field which is just able to return the electrons back to the cathode before reaching the anode, is termed the cut-off field as shown in path 'd'.

Thus when the magnetic field exceeds the cut-off value, then in the absence of oscillations all the emitted electrons return to the cathode and the plate current is zero.

**Favourable condition** – one fundamental condition for existence of oscillations in the resonant structure, when the magnetic field strength exceeds towards the cut-off there is an interaction between the electrons and the electric field. Thus we find a favourable condition causes the oscillations to receive energy from the electrons in the interaction space.

**$\pi$ -Mode Oscillations-** Let the cavity magnetron has 8 cavities, by which it supports varieties of modes depending upon the phase difference between fields in two adjacent cavities. Boundary conditions are

satisfied when total phase shift around the eight cavities is multiplied by  $2\pi$  radians. However, the most important mode for magnetron operation is one where in the phase shift between the fields of adjacent cavities is  $\pi$  radians. This is known as  **$\pi$ -Mode**.

**Frequency Pushing and Pulling-** Similarly to Reflex Klystron, it is possible to change the resonant frequency of magnetron by changing the anode voltage. This process referred to as **Frequency Pushing** is due to the fact that the change in anode voltage results in a change in orbital velocity of electrons. This alters the rate at which the energy is transferred to anode resonators and results in change of oscillation of frequency.

Magnetron is also found the frequency variation due to changes in load impedance. This takes place regardless of whether these load variations are purely resistive or reactive variations. However magnetron frequency variations are more strict for reactive variations. These frequency variations are known as **Frequency Pulling**, caused by load impedance variations reflected into cavity resonators.

To prevent frequency pushing a stabilized power supply is employed. It is also prevented by using a circulator which does not allow backward flow of electromagnetic energy. it is placed before the waveguide connection at the output of the magnetron.

**Mode Jumping:** The various modes differ very little in frequency from each other. Due to this magnetron oscillations have a tendency to jump from one mode to another mode under slight variation in operating conditions. This results in an abrupt change in frequency of oscillations and power output. This is known as Mode Jumping.

#### **Performance characteristics-**

1. Power output- (i) For pulsed mode- 250 kW  
(ii) For UHF band- 10 mW  
(iii) For X band- 2mW  
(iv) For 95 GHz- 8 kW
2. Frequency- 500 MHz to 12 GHz
3. Efficiency- 40 to 70 %
4. Duty cycle- 0.1%

#### **Applications-**

1. Mostly used in pulsed Radar as transmitter value.
2. Voltage tunable Magnetrons (VTMs) are used in sweep oscillator in telemetry and missile applications.
3. Fixed frequency magnetrons are used in industry as heating and in microwave oven.

### Cylindrical or Conventional Magnetron

In a cylindrical magnetron, several reentrant cavities are connected to the gaps. Thus it is also called as Cavity Magnetron. Assume the radius of cathode is 'a' and anode is 'b'. The dc voltage  $V_0$  is applied between the cathode and anode. When the dc voltage and the magnetic flux (i.e. which is in the +ve z-direction) are adjusted properly, the electrons will follow parabolic path in the presence of cross field. These parabolic paths are formed in the cathode-anode space under the combined force of both electric and magnetic fields which are perpendicular to each other.

Let angular displacement of the electron bends is  $\phi$ . The magnetic field is normal to the path of electron; hence it creates a no work force. The magnetic field does not work on the electrons.

1. Since Angular momentum = Angular velocity  $\times$  Moment of inertia =  $v_{ang} \times M$   

$$= (d\phi/dt) \times (mr^2)$$

Where, r = Radial distance from the centre of the cathode

The time rate for angular momentum =  $\frac{d}{dt} (d\phi/dt) \times (mr^2) \left[ \dots\dots\dots(i) \right]$   

$$= \text{Torque in } \phi \text{ direction}$$

2. Since Torque in  $\phi$  direction,  $T_\phi = r.F(\phi)$

Where,  $F(\phi)$  = force component in the direction of  $\phi = ev_p B = e(dr/dt)B$

$$\therefore T_\phi = r. e(dr/dt)B \dots\dots\dots(ii)$$

Equating equation (i) and (ii)  $\Rightarrow \frac{d}{dt} (d\phi/dt) \times (mr^2) = r. e(dr/dt)B$

Now integrating above equation w. r. t 't'

$$\Rightarrow (d\phi/dt) \times (mr^2) = (eBr^2/2) + C \dots\dots\dots(iii)$$

Where,  $\int (rdr/dt).dt = r^2/2$  and C = integrating constant

3. Now applying boundary condition- At the surface of the cathode  $r = a$  and angular velocity at emission  $v_{ang} = d\phi/dt = 0$

$$\therefore \text{equation (iii)} \Rightarrow C = -(eBa^2/2)$$

Put the value of C in equation (iii)  $\Rightarrow$

$$(d\phi/dt) \times (mr^2) = (eBr^2/2) - (eBa^2/2)$$

$$\Rightarrow (d\phi/dt) \times (mr^2) = (eB/2)[r^2 - a^2]$$

$$\Rightarrow (d\phi/dt) = (eB/2m)[1 - a^2/r^2] \dots\dots\dots(iv)$$

4. At cathode, boundary condition, when  $r = a$  then  $d\phi/dt = 0$

Since  $r \gg a$ , then equation (iv)  $\Rightarrow (d\phi/dt) = (eB/2m)[1 - 0] = (eB/2m)$

At  $B = B_c$  = cut-off magnetic flux density

$$(d\phi/dt)_{max} = (eB_c/2m) = \omega_c/2 \dots\dots\dots(v)$$

Where,  $\omega_c = eB_c/m$

5. In equilibrium condition,

Potential Energy = Kinetic Energy

$$eV_0 = \frac{1}{2} m v^2$$

$$\Rightarrow eV_0 = \frac{1}{2} m (v_r^2 + v_\phi^2)$$

$$\Rightarrow eV_0 = \frac{1}{2} m [(dr/dt)^2 + r^2(d\phi/dt)^2] \dots\dots\dots(vi)$$

Where,  $v_r$  and  $v_\phi$  are components in  $r$  and  $\phi$  directions in cylindrical co-ordinates.  $v_r = dr/dt$  and  $v_\phi = r.(d\phi/dt)$

now put equation(v) in equation (vi) $\Rightarrow$

$$eV_0 = \frac{1}{2} m [(dr/dt)^2 + r^2 \{ (eB_0/2m)(1 - a^2/r^2) \}^2]$$

$$\Rightarrow eV_0 = \frac{1}{2} m [(dr/dt)^2 + r^2 (eB_0/2m)^2 (1 - a^2/r^2)^2] \dots\dots\dots(vii)$$

6. Boundary condition at anode, if  $r = b$ , it implies  $dr/dt = 0$

$$\text{Equation (vii)} \Rightarrow eV_0 = \frac{1}{2} m [0 + b^2 (eB_0/2m)^2 (1 - a^2/b^2)^2]$$

$$\Rightarrow eV_0 = \frac{1}{2} m b^2 (eB_0/2m)^2 (1 - a^2/b^2)^2 \dots\dots\dots(viii)$$

Hence Hull's cut-off voltage is

$$\Rightarrow \boxed{B_c = [1/b \{1 - (a^2/b^2)\}] \cdot \sqrt{(8mV_0/e)}} \dots\dots\dots(ix)$$

Above equation is called the **Hull's cut-off magnetic field equation**.

Since  $b \gg a$ , so  $a^2/b^2$  may be neglected, equation (viii) becomes

$$\boxed{B_c = (1/b) \cdot \sqrt{(8mV_0/e)}}$$

7. If  $B > B_c$  for a given  $V_0$ , the electron grazes or will not reach the anode, it means anode current is zero. For the Hull's cut-off voltage  $V_c$ , if  $B_c = B$  then  $V_0 = V_c$  and thus cut-off voltage for a given  $B$  is found from the equation (viii),

$$\boxed{V_c = (eB_0^2 b^2 / 8m) (1 - a^2/b^2)^2}$$

$\dots\dots\dots(x)$

Where,  $e/m = 1.759 \times 10^{11}$  ,C/kg

If  $V_0 < V_c$  for given  $B$ , the electrons will not reach the anode. Equation (x) is referred to as **Hull's cut-off voltage equation**.

## UNIT-IV

### MICROWAVE SOLID-STATE DEVICES

#### 4.0 INTRODUCTION

The early generations of microwaves during the Second World War were vacuum devices, viz. klystrons, magnetrons and travelling wave tubes, which depended on the motion of electrons in vacuum for various configurations of electric and magnetic fields. These tubes were heavy and bulky and required high voltage for their operation and occupied large space. A wide range of microwave semiconductor devices have been developed since 1960s for detection, mixing, frequency multiplication, phase-shifting, attenuating, switching, limiting, amplification and oscillation. In most of the low power applications, solid-state devices have replaced electron beam devices because of the advantages of their small size, light weight, high reliability, low cost and capability of being incorporated into microwave integrated circuits. Some of the widely used microwave diodes like PIN diode, varactor diode, gunn diode IMPATT and TRAPATT are discussed in this chapter. PIN diodes are used for attenuation, modulation, switching, phase shifting and limiting. Varactor diodes are used for frequency multiplication, parametric amplification and tuning. Tunnel diodes and Gunn diodes are used for oscillation IMPATT and TRAPATT are used for amplification and oscillations.

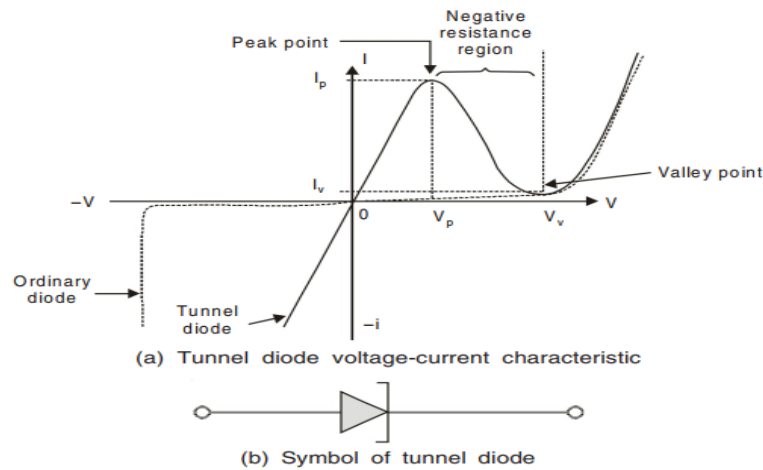
Microwave semiconductor devices have advantages like low cost, small size, less weight, high reliability and also employed in microwave integrated Circuits.

#### 4.1 TUNNEL DIODE (ESAKI DIODE)

Tunnel diode was discovered by a Japanese scientist named Esaki in 1958. The tunnel diode is a pn junction with a extremely heavy doping on the both sides of the junction and an abrupt transition from the p-side to n-side. Due to heavy doping, the width of the depletion region becomes very thin and an overlap occurs between the conduction band level on the n-side and the valance band level on the p-side. The tunnel diode are heavily doped pn junction that have a negative resistance over a portion of its VI characteristics as shown in Fig. It is clear that both in the forward as well as reverse directions, the diode responds with a huge current for a very small applied voltage. In the forward direction, the current reaches a maximum value equal to  $I_p$ (called peak current) at a voltage  $V_p$ (called peak voltage). At this point  $dI/dV$  is zero. After this, the current starts decreasing with increase in voltage. At the peak point, the slope of the characteristics changes from positive to negative while at the valley point, it changes from negative to positive again. Between the points, the device exhibits negative resistance.

**VI characteristics and symbol of tunnel diode:**





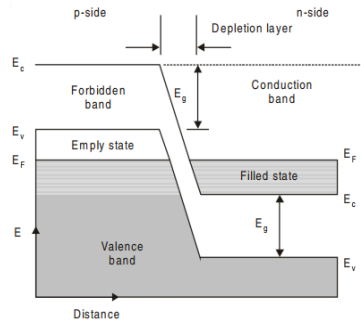
The tunnel diodes are useful to many circuit applications in the microwave amplification and microwave oscillation because of their low cost, light weight, high speed, low noise and high peak current to valley-current ratio. Because of the thin junction and short transit time, it is also useful for microwave application in fast switching circuits.

#### 4.1.1 Operation of Tunnel Diode (Quantum Mechanical Tunnelling):

The tunneling phenomenon in tunnel diode is known as quantum mechanical tunneling. As we know that tunnel diode is a negative resistance semiconductor pn junction diode. This negative resistance is created by the tunnel effect of electron in the pn junction. The tunnel effect is a majority carrier effect. The doping of both the p and n regions of the tunnel diode is very high, due to this the depletion-barrier at the junction is very thin (on the order of  $100 \text{ \AA}$ ). Classically, it is possible for those particles to pass over the barrier if and only if they have energy equal to or greater than the height of the potential barrier. But according to quantum-mechanical theory, if the barrier is less than  $3 \text{ \AA}$  there is an appreciable probability that particles will tunnel through the potential barrier even though they do not have enough kinetic energy to pass over the same

barrier. In addition to the barrier thickness, there must also be filled energy states on the side from which particles will tunnel and allow empty states on the other side into which particles penetrate through at the same energy level. To understand the tunnel effect, we analyze the energy band diagram of tunnel diode

The equilibrium energy level diagram of tunnel diode with no bias applied. Due to the heavy doping, the valence band of the p-side overlaps the conduction band of the n-side. Under unbiased condition the upper level of the electron energy of both the p-side and n-side are lined up at the same Fermi level as shown in Fig. 7.23. So there is just the same probability of electrons going from states in the conduction band on the n-side to the state in the valence band on the p-side, as in the opposite direction. Thus net tunneling on the thin barrier is zero.

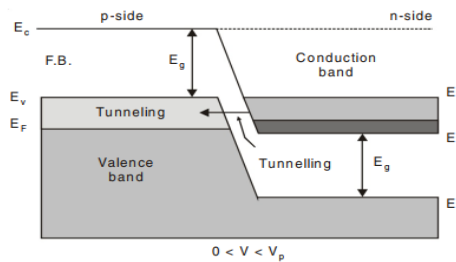


**Energy band diagram under no-bias**

It should be noted that in the ordinary diodes Fermi level exists in the forbidden band. Since the tunnel diode is heavily doped, the Fermi level exist in the valance band in *p*-side and in conduction band in the *n*-side.

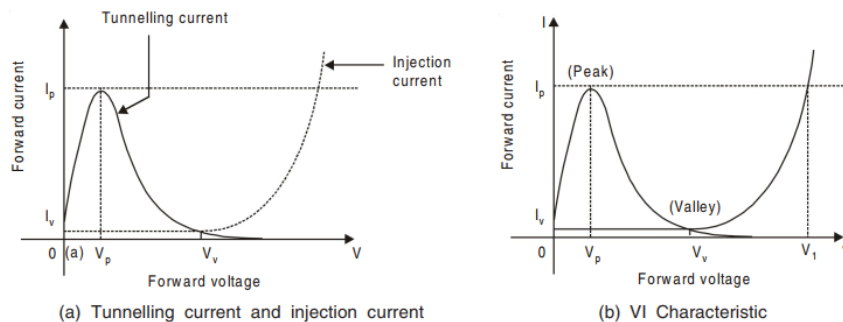
**2. When  $0 < V < V_p$**

When a forward bias is applied to the diode then the potential barrier is decreased. Hence, *n*-side level must shift upward with respect to those on the *p*-side as shown in Fig. 7.24. The electrons in the conduction band (filled state) on the *n*-side see empty states just across the barrier and tunneling takes place. Due to this tunneling (flow of electron from *n*-side to *p*-side) the forward current will increases as bias voltage increases



*Energy band diagram when  $0 < V < V_p$*

This injection current is increased exponentially with the forward voltage as indicated by dashed line Fig. (a). The total current is given by the sum of the tunnelling current and the injection current, results in the VI characteristic of the tunnel diode



**Fig. 7.28.** VI characteristic of tunnel diode.

**Equivalent Circuit of Tunnel Diode:**

**Advantages of tunnel diode:**

- High speed of operation due to the fact that the tunnelling takes place at the speed of light.
- Low cost
- Low noise
- Environmental immunity
- Low power dissipation
- Simplicity in fabrication
- Longevity

**Disadvantages of tunnel diode:**

- Low output voltage swing
- Because it is a two terminal device, there is no isolation between input and output.

**Applications of tunnel diode**

Some of the applications of Tunnel diode are

- Tunnel diodes are used as very high speed switches
- Used as high frequency micro wave oscillator

**TRANSFERRED ELECTRON DEVICES**

The application of two-terminal semiconductor devices at microwave frequencies has been increased usage during the past decades. The CW, average, and peak power outputs of these devices at higher microwave frequencies are much larger than those obtainable with the best power transistor. The common

characteristic of all active two-terminal solid-state devices is their negative resistance. The real part of their impedance is negative over a range of frequencies. In a positive resistance the current through the resistance and the voltage across it are in phase. The voltage drop across a positive resistance is positive and a power of  $(I^2 R)$  is dissipated in the resistance. In a negative resistance, however, the current and voltage are out of phase by  $180^\circ$ . The voltage drop across a negative resistance is negative, and a power of  $(-I^2 R)$  is generated by the power supply associated with the negative resistance. In other words, positive resistances absorb power (passive devices), whereas negative resistances generate power (active devices). In this chapter the transferred electron devices (TEDs) are analyzed. The differences between microwave transistors and transferred electron devices (TEDs) are fundamental. Transistors operate with either junctions or gates, but TEDs are bulk devices having no junctions or gates.

The majority of transistors are fabricated from elemental semiconductors, such as silicon or germanium, whereas TEDs are fabricated from compound semiconductors, such as gallium arsenide (GaAs), indium phosphide (InP), or cadmium telluride (CdTe). Transistors operate with "warm" electrons whose energy is not much greater than the thermal energy (0.026 eV at room temperature) of electrons in the

semiconductor, whereas TEDs operate with "hot" electrons whose energy is very much greater than the thermal energy. Because of these fundamental differences, the theory and technology of transistors cannot be applied to TEDs.

## GUNN DIODE

### *Gunn Effect:*

Gun effect was first observed by GUNN in n\_type GaAs bulk diode. According to GUNN, above some critical voltage corresponding to an electric field of 2000-4000v/cm, the current in every specimen became a fluctuating function of time. The frequency of oscillation was determined mainly by the specimen and not by the external circuit.

### *RIDLEY-WATKINS-HILSUM (RWH) THEORY*

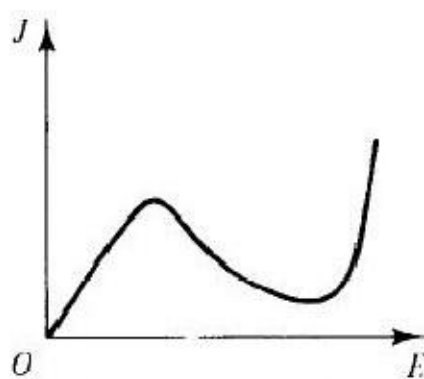
#### **Differential Negative Resistance**

The fundamental concept of the Ridley-Watkins-Hilsum (RWH) theory is the differential negative resistance developed in a bulk solid-state III-V compound when either a voltage (or electric field) or a current is applied to the terminals of the sample.

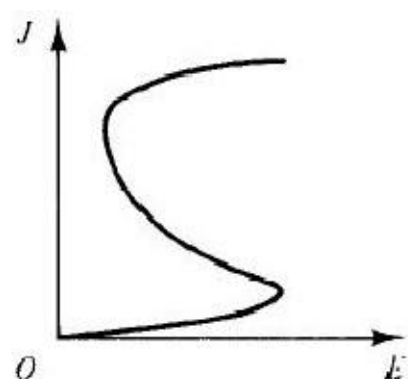
There are two modes of negative-resistance devices:

*i) Voltage-controlled and*

*ii) current controlled modes as shown in Fig.*

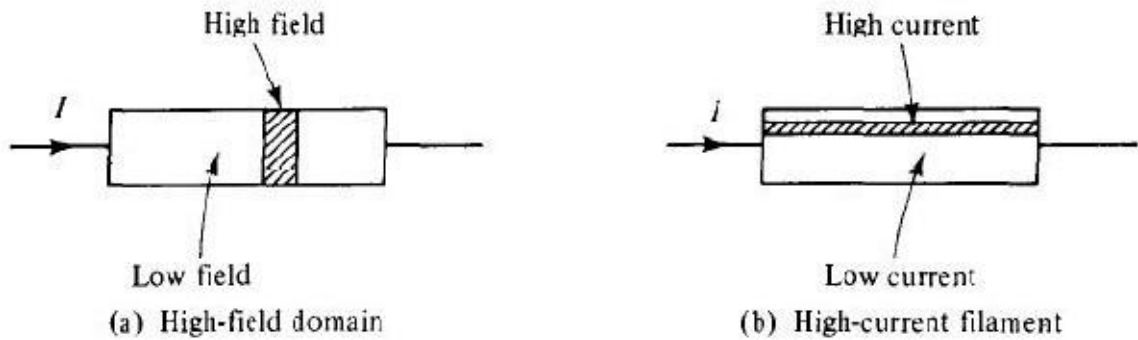


(a) Voltage-controlled mode



(b) Current-controlled mode

In the voltage-controlled mode the current density can be multivalued, whereas in the current- controlled mode the voltage can be multivalued.



The major effect of the appearance of a differential negative-resistance region in the current-density-field curve is to render the sample electrically unstable. As a result, the initially homogeneous sample becomes electrically heterogeneous in an attempt to reach stability.

In the voltage-controlled negative-resistance mode high-field domains are formed, separating two low-field regions. The interfaces separating low and high-field domains lie along equipotentials; thus they are in planes perpendicular to the current direction as shown in Fig. 7-2-2(a). In the current-controlled negative-resistance mode splitting the sample results in high-current filaments running along the field direction as shown in Fig. 7-2-2(b).

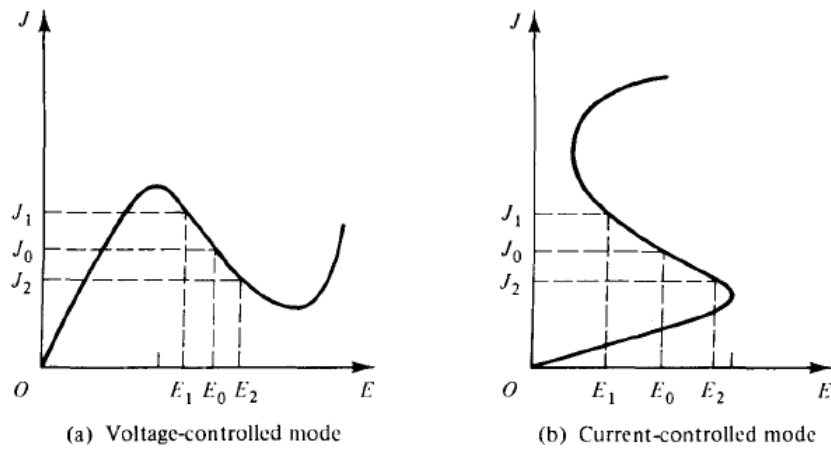
Expressed mathematically, the negative resistance of the sample at a particular region is

$$\frac{dI}{dV} = \frac{dJ}{dE} = \text{negative resistance} \quad (7-2-1)$$

If an electric field  $E_0$  (or voltage  $V_0$ ) is applied to the sample, for example, the current density is generated. As the applied field (or voltage) is increased to  $E_2$  (or  $V_2$ ), the current density is decreased to  $J_2$ .

When the field (or voltage) is decreased to  $E_1$  (or  $V_1$ ), the current density is increased to  $J_1$ . These phenomena of the voltage controlled negative resistance are shown in Fig. 7-2-3(a).

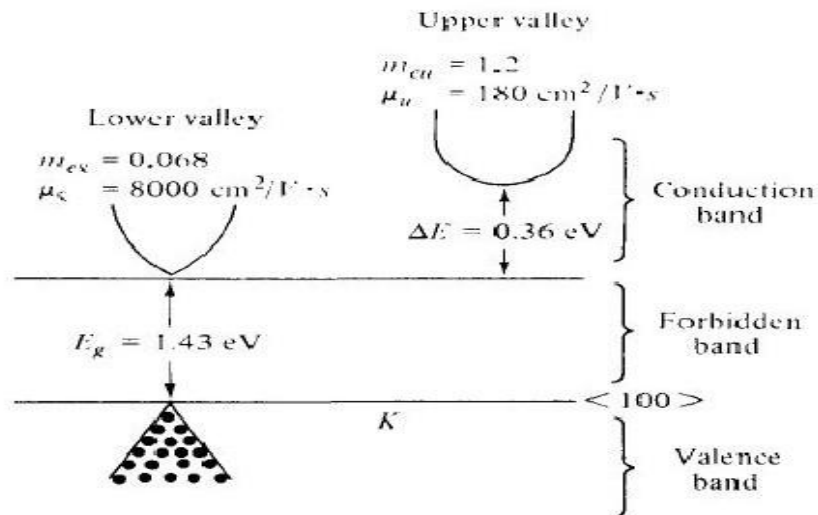
Similarly, for the current controlled mode, the negative-resistance profile is as shown in Fig. 7-2-3(b).



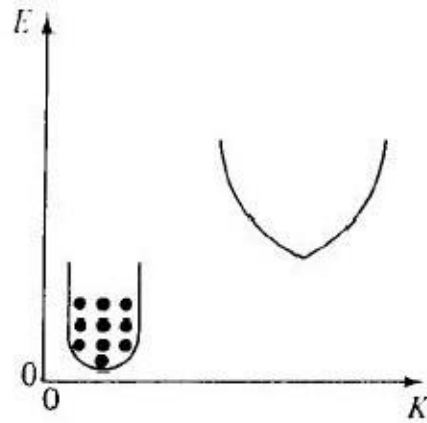
### Two-Valley Model Theory

According to the energy band theory of then-type GaAs, a high-mobility lower valley is separated by an energy of 0.36 eV from a low-mobility upper valley

Valley	Effective Mass $M_e$	Mobility $\mu$	Separation $\Delta E$
Lower	$M_{e\ell} = 0.068$	$\mu_{\ell} = 8000 \text{ cm}^2/\text{V}\cdot\text{sec}$	$\Delta E = 0.36 \text{ eV}$
Upper	$M_{eu} = 1.2$	$\mu_u = 180 \text{ cm}^2/\text{V}\cdot\text{sec}$	$\Delta E = 0.36 \text{ eV}$

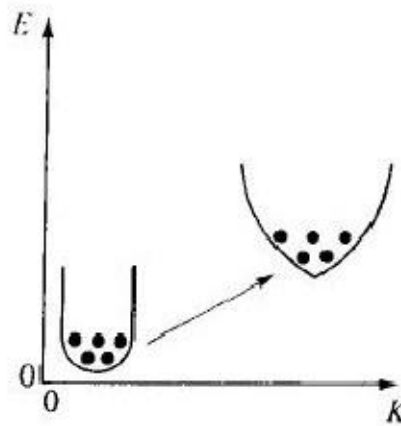


When the applied electric field is lower than the electric field of the lower valley ( $\mathcal{E} < E_c$ ), no electrons will transfer to the upper valley as show in Fig. 7-2-S(a).



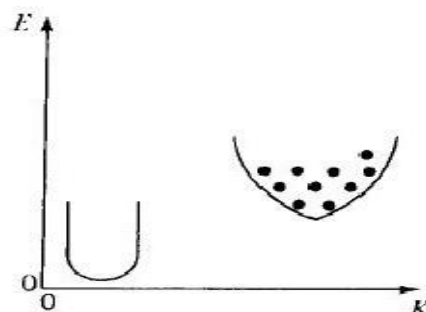
(a)  $E < E_v$

When the applied electric field is higher than that of the lower valley and lower than that of the upper valley ( $E_c < E < E_u$ ), electrons will begin to transfer to the upper valley as shown in Fig. 7-2-S(b).



(b)  $E_v < E < E_u$

And when the applied electric field is higher than that of the upper valley ( $E_u < E$ ), all electrons will transfer to the upper valley as shown in Fig. 7-2-S(c).



(c)  $E_u < E$

If electron densities in the lower and upper valleys are  $n_c$  and  $n_u$ , the conductivity of the  $n$ -type GaAs is

$$\sigma = e(\mu_{\epsilon}n_{\epsilon} + \mu_u n_u) \quad (7-2-2)$$

where  $e$  = the electron charge  
 $\mu$  = the electron mobility  
 $n = n_{\epsilon} + n_u$  is the electron density

When a sufficiently high field  $E$  is applied to the specimen, electrons are accelerated and their effective temperature rises above the lattice temperature. Furthermore, the lattice temperature also increases. Thus electron density  $n$  and mobility  $f$ - $L$  are both functions of electric field  $E$ . Differentiation of Eq. (7-2-2) with respect to  $E$  yields

$$\frac{d\sigma}{dE} = e\left(\mu_{\epsilon} \frac{dn_{\epsilon}}{dE} + \mu_u \frac{dn_u}{dE}\right) + e\left(n_{\epsilon} \frac{d\mu_{\epsilon}}{dE} + n_u \frac{d\mu_u}{dE}\right) \quad (7-2-3)$$

If the total electron density is given by  $n = n_{\epsilon} + n_u$  and it is assumed that  $f$ - $L$  and  $\mu$  are proportional to  $E^p$ , where  $p$  is a constant, then

$$\frac{d}{dE} (n_{\epsilon} + n_u) = \frac{dn}{dE} = 0 \quad (7-2-4)$$

$$\frac{dn_{\epsilon}}{dE} = -\frac{dn_u}{dE} \quad (7-2-5)$$

$$\frac{d\mu}{dE} \propto \frac{dE^p}{dE} = pE^{p-1} = p \frac{E^p}{E} \propto p \frac{\mu}{E} = \mu \frac{p}{E} \quad (7-2-6)$$

Substitution of Eqs. (7-2-4) to (7-2-6) into Eq. (7-2-3) results in

$$\frac{d\sigma}{dE} = e(\mu_{\epsilon} - \mu_u) \frac{dn_{\epsilon}}{dE} + e(n_{\epsilon}\mu_{\epsilon} + n_u\mu_u) \frac{p}{E} \quad (7-2-7)$$

Then differentiation of Ohm's law  $J = aE$  with respect to  $E$  yields



$$\frac{dJ}{dE} = \sigma + \frac{d\sigma}{dE} E \quad (7-2-8)$$

Equation (7-2-8) can be rewritten

$$\frac{1}{\sigma} \frac{dJ}{dE} = 1 + \frac{d\sigma/dE}{\sigma/E} \quad (7-2-9)$$

Clearly, for negative resistance, the current density  $J$  must decrease with increasing field  $E$  or the ratio of  $dJ/dE$  must be negative. Such would be the case only if the right-hand term of Eq. (7-2-9) is less than zero. In other words, the condition for negative resistance is

$$-\frac{d\sigma/dE}{\sigma/E} > 1 \quad (7-2-10)$$

Substitution of Eqs. (7-2-2) and (7-2-7) with  $= nu/ne$  results in [2]

$$\left[ \left( \frac{\mu_e - \mu_u}{\mu_e + \mu_u f} \right) \left( -\frac{E}{n_e} \frac{dn_e}{dE} \right) - p \right] > 1 \quad (7-2-11)$$

#### **AVALANCE TRANSIT TIME DEVICES:**

Avalanche transit-time diode oscillators rely on the effect of voltage breakdown across a reverse-biased p-n junction to produce a supply of holes and electrons. Ever since the development of modern semiconductor device theory scientists have speculated on whether it is possible to make a two-terminal negative-resistance device.

The tunnel diode was the first such device to be realized in practice. Its operation depends on the properties of a forward-biased p-n junction in which both the p and n regions are heavily doped. The other two devices are the transferred electron devices and the avalanche transit-time devices. In this chapter the latter type is discussed.

The transferred electron devices or the Gunn oscillators operate simply by the application of a dc voltage to a bulk semiconductor. There are no  $p-n$  junctions in this device. Its frequency is a function of the load and of the natural frequency of the circuit. The avalanche diode oscillator uses carrier impact ionization and drift in the high-field region of a semiconductor junction to produce a negative resistance at microwave frequencies.

The device was originally proposed in a theoretical paper by Read in which he analyzed the negative-resistance properties of an idealized  $n+p-i-p+$  diode. Two distinct modes of avalanche oscillator have been observed. One is the IMPATT mode, which stands for *impact ionization avalanche transit-time* operation. In this mode the typical dc-to-RF conversion efficiency is 5 to 10%, and frequencies are as high as 100 GHz with silicon diodes.

The other mode is the TRAPATT mode, which represents *trapped plasma avalanche triggered transit* operation. Its typical conversion efficiency is from 20 to 60%. Another type of active microwave device is the BARITT (*barrier injected transit-time*) diode [2]. It has long drift

regions similar to those of IMPATT diodes. The carriers traversing the drift regions of BARITT diodes, however, are generated by minority carrier injection from forward-biased junctions rather

than being extracted from the plasma of an avalanche region. Several different structures have been operated as BARITT diodes, such as  $p-n-p$ ,  $p-n-v-p$ ,  $p-n$ -metal, and metal- $n$ -metal. BARITT diodes have low noise figures of 15 dB, but their bandwidth is relatively narrow with low output power.

#### IMPATT AND TRAPATT DIODE:

##### ***Physical Structures***

A theoretical Read diode made of  $n-p-i-p+$  or  $p+n-i-n+$  structure has been analyzed. Its basic physical mechanism is the interaction of the impact ionization avalanche and the transit time of charge carriers. Hence the Read-type diodes are called IMPATT diodes. These diodes exhibit a differential negative resistance by two effects:

1) The impact ionization avalanche effect, which causes the carrier current  $i_o(t)$  and the ac voltage to be out of phase by  $90^\circ$

2) The transit-time effect, which further delays the external current  $i_o(t)$  relative to the ac voltage by  $90^\circ$

The first IMPATT operation as reported by Johnston et al. [4] in 1965, however, was obtained from a simple  $p-n$  junction. The first real Read-type IMPATT diode was reported by Lee et al. [3], as described previously.

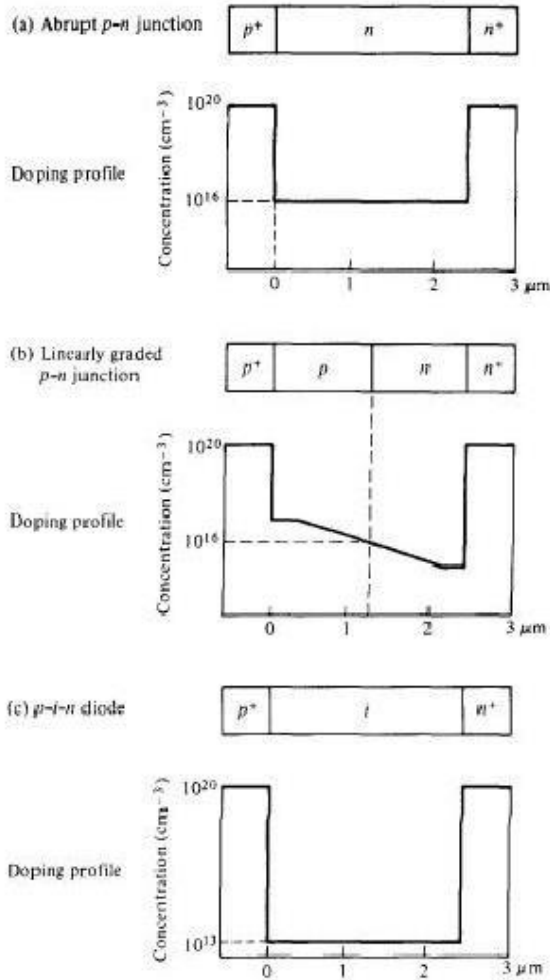
From the small-signal theory developed by Gilden [5] it has been confirmed that a negative resistance of the IMPATT diode can be obtained from a junction diode with any doping profile. Many IMPATT diodes consist of a high doping avalanching region followed by a drift region where the field is low enough that

the carriers can traverse through it without avalanching The Read diode is the basic type in the IMPATT diode family. The others are the one-sided abrupt  $p-n$  junction, the linearly graded  $p-n$  junction (or double-drift region), and the  $p-i-n$  diode, all of which are shown in Fig. 8-2-1. The principle of operation of these devices, however, is essentially similar to the mechanism described for the Read diode.

Small-signal analysis of a Read diode results in the following expression for the real part of the diode terminal impedance [5]:

$$R = R_i + \frac{2L^2}{v_d \epsilon_s A} \frac{1}{1 - \omega^2/\omega_c^2} \frac{1 - \cos \theta}{\theta} \quad (8-2-1)$$

where  $R_i$  = passive resistance of the inactive region  
 $v_d$  = carrier drift velocity  
 $L$  = length of the drift space-charge region  
 $A$  = diode cross section  
 $\epsilon_s$  = semiconductor dielectric permittivity



Moreover,  $\theta$  is the transit angle, given by

$$\theta = \omega\tau = \omega \frac{L}{v_d} \quad (8-2-2)$$

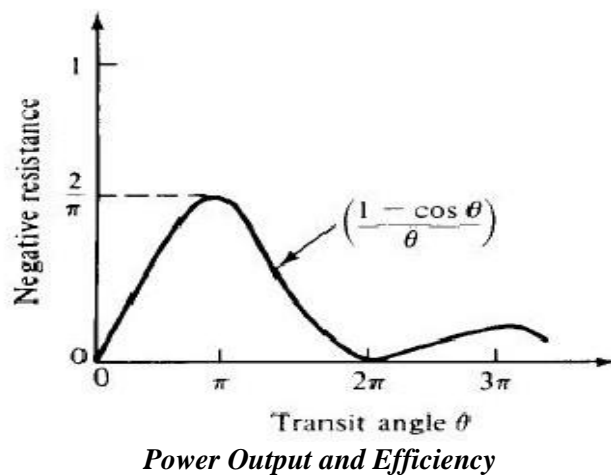
and  $\omega_r$  is the avalanche resonant frequency, defined by

$$\omega_r = \left( \frac{2\alpha' v_d I_0}{\epsilon_s A} \right)^{1/2} \quad (8-2-3)$$

The variation of the negative resistance with the transit angle when  $\omega > \omega_r$  is plotted in Fig. 8-2-

2. The peak value of the negative resistance occurs near  $\theta = \pi$ . For transit angles larger than  $\pi$  and approaching  $3\pi/2$ , the negative resistance of the diode decreases rapidly. For practical purposes, the Read-type IMPATT diodes work well only in a frequency range around the  $\pi$  transit angle. That is,

$$f = \frac{1}{2\tau} = \frac{v_d}{2L} \quad (8-2-4)$$



For a uniform avalanche, the maximum voltage that can be applied across the diode is given by

$$V_m = E_m L \quad (8-2-5)$$

where

$L$  is the depletion length

$E_m$  is the maximum electric field.

This maximum applied voltage is limited by the breakdown voltage. Furthermore, the maximum current that can be carried by the diode is also limited by the avalanche breakdown process, for the current in the space-charge region causes an increase in the electric field. The maximum current is given by

$$I_m = J_m A = \sigma E_m A = \frac{\epsilon_s}{\tau} E_m A = \frac{v_d \epsilon_s E_m A}{L} \quad (8-2-6)$$

Therefore the upper limit of the power input is given by

$$P_m = I_m V_m = E_m^2 \epsilon_s v_d A \quad (8-2-7)$$

$$C = \frac{\epsilon_s A}{L} \quad (8-2-8)$$

The capacitance across the space-charge region is defined as

Substitution of Eq. (8-2-8) in Eq. (8-2-7) and application of  $2\pi f T = 1$  yield

$$P_m f^2 = \frac{E_m^2 v_d^2}{4\pi^2 X_c} \quad (8-2-9)$$

It is interesting to note that this equation is identical to Eq. (5-1-60) of the power-frequency limitation for the microwave power transistor. The maximum power that can be given to the mobile carriers decreases

as  $1/f$ . For silicon, this electronic limit is dominant at frequencies as high as 100 GHz. The efficiency of the IMPATT diodes is given by

$$\eta = \frac{P_{ac}}{P_{dc}} = \left(\frac{V_a}{V_d}\right) \left(\frac{I_a}{I_d}\right) \quad (8-2-10)$$

## 5.2 VARACTOR DIODE

Varactor diode is the short name of the “Variable reactor diode”, referred as voltage variable capacitance of a reversed bias pn junction. The junction capacitance depends on the applied voltage and the design of the junction. In some cases a junction with fixed reverse bias may be used as a capacitance of a set value. In a wide sense the varactor diode is designed to exploit the voltage variable properties of the junction capacitance. For example, a varactor may be used in tuning stage of a radio receiver to replace the bulky variable plate capacitor. Other applications of varactor diode include use in harmonic generation, microwave frequency multiplication, active filter and very low-noise parametric amplifiers etc.

### *Operation:*

All semiconductor junction diode exhibit a junction capacitance when reverse biased. The charge free depletion region of an pn junction in a semiconductor widens with application of reverse bias since the surfaces of charge free depletion region represent effective plates of the capacitance of the junction, i.e., the diode behaves as a capacitance with the junction acting as a dielectric between the two conducting material. If the magnitude of reverse bias voltage is increased, the depletion layer width ( $w$ ) will increase and the junction capacitance ( $C_j$ ) will decrease in accordance with

$$C_j \propto \frac{1}{w}$$

A varactor diode is so designed to maximize the capacitance variation with applied reverse bias. Figure 7.15 shows biasing of varactor diode and capacitance versus reverse bias characteristics.

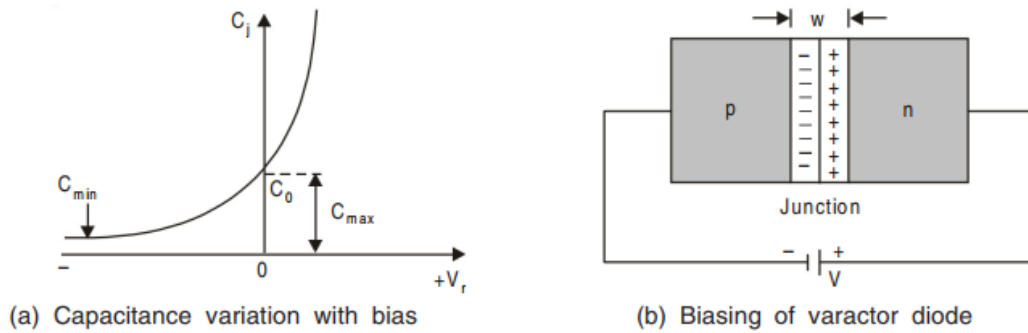


Fig. 7.15. Varactor diode characteristic.

The capacitance is maximum for zero bias and varies inversely with applied reverse bias voltage. The useful reverse voltage swing is between the reverse saturation point (maximum capacitance) and the point just above avalanche (minimum capacitance). Conduction and avalanche are two conditions which limit the voltage swing and hence the capacitance variation. Operation of varactor diode also depend on the types of the junction. We know that the relationship between junction capacitance ( $C_j$ ) and the reverse bias voltage ( $V_r$ ) is given by

### Materials and Construction

Gallium arsenide is the most commonly used semiconductor material for fabricating varactor diode due to their higher mobility of charge carriers. A varactor diode made of gallium arsenide has advantages as a higher maximum operating frequency (up to nearly 1000 GHz) and better functioning at the lowest temperatures. The construction of varactor diode is shown in Fig. 7.17. The diode encapsulation contains electrical lead attached to the semiconductor wafer and a lead to the ceramic case. Diffused junction mesa GaAs diode are widely used at microwave frequencies.

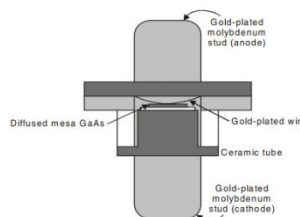


Fig. 7.17. Varactor diode construction.

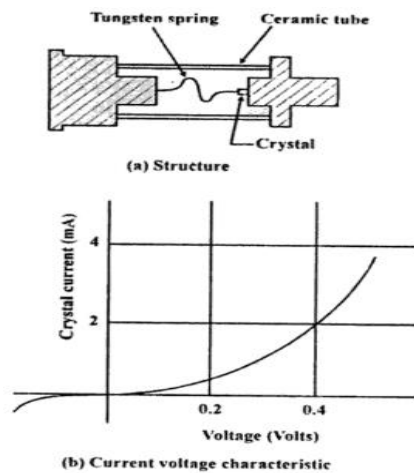
They are capable of handling large power and large breakdown voltages. They have relative independence of ambient temperature change and have low noise. Silicon may be used for varactor diode with the frequency limit up to 25 GHz. However, the manufacturing techniques are easier for silicon.

### Applications of Varactor Diode:

1. Varactor diodes find extensive use in microwave frequency multiplier, parametric amplifiers and as a tuning element.
2. Varactor diodes are widely used within RF circuits.
3. They provide a method of varying the capacitance within a circuit by the application of a control voltage.
4. This gives them an almost unique capability and as a result varactor diodes are widely used within the RF industry.

### 5.3 Crystal Detectors:

Most crystals used are N-type crystal and conduct more readily when the voltage is negative as shown in figure below, These are easy to construct and quite sensitive.



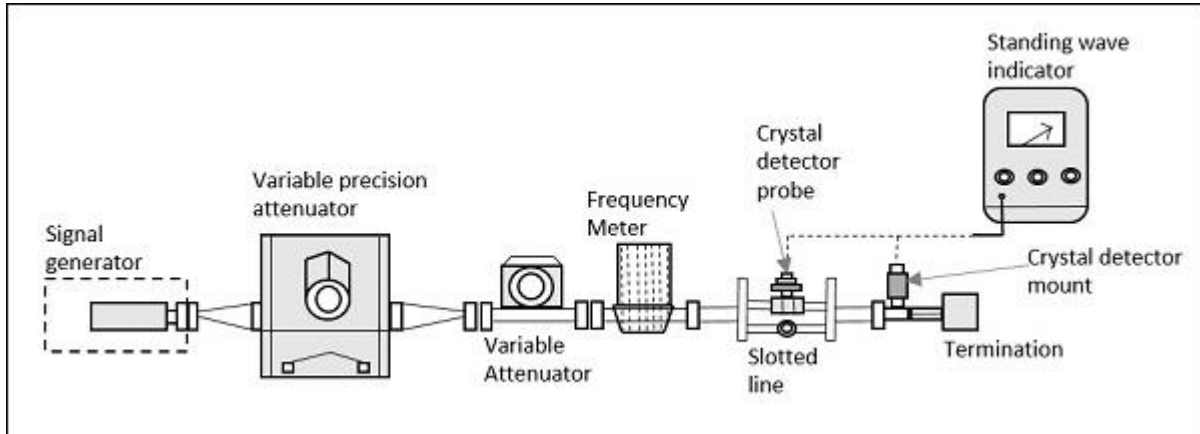


## UNIT-5

### MICROWAVE MEASUREMENTS

#### Microwave Bench General Measurement Setup

This setup is a combination of different parts which can be observed in detail. The following figure clearly explains the setup.



#### Signal Generator

As the name implies, it generates a microwave signal, in the order of a few milliwatts. This uses velocity modulation technique to transfer continuous wave beam into milliwatt power.

A Gunn diode oscillator or a Reflex Klystron tube could be an example for this microwave signal generator.

#### Precision Attenuator

This is the attenuator which selects the desired frequency and confines the output around 0 to 50db. This is variable and can be adjusted according to the requirement.

#### Variable Attenuator

This attenuator sets the amount of attenuation. It can be understood as a fine adjustment of values, where the readings are checked against the values of Precision Attenuator.

#### Isolator

This removes the signal that is not required to reach the detector mount. Isolator allows the signal to pass through the waveguide only in one direction.

#### Frequency Meter

This is the device which measures the frequency of the signal. With this frequency meter, the signal can be adjusted to its resonance frequency. It also gives provision to couple the signal to waveguide.

#### Crystal Detector

A crystal detector probe and crystal detector mount are indicated in the above figure, where the detector is connected through a probe to the mount. This is used to demodulate the signals.

#### Standing Wave Indicator

The standing wave voltmeter provides the reading of standing wave ratio in dB. The waveguide is slotted by

some gap to adjust the clock cycles of the signal. Signals transmitted by waveguide are forwarded through BNC cable to VSWR or CRO to measure its characteristics.

A microwave bench set up in real-time application would look as follows –



Now, let us take a look at the important part of this microwave bench, the slotted line.

#### Slotted Line

In a microwave transmission line or waveguide, the electromagnetic field is considered as the sum of incident wave from the generator and the reflected wave to the generator. The reflections indicate a mismatch or a discontinuity. The magnitude and phase of the reflected wave depends upon the amplitude and phase of the reflecting impedance.

The standing waves obtained are measured to know the transmission line imperfections which is necessary to have a knowledge on impedance mismatch for effective transmission. This slotted line helps in measuring the standing wave ratio of a microwave device.

#### Construction

The slotted line consists of a slotted section of a transmission line, where the measurement has to be done. It has a travelling probe carriage, to let the probe get connected wherever necessary, and the facility for attaching and detecting the instrument.

In a waveguide, a slot is made at the center of the broad side, axially. A movable probe connected to a crystal detector is inserted into the slot of the waveguide.

#### Operation

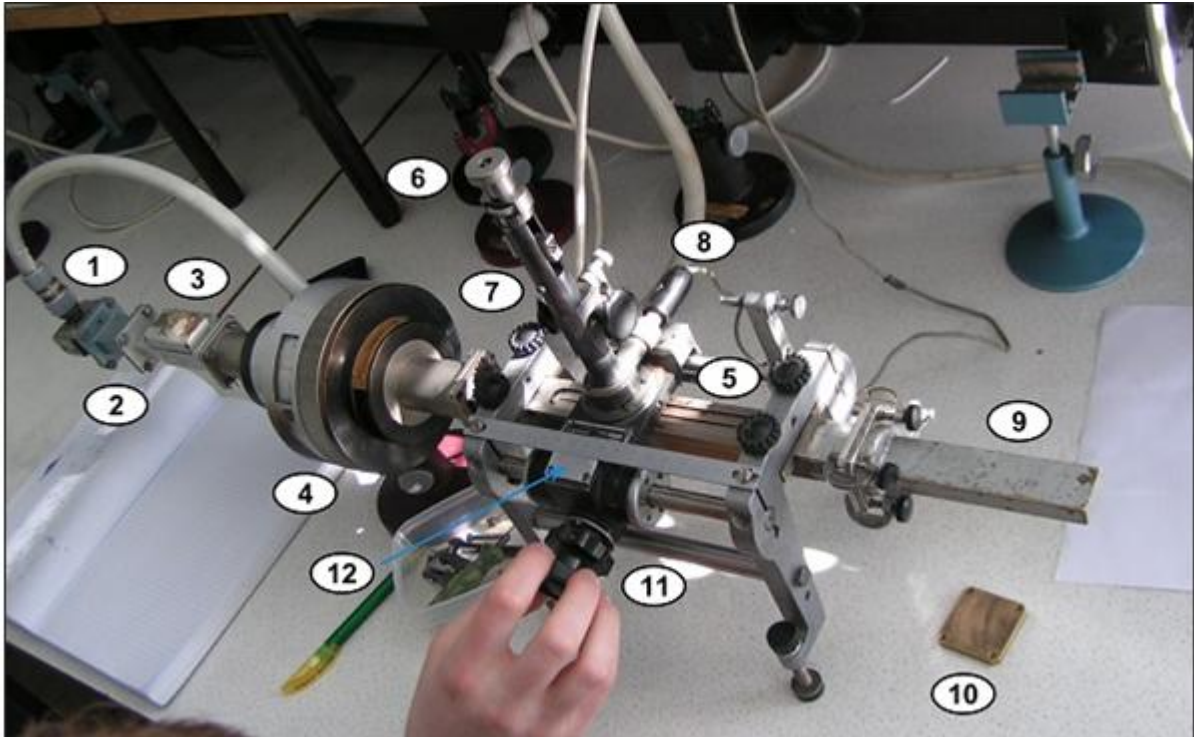
The output of the crystal detector is proportional to the square of the input voltage applied. The movable probe permits convenient and accurate measurement at its position. But, as the probe is moved along, its output is proportional to the standing wave pattern, which is formed inside the waveguide. A variable attenuator is employed here to obtain accurate results.

The output VSWR can be obtained by

$$VSWR = \sqrt{V_{\max}/V_{\min}}$$

Where,  $V$  is the output voltage.

The following figure shows the different parts of a slotted line labelled.



The parts labelled in the above figure indicate the following.

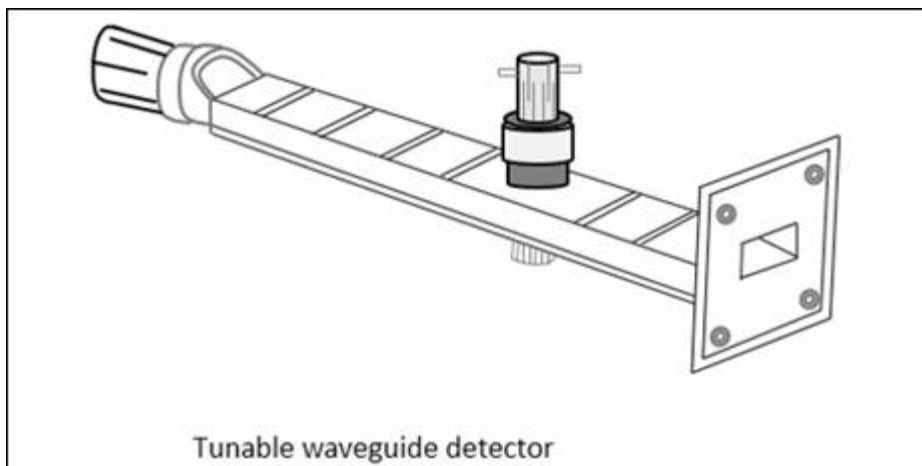
- Launcher – Invites the signal.
- Smaller section of the waveguide.
- Isolator – Prevents reflections to the source.
- Rotary variable attenuator – For fine adjustments.
- Slotted section – To measure the signal.
- Probe depth adjustment.
- Tuning adjustments – To obtain accuracy.
- Crystal detector – Detects the signal.
- Matched load – Absorbs the power exited.
- Short circuit – Provision to get replaced by a load.
- Rotary knob – To adjust while measuring.
- Vernier gauge – For accurate results.

In order to obtain a low frequency modulated signal on an oscilloscope, a slotted line with a tunable detector is employed. A slotted line carriage with a tunable detector can be used to measure the following.

- VSWR (Voltage Standing Wave Ratio)
- Standing wave pattern
- Impedance
- Reflection coefficient
- Return loss
- Frequency of the generator used

#### Tunable Detector

The tunable detector is a detector mount which is used to detect the low frequency square wave modulated microwave signals. The following figure gives an idea of a tunable detector mount.



The following image represents the practical application of this device. It is terminated at the end and has an opening at the other end just as the above one.



To provide a match between the Microwave transmission system and the detector mount, a tunable stub is

often used. There are three different types of tunable stubs.

- Tunable waveguide detector
- Tunable co-axial detector
- Tunable probe detector

Also, there are fixed stubs like –

- Fixed broad band tuned probe
- Fixed waveguide matched detector mount

The detector mount is the final stage on a Microwave bench which is terminated at the end.

In the field of Microwave engineering, there occurs many applications, as already stated in first chapter. Hence, while using different applications, we often come across the need of measuring different values such as Power, Attenuation, Phase shift, VSWR, Impedance, etc. for the effective usage.

### **Measurement of Power**

The Microwave Power measured is the average power at any position in waveguide. Power measurement can be of three types.

- Measurement of Low power (0.01mW to 10mW)

Example – Bolometric technique

- Measurement of Medium power (10mW to 1W)

Example – Calorimeter technique

- Measurement of High power (>10W)

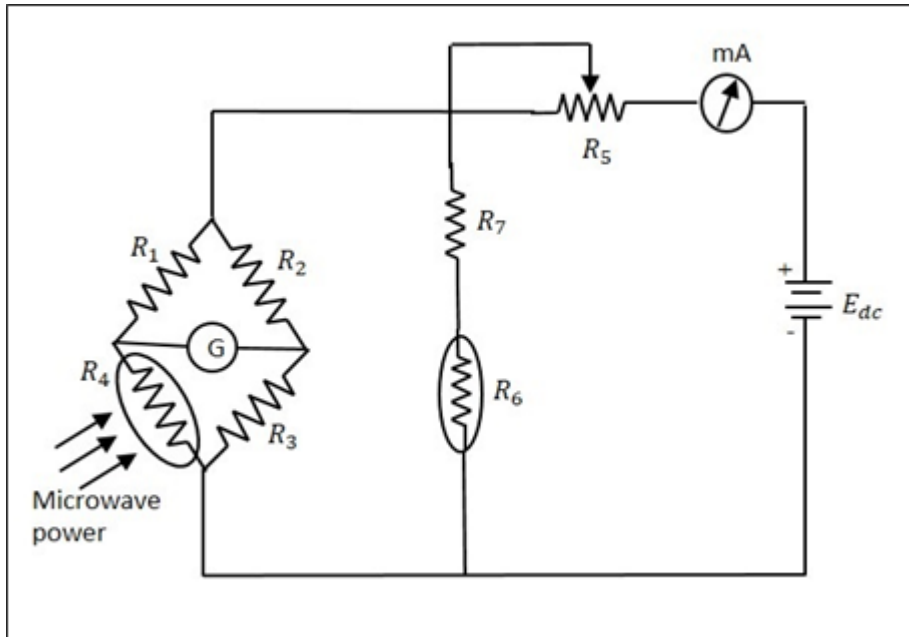
Example – Calorimeter Watt meter

### **Measurement of Low Power**

The measurement of Microwave power around 0.01mW to 10mW, can be understood as the measurement of low power.

**Bolometer** is a device which is used for low Microwave power measurements. The element used in bolometer could be of positive or negative temperature coefficient. For example, a barrater has a positive temperature coefficient whose resistance increases with the increase in temperature. Thermistor has negative temperature coefficient whose resistance decreases with the increase in temperature.

Any of them can be used in the bolometer, but the change in resistance is proportional to Microwave power applied for measurement. This bolometer is used in a bridge of the arms as one so that any imbalance caused, affects the output. A typical example of a bridge circuit using a bolometer is as shown in the following figure.



The milliammeter here, gives the value of the current flowing. The battery is variable, which is varied to obtain balance, when an imbalance is caused by the behavior of the bolometer. This adjustment which is made in DC battery voltage is proportional to the Microwave power. The power handling capacity of this circuit is limited.

#### Measurement of Medium Power

The measurement of Microwave power around 10mW to 1W, can be understood as the measurement of medium power.

A special load is employed, which usually maintains a certain value of specific heat. The power to be measured, is applied at its input which proportionally changes the output temperature of the load that it already maintains. The difference in temperature rise, specifies the input Microwave power to the load.

The bridge balance technique is used here to get the output. The heat transfer method is used for the measurement of power, which is a Calorimetric technique.

#### Measurement of High Power

The measurement of Microwave power around 10W to 50KW can be understood as the measurement of high power.

The High Microwave power is normally measured by Calorimetric watt meters, which can be of dry and flow type. The dry type is named so as it uses a coaxial cable which is filled with di-electric of high hysteresis loss, whereas the flow type is named so as it uses water or oil or some liquid which is a good absorber of microwaves.

The change in temperature of the liquid before and after entering the load, is taken for the calibration of values. The limitations in this method are like flow determination, calibration and thermal inertia, etc.

#### Measurement of Attenuation

In practice, Microwave components and devices often provide some attenuation. The amount of attenuation

offered can be measured in two ways. They are – Power ratio method and RF substitution method.

Attenuation is the ratio of input power to the output power and is normally expressed in decibels.

$$\text{Attenuation in dB} = 10 \log P_{in}/P_{out}$$

Where  $P_{in}$  = Input power and  $P_{out}$  = Output power

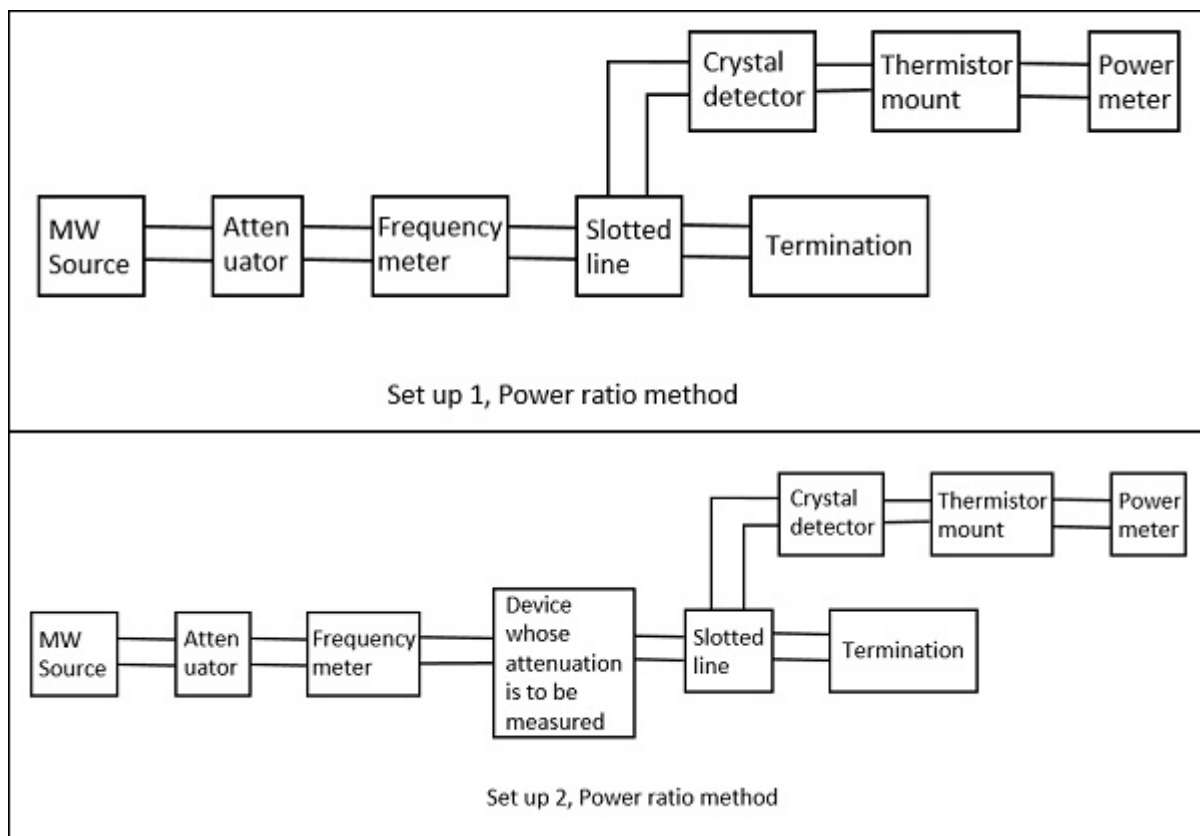
### Power Ratio Method

In this method, the measurement of attenuation takes place in two steps.

- **Step 1** – The input and output power of the whole Microwave bench is done without the device whose attenuation has to be calculated.
- **Step 2** – The input and output power of the whole Microwave bench is done with the device whose attenuation has to be calculated.

The ratio of these powers when compared, gives the value of attenuation.

The following figures are the two setups which explain this.



**Drawback** – The power and the attenuation measurements may not be accurate, when the input power is low and attenuation of the network is large.

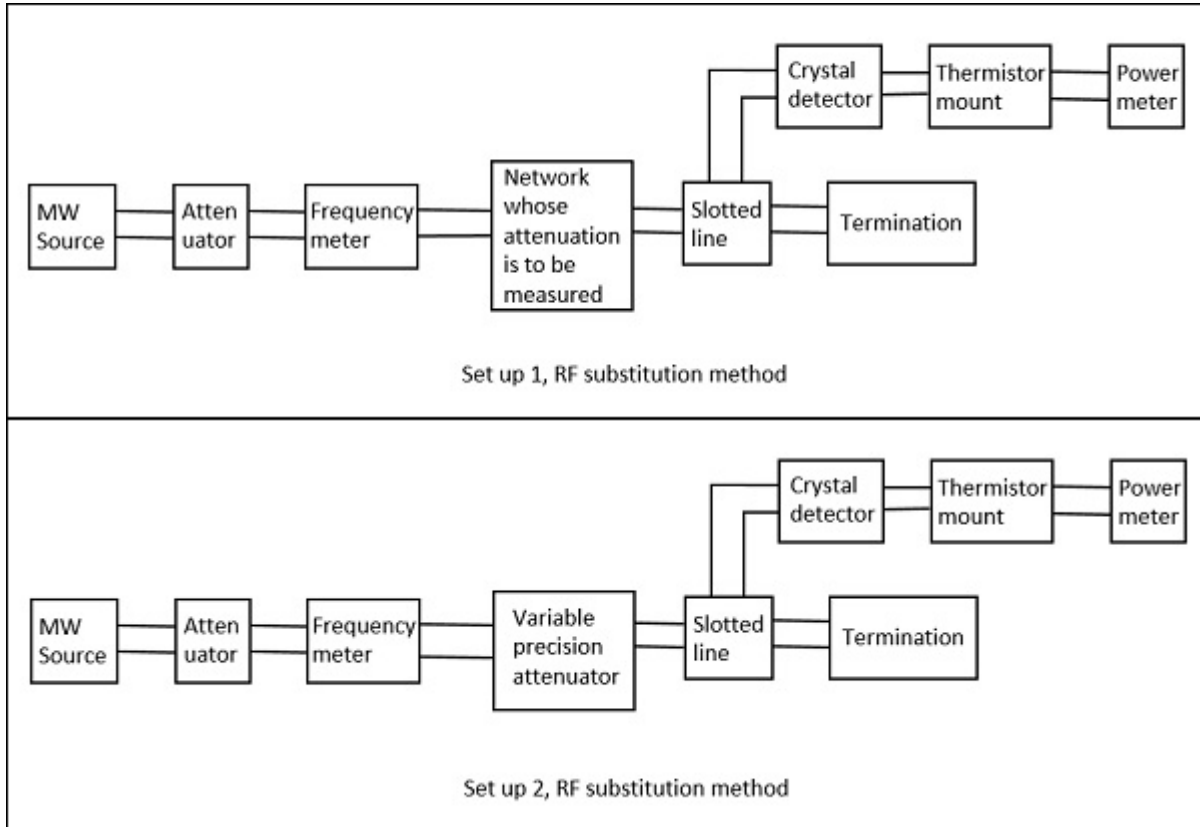
### RF Substitution Method

In this method, the measurement of attenuation takes place in three steps.

- **Step 1** – The output power of the whole Microwave bench is measured with the network whose attenuation has to be calculated.

- **Step 2** – The output power of the whole Microwave bench is measured by replacing the network with a precision calibrated attenuator.
- **Step 3** – Now, this attenuator is adjusted to obtain the same power as measured with the network.

The following figures are the two setups which explain this.



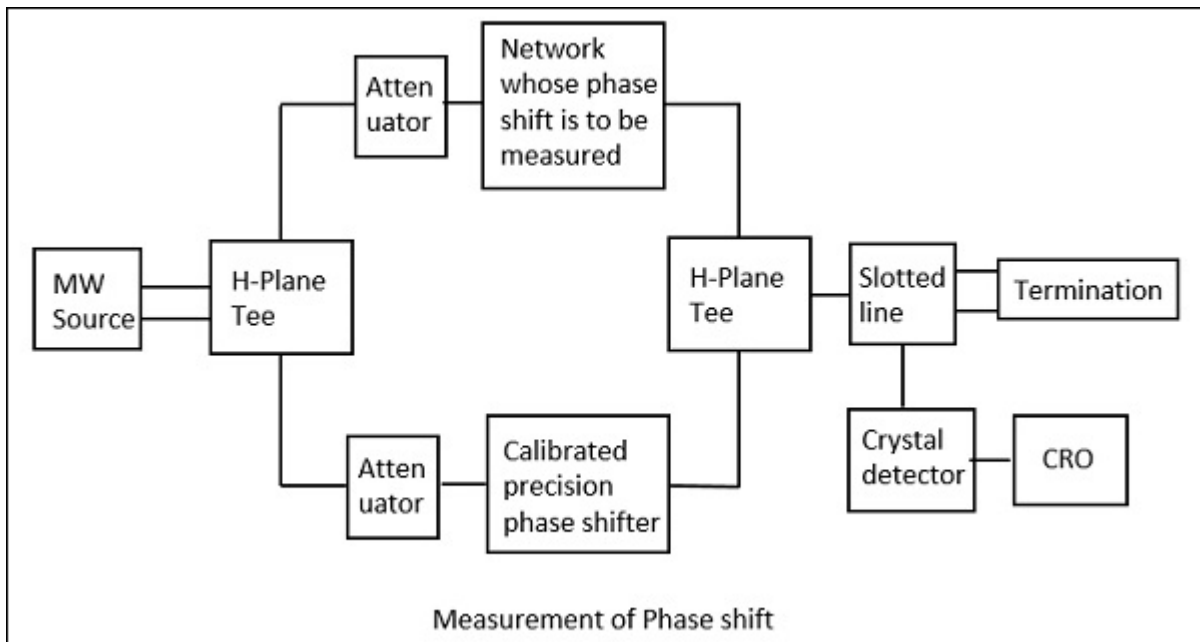
The adjusted value on the attenuator gives the attenuation of the network directly. The drawback in the above method is avoided here and hence this is a better procedure to measure the attenuation.

### Measurement of Phase Shift

In practical working conditions, there might occur a phase change in the signal from the actual signal. To measure such phase shift, we use a comparison technique, by which we can calibrate the phase shift.

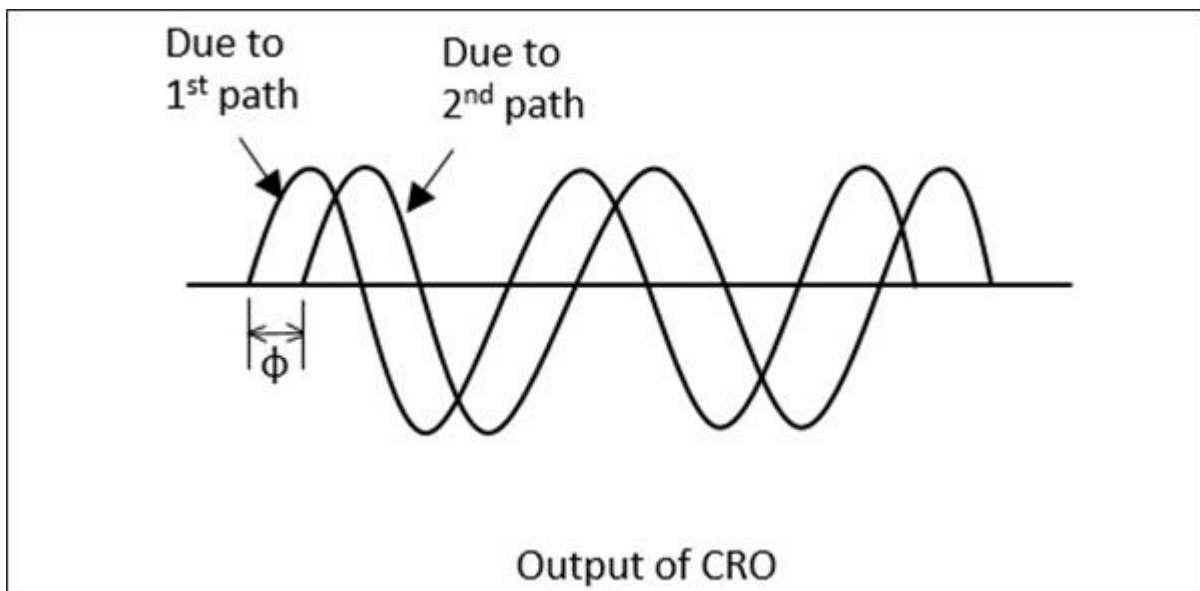
The setup to calculate the phase shift is shown in the following figure.





Here, after the microwave source generates the signal, it is passed through an H-plane Tee junction from which one port is connected to the network whose phase shift is to be measured and the other port is connected to an adjustable precision phase shifter.

The demodulated output is a 1 KHz sine wave, which is observed in the CRO connected. This phase shifter is adjusted such that its output of 1 KHz sine wave also matches the above. After the matching is done by observing in the dual mode CRO, this precision phase shifter gives us the reading of phase shift. This is clearly understood by the following figure.



This procedure is the mostly used one in the measurement of phase shift. Now, let us see how to calculate the VSWR.

### Measurement of VSWR

In any Microwave practical applications, any kind of impedance mismatches lead to the formation of

standing waves. The strength of these standing waves is measured by Voltage Standing Wave Ratio (VSWR). The ratio of maximum to minimum voltage gives the VSWR, which is denoted by S.

$$S = V_{max}/V_{min} = 1 + \rho / 1 - \rho$$

Where,  $\rho$  = reflection coefficient = P reflected / P incident

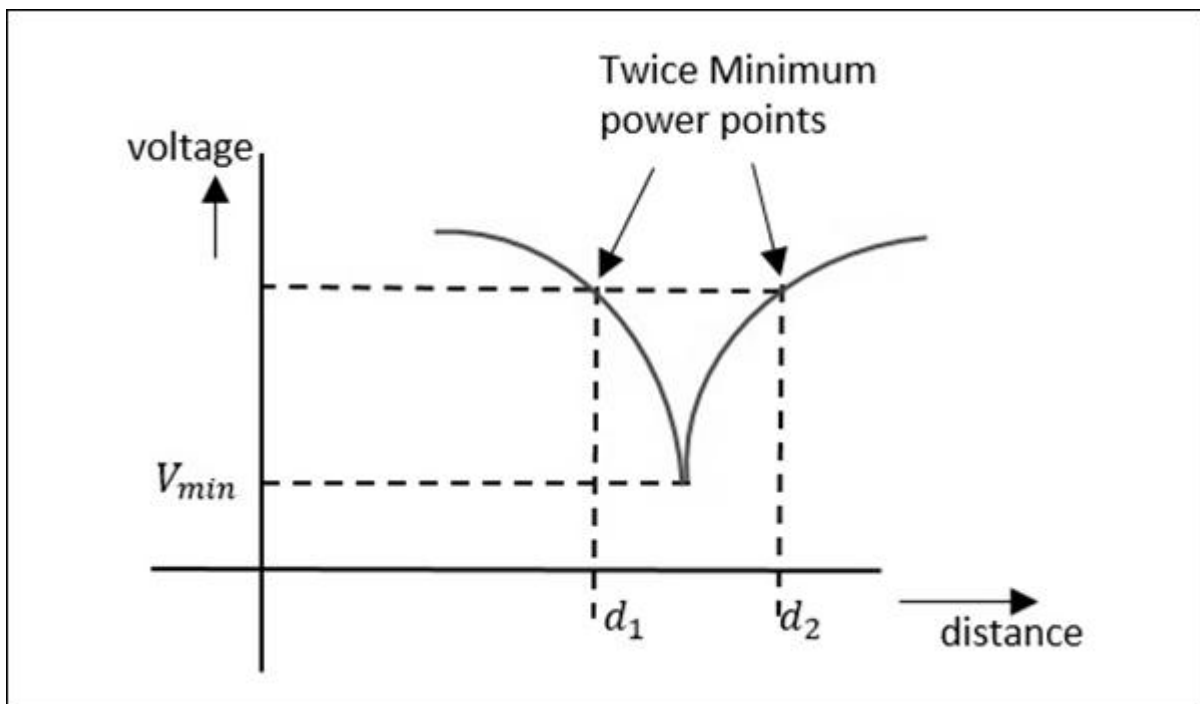
#### Measurement of Low VSWR (S < 10)

The measurement of low VSWR can be done by adjusting the attenuator to get a reading on a DC milli voltmeter which is VSWR meter. The readings can be taken by adjusting the slotted line and the attenuator in such a way that the DC milli voltmeter shows a full scale reading as well as a minimum reading.

Now these two readings are calculated to find out the VSWR of the network.

#### Measurement of High VSWR (S > 10)

The measurement of high VSWR whose value is greater than 10 can be measured by a method called the **double minimum method**. In this method, the reading at the minimum value is taken, and the readings at the half point of minimum value in the crest before and the crest after are also taken. This can be understood by the following figure.



Now, the VSWR can be calculated by a relation, given as –

$$VSWR = \lambda_g / \pi (d_2 - d_1)$$

Where,  $\lambda_g$  is the guided wavelength

As the two minimum points are being considered here, this is called as double minimum method. Now, let us learn about the measurement of impedance.

#### Measurement of Impedance

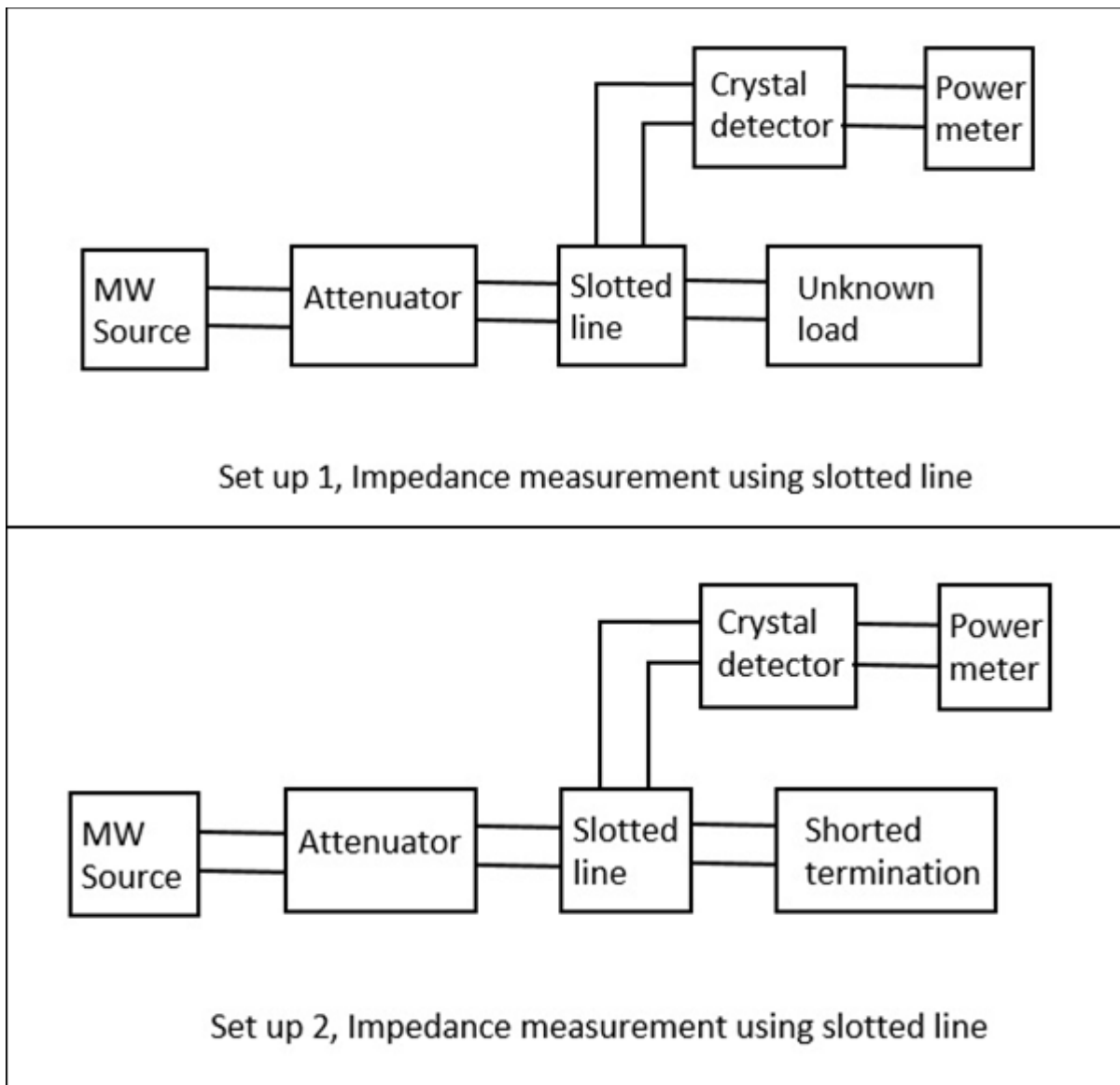
Apart from Magic Tee, we have two different methods, one is using the slotted line and the other is using the reflecto meter.

#### Impedance Using the Slotted Line

In this method, impedance is measured using slotted line and load  $Z_L$  and by using this,  $V_{max}$  and  $V_{min}$  can be determined. In this method, the measurement of impedance takes place in two steps.

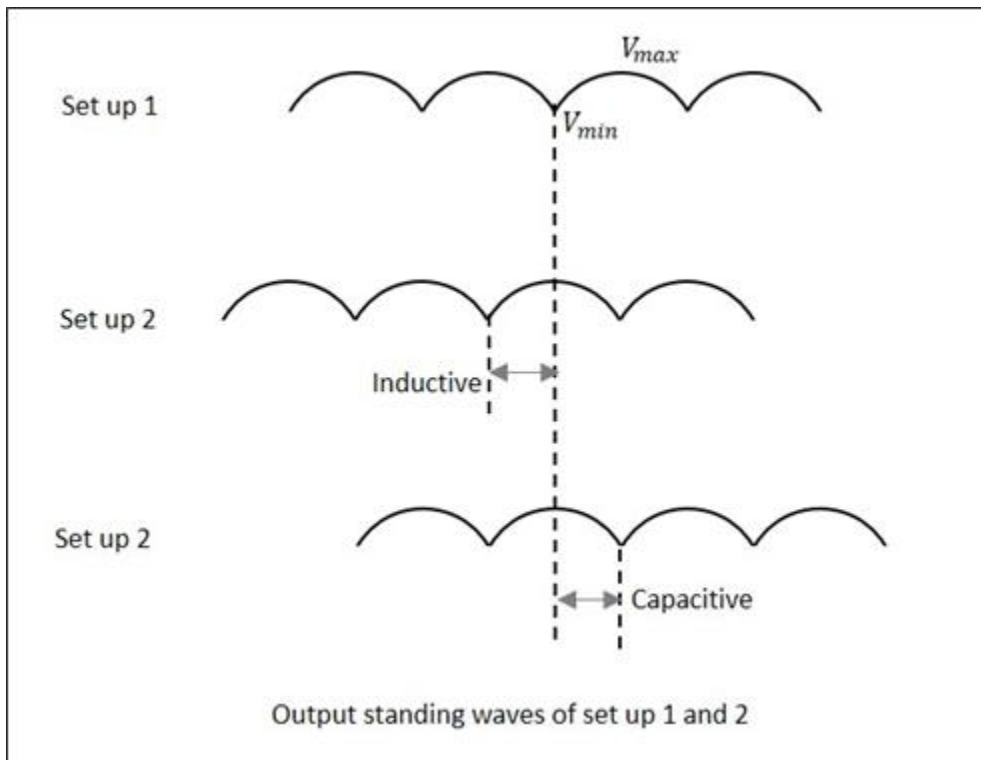
- **Step 1** – Determining  $V_{min}$  using load  $Z_L$ .
- **Step 2** – Determining  $V_{min}$  by short circuiting the load.

This is shown in the following figures.



When we try to obtain the values of  $V_{max}$  and  $V_{min}$  using a load, we get certain values. However, if the same is done by short circuiting the load, the minimum gets shifted, either to the right or to the left. If this shift is to the left, it means that the load is inductive and if it the shift is to the right, it means that the load is

capacitive in nature. The following figure explains this.

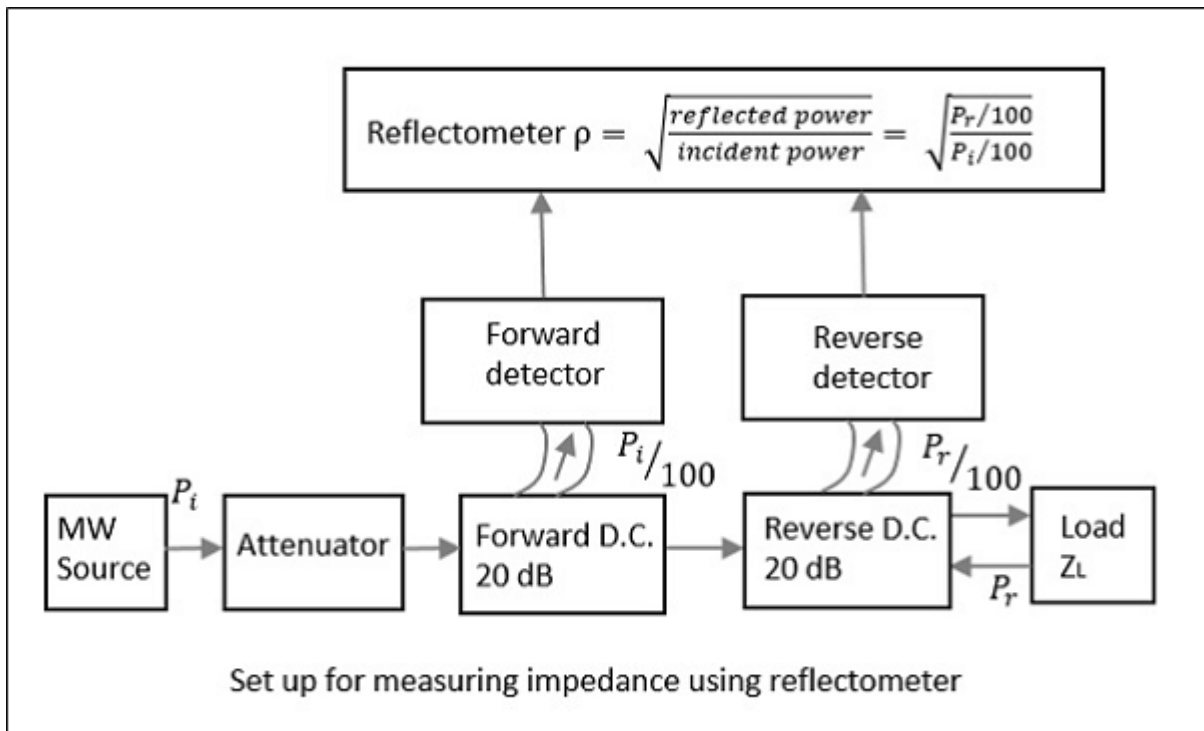


By recording the data, an unknown impedance is calculated. The impedance and reflection coefficient  $\rho$  can be obtained in both magnitude and phase.

### Impedance Using the Reflecto meter

Unlike slotted line, the Reflecto meter helps to find only the magnitude of impedance and not the phase angle. In this method, two directional couplers which are identical but differ in direction are taken.

These two couplers are used in sampling the incident power  $P_i$  and reflected power  $P_r$  from the load. The reflecto meter is connected as shown in the following figure. It is used to obtain the magnitude of reflection coefficient  $\rho$ , from which the impedance can be obtained.



From the reflectometer reading, we have

$$\rho = \sqrt{P_r/P_i}$$

From the value of  $\rho$ , the VSWR, i.e.  $S$  and the impedance can be calculated by

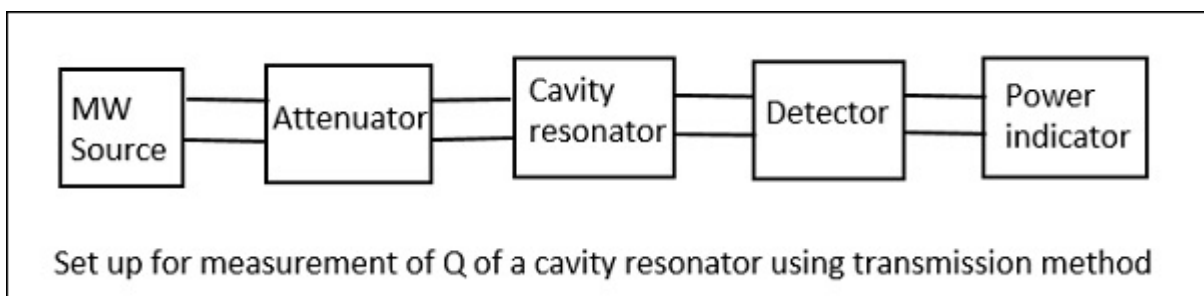
$$S = 1 + \rho / 1 - \rho \quad \& \quad \rho = z - z_0 / z + z_0$$

Where,  $z_0$  is known wave impedance and  $z$  is unknown impedance.

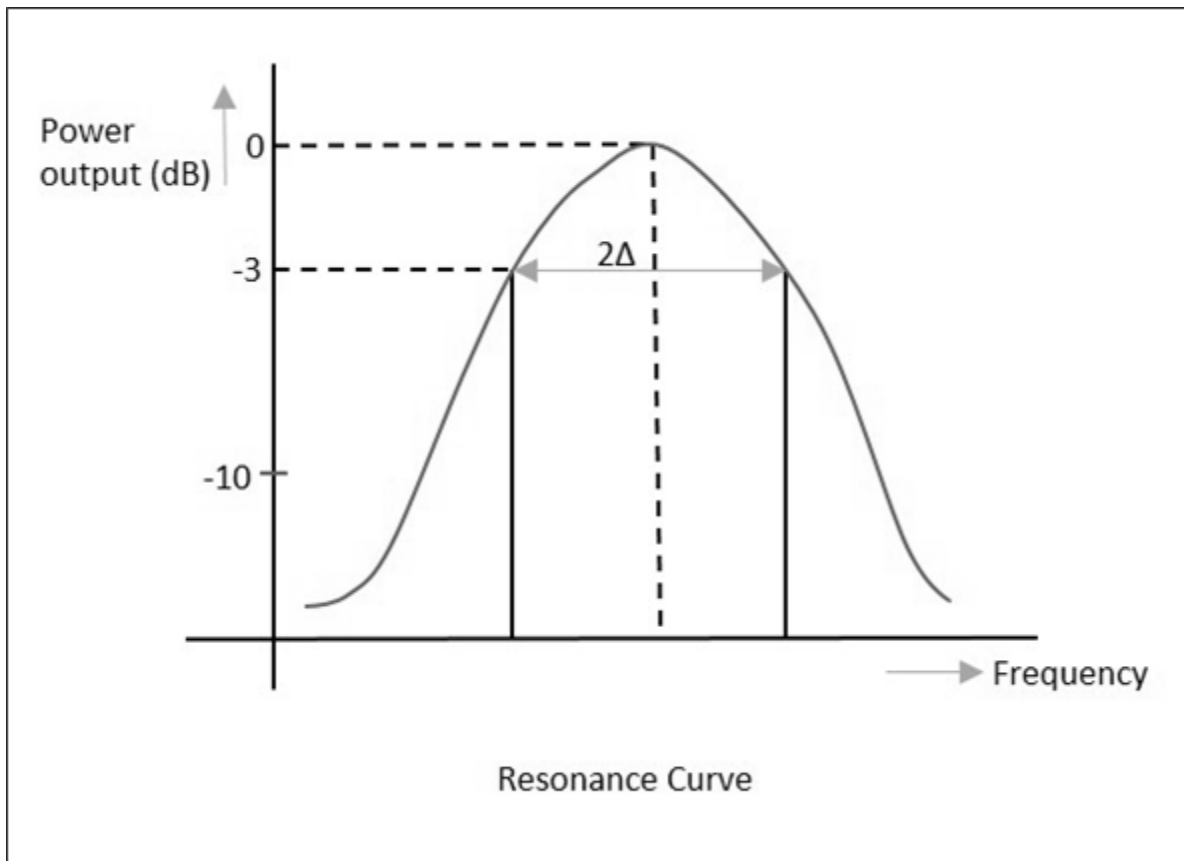
Though the forward and reverse wave parameters are observed here, there will be no interference due to the directional property of the couplers. The attenuator helps in maintaining low input power.

### Measurement of Q of Cavity Resonator

Though there are three methods such as Transmission method, Impedance method, and Transient decay or Decrement method for measuring  $Q$  of a cavity resonator, the easiest and most followed method is the **Transmission Method**. Hence, let us take a look at its measurement setup.



In this method, the cavity resonator acts as the device that transmits. The output signal is plotted as a function of frequency which results in a resonant curve as shown in the following figure.



From the setup above, the signal frequency of the microwave source is varied, keeping the signal level constant and then the output power is measured. The cavity resonator is tuned to this frequency, and the signal level and the output power is again noted down to notice the difference.

When the output is plotted, the resonance curve is obtained, from which we can notice the Half Power Bandwidth (HPBW) ( $2\Delta$ ) values.

$$2\Delta = \pm 1/Q_L$$

Where,  $Q_L$  is the loaded value

Or

$$Q_L = \pm 1/2\Delta = \pm w/2(w-w_0)$$

If the coupling between the microwave source and the cavity, as well the coupling between the detector and the cavity are neglected, then

$$Q_L = Q_0 (\text{unloaded } Q)$$

The main drawback of this system is that, the accuracy is a bit poor in very high  $Q$  systems due to narrow band of operation.