INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

CIVIL ENGINEERING

TUTORIAL QUESTION BANK

Course Name	:	**FINITE ELEMENT METHOD**
Course Code	:	BST005
Class	:	M. Tech (Structural Engineering) – II semester
Department	:	Civil Engineering
Academic Year	:	2017 – 2018
Course Faculty	:	Dr. M. Venu, Professor

COURSE OVERVIEW:

The Finite Element Method (FEM) is widely used in industry for analysing and modelling structures and continua, whose physical behaviour is described by ordinary and partial differential equations. The FEM is particularly useful for engineering problems that are too complicated to be solved by classical analytical methods. The main objective of this course is to introduce the mathematical concepts of the Finite Element Method for obtaining an approximate solution of ordinary and partial differential equations. In this course you will attend lectures on the fundamentals of the Finite Element Method. The learning process will be enhanced by completing assignments using mathematical software. You will also be introduced to a commercial Finite Element software package – ANSYS – during lectures with computer laboratories providing opportunities to practice on, and to complete practical assignments, using ANSYS.

COURSE OBJECTIVES:

The course should enable the students to:

I	Equip the students with the Finite Element Analysis fundamentals.
II	Enable the students to formulate the design problems into Finite Element Method (FEM).
III	Develop the ability to generate the governing finite element equations for systems.
IV	Enable to understand the different kinds of elements used while analysing the structure.
V	Understand the use of the basic finite elements for structural applications using truss, beam, frame and plate elements.

COURSE LEARNING OUTCOMES:

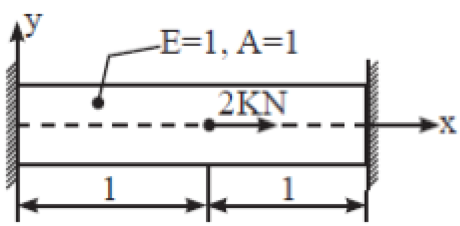
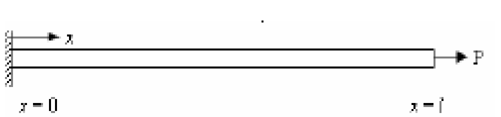
Students, who complete the course, will have demonstrated the ability to do the following:

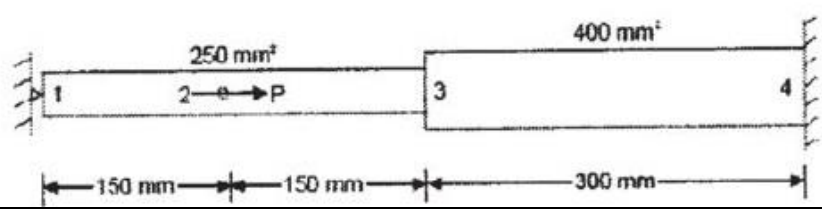
CBST005.01	Understand the Concepts of FEM, steps involved merits and demerits.
CBST005.02	Understand the concept of energy principles, discrimination.
CBST005.03	Solve the problems using Raleigh-Ritz method of functional approximation.
CBST005.04	Know the Stress equations, strain displacement relationships in matrix form plane stress.
CBST005.05	Understand the concept of plane strain and axisymmetric bodies of revolution with axisymmetric loading.
CBST005.06	Understand the concept of One dimensional FEM Stiffness matrix for beam and bar elements.
CBST005.07	Different types of elements for plane stress and plane strain analysis, displacement models.
CBST005.08	Know the generalized coordinates, shape functions.
CBST005.09	Concept of convergent and compatibility requirements, geometric invariance.
CBST005.10	Know the natural coordinate system, area and volume coordinates.

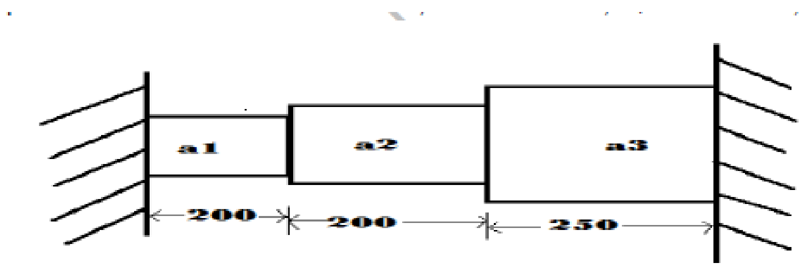
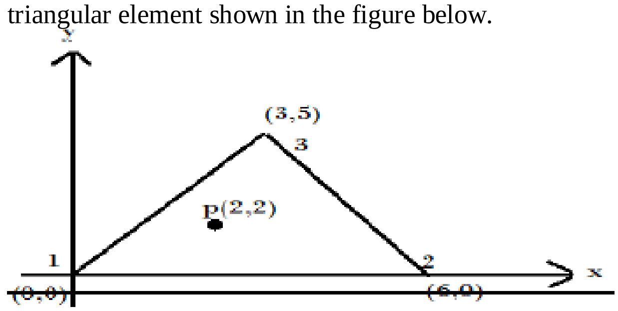
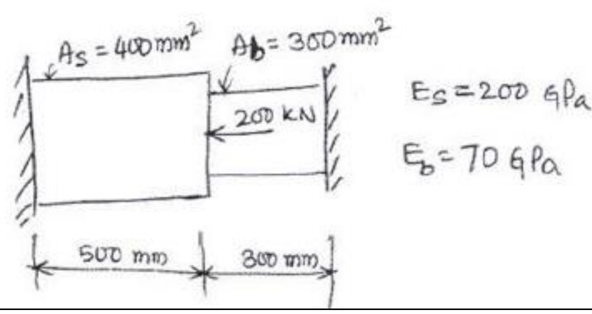
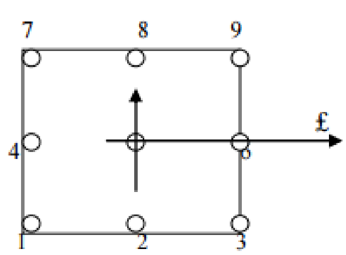
CBST005.11	Understand the generation of element stiffness and nodal load matrices.
CBST005.12	Concept of isoparametric formulation, different isoparametric elements for 2D analysis.
CBST005.13	Understand the formulation of 4- noded and 8-noded isoparametric quadrilateral elements.
CBST005.14	Understand the lagrange elements, serendipity elements.
CBST005.15	Concept of Axisymmetric bodies of revolution, axisymmetric modeling.
CBST005.16	Understand the strain displacement relationship, formulation of axisymmetric elements.
CBST005.17	Know the different 3-D elements strain, displacement relationship.
CBST005.18	Understand the formulation of hexahedral and isoparametric solid element.
CBST005.19	Understand the basic theory of plate bending, thin plate theory.
CBST005.20	Stress resultants, mindlin's approximations.
CBST005.21	Understand the formulation of 4-noded isoperimetric quadrilateral plate element, shell element.
CBST005.22	Introduction to nonlinear analysis: basic methods, application to special structures.

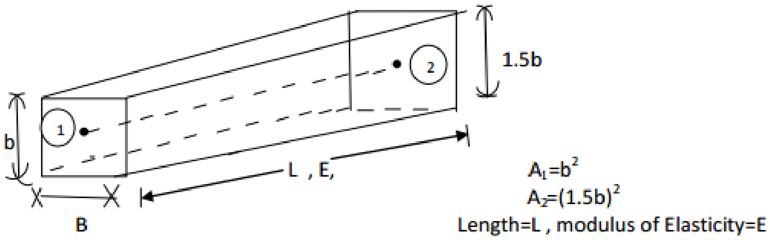
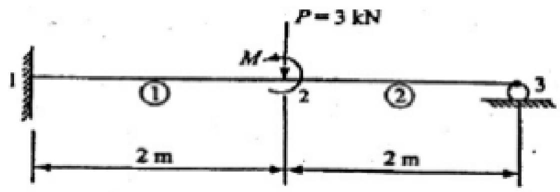
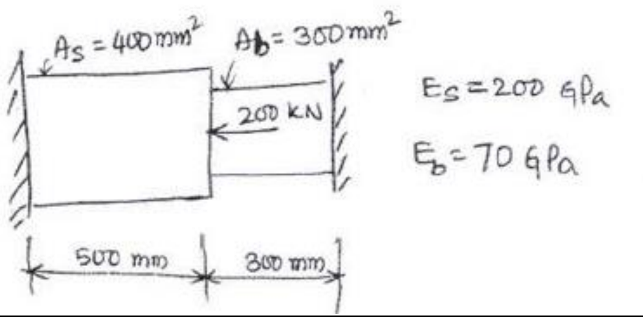
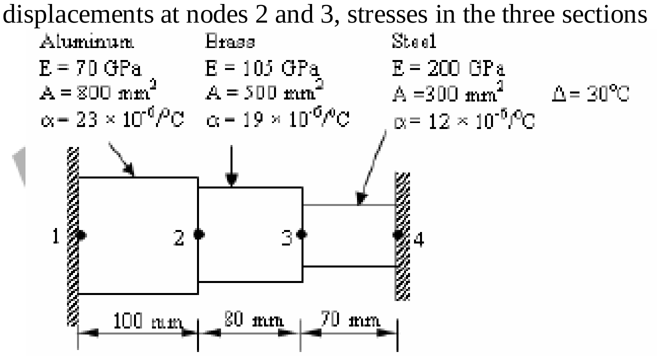
TUTORIAL QUESTION BANK

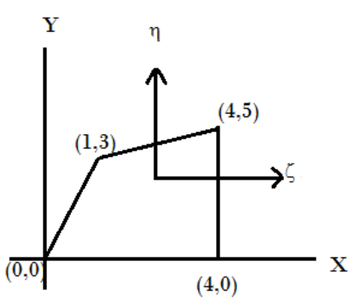
UNIT – I			
INTRODUCTION TO FEM AND PRINCIPLES OF ELASTICITY			
Part - A (Short Answer Questions)			
1	What is meant by Finite Element Analysis?	Remember	CBST005.01
2	What is Raleigh-Ritz method?	Remember	CBST005.03
3	What are the properties of stiffness matrix?	Remember	CBST005.01
4	Define plane stress with a suitable example	Remember	CBST005.05
5	Distinguish between essential boundary condition and natural boundary condition.	Remember	CBST005.04
6	What are the general constituents of finite element software?	Remember	CBST005.02
7	What are the merits and demerits of finite element methods?	Remember	CBST005.02
8	What is the principle of finite element method?	Remember	CBST005.01
9	Explain the terms 'Plane stress' and 'Plane strain' problems. Give constitutive laws for these cases.	Remember	CBST005.02
10	List and briefly describe the general steps of the finite element method.	Remember	CBST005.01
Part - B (Long Answer Questions)			
1	Find out deflection at centre of a simply supported beam of span length (L) subjected to uniformly distributed load throughout its length of intensity w per unit length. Use Rayleigh Ritz method. Take EI is constant	Apply	CBST005.01
2	If a displacement field is described by $u = (x^2 - 2y^2 + 6xy)10^{-4}$ and $v = (6x + 3y)$ $10\text{-}^{4,}$ Determine ε_x, ε_y and γ_{xy} at the point x = 2 and y =1.	Apply	CBST005.01
3	Write the potential energy for beam of span 'L' simply supported at ends, subjected to a concentrated 'P' at midspan. Assume EI constant.	Apply	CBST005.01
4	Solve the following differential equation using Ritz method. $d^2 y/dx_2 = -\sin(\pi x)$ boundary conditions u(0) = 0 and u(1) = 0.	Apply	CBST005.02
5	Using Rayleigh Ritz method, find the maximum deflection of simply supported beam with point load at center.	Apply	CBST005.02
6	Determine the deflection of cantilever beam of length 'L and loaded with a vertical load at the free end by Rayleigh-Ritz method. Use a trail function	Apply	CBST005.03
7	Determine the stiffness matrix, for the plane stress element as shown in figure above. Take E = 200 GPa, and μ = 0.3, thickness of element = 10 mm.	Apply	CBST005.03
8	Obtain an expression for deflection at free end for a cantilever beam of span 'l' meter Subjected to a point load at free end. Using Rayleigh- ritz method. Choose polynomial as a trail function.	Apply	CBST005.04
9	Derive the finite element equation using the potential energy approach.	Apply	CBST005.04
10	State the principle of minimum potential energy .Explain the potential , with	Apply	CBST005.05

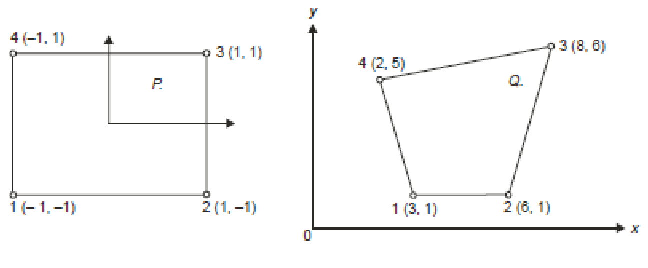
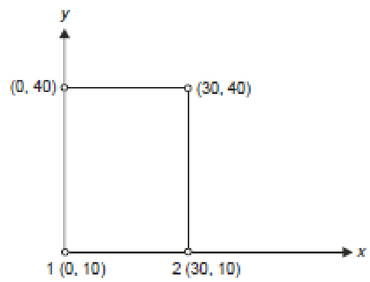
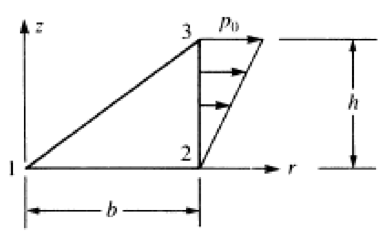
	usual notation		
11	Determine the deflection of cantilever beam of length l and loaded with vertical load p at the free end by Rayleigh Ritz method	Apply	CBST005.05
	Part - C (Problem Solving and Critical Thinking Questions)		
1	For the spring assemblage with arbitrarily numbered nodes shown in Figure obtain (a) the global stiffness matrix, (b) the displacements of nodes 3 and 4, (c) the reaction forces at nodes 1 and 2, and (d) the forces in each spring. A force of 5000 lb is applied at node 4 in the x direction. The spring constants are given in the figure. Nodes 1 and 2 are fixed. 	Analyze & evaluate	CBST005.01
2	A simply supported beam is subjected to uniformly distributed load over entire span. Determine the bending moment and deflection at the mid span using Rayleigh –Ritz method and compare with exact solution. Use a two term trial function y=a1sin (πx/l)+a2sin(3πx/l).	Analyze & evaluate	CBST005.02
3	A simply supported beam of span L, young's modulus, moment of inertia I is subjected to a uniformly distributed load of P/unit length. Determine the deflection W at the mid span. Use Rayleigh Ritz method.	Analyze & evaluate	CBST005.02
4	What are the steps involved in Rayleigh-Ritz method? Determine the displacement at midpoint and stress in linear one-dimensional rod as shown in figure. Use second degree polynomial approximation, for the displacement. 	Analyze & evaluate	CBST005.03
5	Determine the deflection of cantilever beam of length 'L and loaded with a vertical load P at the free end by Rayleigh-Ritz method. Use a trail function $y = a\left[1 - cos\left(\frac{\pi x}{2L}\right)\right]$	Analyze & evaluate	CBST005.03
6	For a simply supported Beam of uniformly distributed load of Intensity Po per unit length and a concentrated load P at center, Find the Transverse deflection using Raleigh-Ritz method of Functional Evaluation and compare the result with exact Analytical solution.	Analyze & evaluate	CBST005.04
7	In a plane stress problem σx= 60 MPa, σy= - 35 MPa, τ xy = 50 MPa, E = 200 , GPa, μ= 0.3. i) Determine strain component εz ii) If the problem is a case of plane strain case determine stress component σz	Analyze & evaluate	CBST005.04
8	Illustrate the Rayleigh-Ritz method in detail by applying it on an axially loaded bar at one end and fixed at one end as shown on 	Analyze & evaluate	CBST005.05
9	Derive the constitutive matrix for Plane Stress and Plane strain elements. Give at least two practical examples for Plane stress s and Plane strain analysis.	Analyze & evaluate	CBST005.05

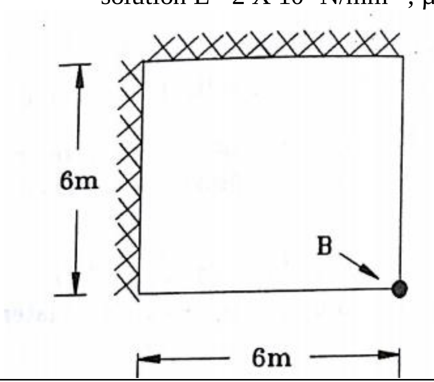
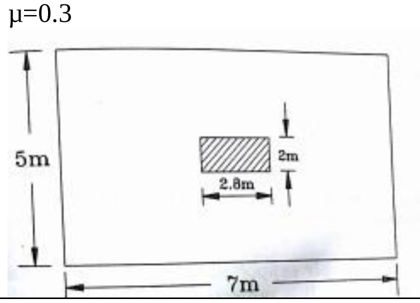
UNIT – II			
1D AND 2D FEM			
Part - A (Short Answer Questions)			
1	What are the properties of shape functions?	Remember	CBST005.06
2	Write down the shape functions for four noded rectangular elements?	Remember	CBST005.06
3	Write down the expression for stiffness matrix for 1D bar element.	Remember	CBST005.06
4	Write the natural co-ordinates for the point P of the triangular element. The point P is the C.G of the triangle.	Remember	CBST005.07
5	Write the shape function for constant strain triangle by using polynomial function?	Remember	CBST005.07
6	What are equivalent nodal forces?	Remember	CBST005.08
7	What is displacement and shape function?	Remember	CBST005.09
8	Determine the shape functions for a constant strain triangular element using area co-ordinates.	Remember	CBST005.09
9	Derive shape functions for a 2D beam element.	Remember	CBST005.09
10	Distinguish between 1D bar element and 1D beam element	Remember	CBST005.10
11	Write the shape function for constant strain triangle by using polynomial function?	Remember	CBST005.10
12	In an element the geometry is defined using 4 nodes and the displacement is defined using 8 nodes. What is this element called?	Remember	CBST005.11
Part - B (Long Answer Questions)			
1	Derive the strain-displacement matrix (B-matrix) for plane stress analysis of three node triangular element	Apply	CBST005.06
2	Determine nodal displacement , element stresses and support reactions of the axially loaded bar as shown in fig 	Apply	CBST005.06
3	The nodal coordinates of the triangular element are 1 (1,1), 2 (4,2), 3 (3,5). At the interior point P, the x coordinate is 3.5 and N1 is 0.4. Determine N2, N3 and y coordinate at point P.	Apply	CBST005.07
4	The nodal coordinates of the triangular element are 1 (1,1), 2 (4,2), 3 (3,5). At the interior point P, the x coordinate is 3.5 and N1 is 0.4. Determine N2, N3 and y coordinate at point P.	Apply	CBST005.07
5	For the three stepped bar shown in fig. 2, the fits snugly between the rigid walls at room temperature. The temperature is then raised by 300C. Determine the displacements at nodes 2 and 3, stresses in the three sections	Apply	CBST005.07
6	Drive the shape function for a Quadratic model for triangular element with neat sketch	Apply	CBST005.08
7	Derive the shape functions for ID cubic element. Shape functions should specified in both natural and global coordinate systems.	Apply	CBST005.09
8	Derive shape functions and their derivatives for a line element with quadratic interpolation function.	Apply	CBST005.10
9	A two noded line element with one translational degree of freedom is subjected to uniformly varying load of intensity P1 at node 1 and P2 at node 2. Evaluate the nodal load vector using numerical integration.	Apply	CBST005.11

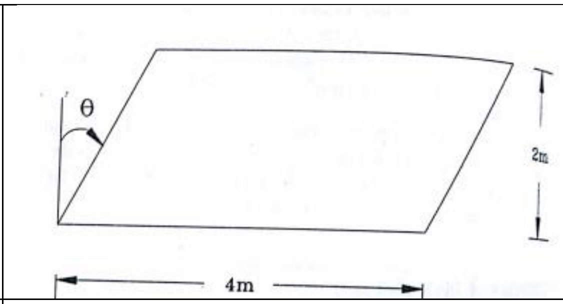
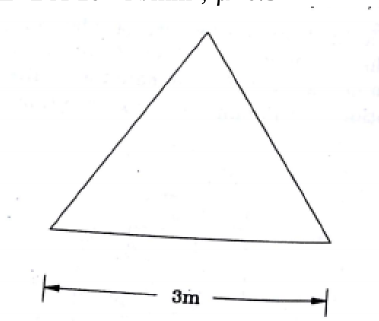
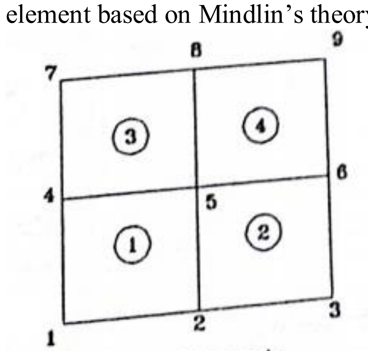
Part - C (Problem Solving and Critical Thinking Questions)			
1	For the stepped bar shown in the figure below, determine the nodal displacements, element stress and support reactions. Take P = 500 kN, E = 210 GPa, a1 = 200 mm2, a2 = 300 mm2 and a3 = 500 mm2. 	Analyze & evaluate	CBST005.06
2	Evaluate the shape functions N1, N2 and N3 at the interior point P for the triangular element shown in the figure below. 	Analyze & evaluate	CBST005.06
3	Find the nodal displacement, stresses and reaction of a Fig. use a penalty approach method 	Analyze & evaluate	CBST005.07
4	Derive the shape functions for element shown in fig. Shape functions should be specified in natural coordinate system. 	Analyze & evaluate	CBST005.07
5	Explain what you understand by convergence requirements; and conditions to be satisfied by the assumed displacement function. What are compatibility requirements and geometric isotropy?	Analyze & evaluate	CBST005.07
6	Explain formulation of one dimensional non prismatic bar element. Assume cross sectional dimensions b × b at node 1) And 1.5b ×1.5b at node 2) And straight	Analyze & evaluate	CBST005.08

<table>
<tr><td></td><td>edges connections the cross section as shown in figure(1)
</td><td></td><td></td></tr>
<tr><td>7</td><td>Determine the displacements and slopes at the nodes for the beam shown in figure. Find the moment at the midpoint of element 1.
</td><td>Analyze & evaluate</td><td>CBST005.09</td></tr>
<tr><td>8</td><td>Evaluate the shape functions N1, N2,N3 at the interior point P(3.85,4.8) for the triangular element shown in fig 1
</td><td>Analyze & evaluate</td><td>CBST005.10</td></tr>
<tr><td>9</td><td>For the three stepped bar shown in fig. 2, the fits snugly between the rigid walls at room temperature. The temperature is then raised by 300C. Determine the displacements at nodes 2 and 3, stresses in the three sections
</td><td>Analyze & evaluate</td><td>CBST005.11</td></tr>
<tr><td colspan="4">UNIT – III</td></tr>
<tr><td colspan="4">DIFFERENT FORMULATIONS AND 3D FEM</td></tr>
<tr><td colspan="4">Part - A (Short Answer Questions)</td></tr>
<tr><td>1</td><td>Briefly explain the types of elements based on geometry.</td><td>Remember</td><td>CBST005.12</td></tr>
<tr><td>2</td><td>List some disadvantages of using 3-D elements.</td><td>Remember</td><td>CBST005.12</td></tr>
<tr><td>3</td><td>What are the conditions for a problem to be axisymmetric?</td><td>Remember</td><td>CBST005.13</td></tr>
<tr><td>4</td><td>Explain Lagrange elements and Serendipity elements.</td><td>Remember</td><td>CBST005.13</td></tr>
</table>

5	Explain the isoparametric elements and its types.	Remember	CBST005.14
6	Obtain the shape functions of a nine noded quadrilateral element.	Remember	CBST005.14
7	State and explain the three basic laws on which isoparametric concept is developed	Remember	CBST005.15
8	Discuss the convergence criteria for isoparametric elements.	Remember	CBST005.16
8	Explain the terms isoparametric, sub parametric and super parametric elements	Remember	CBST005.17
9	Explain the isoparametric elements and their advantages	Remember	CBST005.18
10	What is the difference between natural coordinate and simple natural coordinate?	Remember	CBST005.18
	Part - B (Long Answer Questions)		
1	Derive the shape functions for the four noded quadrilateral isoparametric element and indicate the purpose for the computing its stiffness matrix.	Apply	CBST005.12
2	Write a note on isoparametric formulations and how the geometric as well as field variables are taken into account?	Apply	CBST005.12
3	Derive the shape functions for the four noded quadrilateral isoparametric element and indicate the purpose for the computing its stiffness matrix.	Apply	CBST005.13
4	Using the Lagrange interpolation formula construct the shape function in natural coordinate for one dimensional axial element with 4 nodes. Sketch the shape function.	Apply	CBST005.14
5	Explain with suitable examples why we resort to isoparametric transformation. Differentiate between isoparametric, sub parametric and super parametric elements.	Understand	CBST005.15
6	Derive the stress-strain relationship matrix (D) for the axisymmetric triangular element.	Understand	CBST005.16
7	Derive the element stiffness matrix for a four noded isoparametric plan stress element.	Understand	CBST005.17
8	Derive the shape functions for two noded one dimensional element using Lagrange interpolation formula	Understand	CBST005.18
	Part - C (Problem Solving and Critical Thinking Questions)		
1	Evaluate the Jacobian matrix at the local co-ordinates ζ, η are (0, 0) for the element shown in the below 	Apply	CBST005.13
2	For the isoparametric quadrilateral element shown in fig,determine a) Cartesian coordinate of the point P which has local coordinate ζ =0.57335 and η=0.57735. b) Local coordinate of the point Q which has Cartesian coordinate (7,4)	Apply	CBST005.14

			
3	For the element shown in the fig assemble Jacobian matrix and strain displacement matrix for the Gaussian point (0.57735, 0.57735) *Isoparan* 	Apply	CBST005.15
4	Explain finite element formulation for 8-noded isoperimetric solid element. Explain step by step procedure and elaborate all the steps.	Apply	CBST005.16
5	Evaluate the nodal forces used to replace the linearly varying surface traction shown in Figure 	Apply	CBST005.17
6	The nodal co-ordinates for an axisymmetric triangular element are given as: r1 = 0, r2 = 25, r3 = 30, z1 = 0, z2 = 0 and z3 = 40 mm respectively. Determine the strain-displacement matrix for the element	Apply	CBST005.18
UNIT – IV			
ANALYSIS OF PLATES			
Part - A (Short Answer Questions)			
1	State the assumption of Kirchhoff plate theory.	Remember	CBST005.19
2	Explain Mindlin's approximations for bending of plates in brief.	Remember	CBST005.19
3	Explain the term 'Shear locking'. How this problem is overcome?	Remember	CBST005.20
4	Give a brief account of classification of plates.	Remember	CBST005.20
5	What are the assumptions made in thin plates with small deflections?	Remember	CBST005.20
6	Discuss the use of triangular plate bending elements	Remember	CBST005.21
7	Draw the thin plate with dimension showing transverse loading.	Remember	CBST005.21
8	Give the relation between forces and stresses action on a thin plate.	Remember	CBST005.21
9	Define Shell and give the examples.	Remember	CBST005.21

	Part - B (Long Answer Questions)		
1	Derive element stiffness matrix typical for plate bending with neat figure.	Understand	CBST005.19
2	Derive the element stiffness matrix for Mindlin's plate element	Understand	CBST005.19
3	Explain the different classification of shells with neat sketches.	Understand	CBST005.19
4	Explain shear locking and describe the methods to prevent shear locking	Understand	CBST005.19
5	Discuss Love-Kirchhoff 's and Mindlin's plate bending theories in detail	Understand	CBST005.20
6	Write short note on finite elements for shell analysis.	Understand	CBST005.20
7	Write short notes on finite elements for plate analysis.	Understand	CBST005.21
8	Derive the 4 noded isoparametric element for plates by displacement method.	Understand	CBST005.21
9	Derive the elemental nodal load vector for four noded isoparametric quadrilateral plate element.	Understand	CBST005.21
10	Write short notes on numerical integration and stress smoothening in the case of four noded quadrilateral plate element	Understand	CBST005.21
	Part - C (Problem Solving and Critical Thinking Questions)		
1	Explain the term Mindlin's C^0-continuity plate element and briefly explain stiffness matrix formula for such elements	Apply	CBST005.19
2	Analysis the plate shown in fig. the two adjacent edges are fixed against rotation and translation, and the other two edges are free. Consider the following two load cases (a) Uniform load of 12 kn/m^2 (b) concentrated load of 10 KN at B. compare the result with analytical solution E= 2 X 10^5 N/mm ^{2} , μ=0.3, h=25 mm  	Apply	CBST005.19
3	Analysis the rectangular plate subjected to uniform load of 4 KN/m^2 acting over a rectangular area. All edges are simply supported, h=200 mm , E=2 X 10 ^{4} n/mm^2, μ=0.3  	Apply	CBST005.20
4	Figure show the simply supported skew plate and it is subjected to uniformly distributed load of 4 KN/m^2. Analyze the plate and compare the result with theoretical solution, h= 200 mm, E=2 X 10 ^{4} n/mm^2, μ=0.3,θ=30°.	Apply	CBST005.20

			
5	Derive an expression to compute the nodal load vector {Q} for a four noded element when it is subjected to varying pressure load. Indicate the numerical integration procedure that can be used for computation of {Q}	Apply	CBST005.21
6	Analyze a simply supported equilateral triangular plate shown in fig.It is subjected to an uniformly distributed load of 5 KN/m^2, h=25 mm, E=2 X 10^5 N/mm^2, μ=0.3 	Apply	CBST005.21
7	For the element shown in fig, derive the [B] matrix using 4 noded plate bending element based on Mindlin's theory.  (c) 4 Elements	Apply	CBST005.21
UNIT – V			
THEORY OF PLASTICITY			
Part - A (Short Answer Questions)			
1	What are the types of non-linearity in structural analysis?	Remember	CBST005.22
2	State the difference between Material non-linearity and Geometric non-linearity	Remember	CBST005.22
3	Discuss about Material and Geometric nonlinearity.	Remember	CBST005.22
4	Give two examples of geometric nonlinear problems?	Remember	CBST005.22
5	How is geometry nonlinearity taken care in finite element analysis?	Remember	CBST005.22
6	Define contact nonlinearity.	Remember	CBST005.22
Part - B (Long Answer Questions)			

1	Describe Newton – Raphson iteration technique for solving material non-linearity problems.	Understand	CBST005.22
2	Explain the solution methods for nonlinear algebraic equations.	Understand	CBST005.22
3	Discuss about Material and Geometric nonlinearity.	Understand	CBST005.22
4	Explain incremental procedure to handle material non – linear problems.	Understand	CBST005.22
5	Explain mid-point Runge kutta incremental procedure and discuss its advantage and disadvantage	Understand	CBST005.22
6	Explain iterative procedure and modified iterative procedure for the analysis of material Non-linearity problems.	Understand	CBST005.22
7	Explain the iterative procedure of handling geometric non-linearity problems in structural mechanics.	Understand	CBST005.22
8	Explain the solution methods for nonlinear algebraic equations.	Understand	CBST005.22

Prepared By: Dr. M. Venu, Professor

HEAD
DEPARTMENT OF CIVIL ENGINEERING