



## INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad-500043

### CIVIL ENGINEERING

#### TUTORIAL QUESTION BANK

<b>Course Name</b>	:	<b>FINITE ELEMENT METHOD IN STRUCTURAL ENGINEERING</b>
<b>Course Code</b>	:	<b>BST001</b>
<b>Class</b>	:	M. Tech(Structural Engineering) – II semester
<b>Department</b>	:	Civil Engineering
<b>Academic Year</b>	:	2018–2019
<b>Course Faculty</b>	:	Mr. Gude Rama Krishna, Professor

#### I. COURSE OVERVIEW:

The Finite Element Method (FEM) is widely used in industry for analysing and modelling structures and continua, whose physical behaviour is described by ordinary and partial differential equations. The FEM is particularly useful for engineering problems that are too complicated to be solved by classical analytical methods. The main objective of this course is to introduce the mathematical concepts of the Finite Element Method for obtaining an approximate solution of ordinary and partial differential equations. In this course you will attend lectures on the fundamentals of the Finite Element Method. The learning process will be enhanced by completing assignments using mathematical software. You will also be introduced to a commercial Finite Element software package – ANSYS – during lectures with computer laboratories providing opportunities to practice on, and to complete practical assignments, using ANSYS.

#### II. COURSE OBJECTIVES:

The course should enable the students to:

I	Equip the students with the Finite Element Analysis fundamentals.
II	Enable the students to formulate the design problems into Finite Element Method (FEM).
III	Develop the ability to generate the governing finite element equations for systems.
IV	Develop the ability to generate the governing finite element equations for systems.
V	Understand the use of the basic finite elements for structural applications using truss, beam, frame and plate elements.

#### COURSE LEARNING OUTCOMES:

Students who complete the course, will have demonstrated the ability to do the following:

CLO Code	CLO's	At the end of the course, the student will have the ability to:	PO's Mapped	Strength of Mapping
CBST001.01	CLO 1	Understand the Concepts of FEM, steps involved merits and demerits.	PO 1	3
CBST001.02	CLO 2	Solve the problems using Raleigh-Ritz method of functional approximation.	PO 1	3

CBST001.03	CLO 3	Understand the concept of One dimensional FEM Stiffness matrix for beam and bar elements.	PO 1	3
CBST001.04	CLO 4	Know the generalized coordinates, shape functions.	PO 3	2
CBST001.05	CLO 5	Different types of elements for plane stress and plane strain analysis, displacement models.	PO 3	2
CBST001.06	CLO 6	Generalized coordinates, shape functions, convergent and compatibility requirements.	PO 4	2
CBST001.07	CLO 7	Understand the generation of element stiffness and nodal load matrices.	PO 4	2
CBST001.08	CLO 8	Know the natural coordinate system, area and volume coordinates.	PO 4	2
CBST001.09	CLO 9	Concept of Axisymmetric bodies of revolution, axisymmetric modeling.	PO 1	2
CBST001.10	CLO 10	Know the different 3-D elements strain, displacement relationship.	PO 4	2
CBST001.11	CLO 11	Concept of Axisymmetric bodies of revolution, axisymmetric modeling.	PO 1	3
CBST001.12	CLO 12	Understand the strain displacement relationship, formulation of axisymmetric elements.	PO 4	3
CBST001.13	CLO 13	Concept of isoparametric formulation, different isoparametric elements for 2D analysis.	PO 4	2
CBST001.14	CLO 14	Concept of Axisymmetric bodies of revolution, axisymmetric modeling.	PO 4	3
CBST001.15	CLO 15	Understand the strain displacement relationship, formulation of axisymmetric elements.	PO 3	2
CBST001.16	CLO 16	Concept of Plane Stress, CST Element, Plane Strain Rectangular Element	PO 3	3
CBST001.17	CLO 17	Understand the Principle Isoperimetric Formulation of the Plane Quadrilateral Element, Axi-Symmetric element.	PO 1, PO 4	2
CBST001.18	CLO 18	Introduction to Computer Implementation of FEM ,use of commercial FEA Software.	PO 1, PO 3	2

## TUTORIALQUESTIONBANK

UNIT-I			
<b>INTRODUCTION</b>			
<b>Part-A(ShortAnswerQuestions)</b>			
1	Briefly explain about Finite Element Method?	Remember	CBST001.01
2	Define about volume co-ordinates.	Remember	CBST001.02
3	What is Raleigh-Ritz method Finite Element Method?	Remember	CBST005.03
4	What are the steps involved in Finite Element Method?	Remember	CBST001.02
5	Define Degree of freedom and principle of virtual work.	Remember	CBST001.04
6	Explain about potential energy principle.	Remember	CBST001.02
7	Explain about discretization of finite element method?	Remember	CBST001.02
8	Explain about discretization of finite element method?	Remember	CBST001.01
9	Distinguish between the problems of 'Plane stress' and 'Plane strain'	Remember	CBST001.02
<b>Part-B(LongAnswerQuestions)</b>			
1	Find out deflection at centre of a simply supported beam of span length(L) subjected to uniformly distributed load throughout its length of intensity w per	Apply	CBST001.01

	unit length. Use Rayleigh Ritz method. Take EI is constant		
2	Write the potential energy for beam of span 'L' simply supported at ends, subjected to a concentrated load 'P' at mid span. Assume EI is constant	Apply	CBST001.01
3	Solve the following differential equation using Ritz method. $d^2y/dx^2 = \sin(\pi x)$ boundary condition $u(0)=0$ and $u(1)=0$	Apply	CBST001.02
3	Using Rayleigh Ritz method, find the maximum deflection of simply supported beam with point load at center.	Apply	CBST001.02
5	Determine the deflection of cantilever beam of length 'L' and loaded with a vertical load at the free end by Rayleigh-Ritz method. Use a trial function	Apply	CBST001.03
6	Determine the deflection of cantilever beam of length 'L' and loaded with vertical load at the free end by Rayleigh Ritz method	Apply	CBST001.04
7	Derive the finite element equation using the potential energy approach.	Apply	CBST001.04
8	State the principle of minimum potential energy. Explain the potential, with usual notation	Apply	CBST001.05
9	If a displacement field is described by $u = (x^2 - 2y^2 + 6xy)10^{-4}$ and $v = (6x + 3y)10^{-4}$ . Determine $\epsilon_x, \epsilon_y$ and $\gamma_{xy}$ at the point $x = 2$ and $y = 1$ .	Apply	CBST001.01

### Part-C (Problem Solving and Critical Thinking Questions)

1	For the spring assembly with arbitrarily numbered nodes shown in Figure obtain (a) the global stiffness matrix, (b) the displacements of nodes 3 and 4, (c) the reaction forces at nodes 1 and 2, and (d) the force in each spring. A force of 5000 lb is applied at node 4 in the x direction. The spring constants are given in the figure. Nodes 1 and 2 are fixed.	Analyze & evaluate	CBST001.01
	<p style="text-align: center;"><math>k_1 = 1000 \text{ lb/in.}</math>    <math>k_2 = 2000 \text{ lb/in.}</math>    <math>k_3 = 3000 \text{ lb/in.}</math></p>		
2	Obtain stiffness matrix for truss element with 2-degree of freedom.	Analyze & evaluate	CBST001.01
3	A simply supported beam of span L, young's modulus, moment of inertia I is subjected to a uniformly distributed load of P/unit length. Determine the deflection W at the midspan. Use Rayleigh Ritz method.	Analyze & evaluate	CBST001.01
4	What are the steps involved in Rayleigh-Ritz method? Determine the displacement at midpoint and stress in linear one-dimensional rod as shown in figure. Use second degree polynomial approximation, for	Analyze & evaluate	CBST001.03
5	Determine the deflection of cantilever beam of length 'L' and loaded with a vertical load P at the free end by Rayleigh-Ritz method. Use a trial function	Analyze & evaluate	CBST001.03
	$y = a \left[ 1 - \cos \left( \frac{\pi x}{2L} \right) \right]$		

6	For a simply supported beam of uniformly distributed load of intensity $P$ per unit length and a concentrated load $P$ at center, find the transverse deflection using Raleigh-Ritz method of functional evaluation and compare the result with exact analytical solution.	Analyze & evaluate	CBST001.04
7	In a plane stress problem $\sigma_x = 60 \text{ MPa}$ , $\sigma_y = -35 \text{ MPa}$ , $\tau_{xy} = 50 \text{ MPa}$ , $E = 200 \text{ GPa}$ , $\mu = 0.3$ . i) Determine strain components ii) If the problem is a case of plane strain case determine stress component $\sigma_z$	Analyze & evaluate	CBST001.04
8	Illustrate the Rayleigh-Ritz method in detail by applying it on an axially loaded bar at one end and fixed at one end as shown in fig.	Analyze & evaluate	CBST001.05
9	Derive the constitutive matrix for Plane Stress and Plane strain elements. Give at least two practical examples for Plane stress and Plane strain analysis.	Analyze & evaluate	CBST001.05

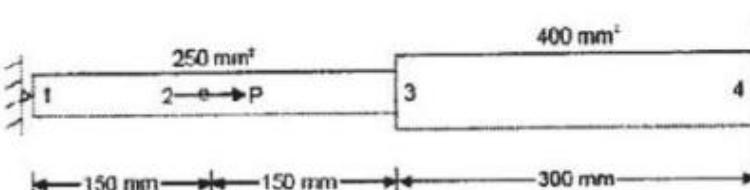
## UNIT-II

### BEAM ELEMENTS

#### Part-A(Short Answer Questions)

1	What are the properties of shape functions?	Remember	CBST001.06
2	Write the shape function for constant strain triangle by using polynomial function?	Remember	CBST001.06
3	Write down the shape functions for four node rectangular elements?	Remember	CBST001.06
4	What is displacement and shape function?	Remember	CBST001.07
5	Derive shape functions for a 2D beam element.	Remember	CBST001.07
6	Distinguish between 1D bar element and 1D beam element	Remember	CBST001.09
7	Write the shape function for constant strain triangle by using polynomial function?	Remember	CBST001.09
8	Write the shape function for constant strain triangle by using polynomial function?	Remember	CBST001.10
9	Write the natural coordinates for the point P of the triangular element. The point P is the C.G. of the triangle.	Remember	CBST001.11

#### Part-B(Long Answer Questions)

1	Derive the strain-displacement matrix (B-matrix) for plane stress analysis of three node triangular element	Apply	CBST001.06
2	Determine nodal displacement, element stresses and support reactions of the axially loaded bar as shown in fig	Apply	CBST001.06
			
3	The nodal coordinates of the triangular element are 1(1,1), 2(4,2), 3(3,5). At the interior point P, the x-coordinate is 3.5 and N1 is 0.4. Determine N2, N3 and y-coordinate at point P.	Apply	CBST001.07
4	The nodal coordinates of the triangular element are 1(2,2), 2(5,3), 3(6,7). At the interior point P, the x-coordinate is 3.5 and N1 is 0.4. Determine N2, N3 and y-coordinate at point P.	Apply	CBST001.07
5	Explain steps involved or procedure to obtain general stiffness matrices and strain-displacement relation.	Apply	CBST001.07
6	Derive the shape function for a Quadratic model for triangular element with neat sketch	Apply	CBST001.08

7	Derive the shape functions for 1D cubic element. Shape functions should be specified in both natural and global coordinate systems.	Apply	CBST001.09
8	Derive shape functions and their derivatives for a line element with quadratic interpolation function.	Apply	CBST001.10
9	A two-node element with one translational degree of freedom is subjected to uniform varying load of intensity $P_1$ at node 1 and $P_2$ at node 2. Evaluate the nodal load vector using numerical integration.	Apply	CBST001.11

**Part-C (Problem Solving and Critical Thinking Questions)**

1	For the stepped bar shown in the figure below, determine the nodal displacements, element stress and support reactions. Take $P=500\text{ kN}$ , $E=210\text{ GPa}$ , $a_1=200\text{ mm}^2$ , $a_2=300\text{ mm}^2$ and $a_3=500\text{ mm}^2$ .	Analyze & evaluate	CBST001.06
2	Evaluate the shape functions $N_1$ , $N_2$ and $N_3$ at the interior point $P$ for the triangular element shown in the figure below.		CBST001.06
3	Find the nodal displacement, stresses and reaction of a Fig. use a penalty approach method	Analyze & evaluate	CBST001.07
4	Explain what you understand by convergence requirements, and conditions to be satisfied by the assumed displacement function. What are compatibility requirements and geometric isotropy?	Analyze & evaluate	CBST001.07
5	Explain formulation of one-dimensional non-prismatic beam element. Assume cross-sectional dimensions $b \times 1.5b$ at node 1) And $1.5b \times 1.5b$ at node 2) And straight edges connection the cross section as shown in figure	Analyze & evaluate	CBST001.08
6	Determine the displacements and slopes at the nodes for the beam shown in figure. Find		

	the moment at the mid point of element 1.		
7	Evaluate the shape functions $N_1, N_2, N_3$ at the interior point $P(3.85, 4.8)$ for the triangular elements shown in fig		
8	For the three stepped bar shown in fig. 2, the fit is snugly between the rigid walls at room temperature. The temperature is then raised by $30^\circ\text{C}$ . Determine the displacements at nodes 2 and 3, stresses in the three sections	<p>Aluminum: <math>E = 70 \text{ GPa}</math>, <math>A = 800 \text{ mm}^2</math>, <math>\alpha = 23 \times 10^{-6}/^\circ\text{C}</math> Brass: <math>E = 105 \text{ GPa}</math>, <math>A = 500 \text{ mm}^2</math>, <math>\alpha = 19 \times 10^{-6}/^\circ\text{C}</math> Steel: <math>E = 200 \text{ GPa}</math>, <math>A = 300 \text{ mm}^2</math>, <math>\alpha = 12 \times 10^{-6}/^\circ\text{C}</math></p>	
9	Using 2 DOF truss element determine the horizontal and vertical displacement of joints and the forces in all the members of the plane pin jointed frame shown in fig. joints B,C,D are at the same level as shown in figure, c/s area = $2400 \text{ mm}^2$ , $E=2 \times 10^5 \text{ N/mm}^2$		

### UNIT-III

#### METHOD OF WEIGHTED RESIDUALS

##### Part-A(Short Answer Questions)

1	Briefly explain the types of elements based on geometry.	Remember	CBST001.12
2	List some disadvantages of using 3-D elements.	Remember	CBST001.12
3	What are the conditions for a problem to be axisymmetric?	Remember	CBST001.13

4	Explain Lagrange elements and Serendipity elements.	Remember	CBST001.13
5	Explain the isoparametric elements and its types	Remember	CBST001.14
6	Obtain the shape functions of a nine noded quadrilateral element.	Remember	CBST001.14
7	State and explain the three basic laws on which isoparametric concept is developed	Remember	CBST001.16
9	Explain the isoparametric elements and their advantages	Remember	CBST001.17
9	Define drilling degree of freedom.	Remember	CBST001.18

### Part-B(Long Answer Questions)

1	Derive the shape functions for the four noded quadrilateral isoparametric element and indicate the purpose for the computing its stiffness matrix.	Apply	CBST001.12
2	Write an note on isoparametric formulations and how the geometric as well as field variables are taken into account	Apply	CBST001.12
3	Derive the shape functions for the four noded quadrilateral isoparametric element and indicate the purpose for the computing its stiffness matrix	Apply	CBST001.13
4	Using the Lagrange interpolation formula construct the shape function in natural coordinate for one dimensional axial element with 4 nodes. Sketch the shape function.	Apply	CBST001.14
5	Explain with suitable examples why we resort to isoparametric transformation. Differentiate between isoparametric, subparametric and superparametric elements.	Understand	CBST001.15
6	Derive the stress-strain relationship matrix (D) for the axisymmetric triangular element.	Understand	CBST001.16
7	Derive the element stiffness matrix for a four noded isoparametric plan stress element.	Understand	CBST001.17
8	Derive the shape functions for two noded one dimensional element using Lagrange interpolation formula	Understand	CBST001.18

### Part-C(Problem Solving and Critical Thinking Questions)

1	Evaluate the Jacobian matrix at the local coordinates $\zeta, \eta$ are (0,0) for the element shown in the below	Apply	
2	Explain finite element formulation for 8-noded isoparametric solid element. Explain step by step procedure and elaborate all the steps	Apply	
3	Evaluate the nodal forces used to replace the linearly varying surface tractions shown in Figure	Apply	
4	a) Explain about plain stress and plane strain in finite element method. b) Explain about drilling degree of freedom and interpolation constant.		
5	Explain briefly about material and geometric non-linearities		
6	Obtain the formulation finite element method using principle of virtual work.		

UNIT-IV			
TYPES			
<b>Part-A(ShortAnswerQuestions)</b>			
1	What are the required conditions for a problem to be axisymmetric, and explain.	Remember	CBST001.19
2	Explain about Interpolation or shape function and polynomial shape function	Remember	CBST001.19
3	Explain about plain stress triangles	Remember	CBST001.20
4	What are the types of elements used in finite element analysis.	Remember	CBST001.20
5	What are the advantages and disadvantages of finite element method	Remember	CBST001.20
6	Write on area coordinates and volume coordinates.	Remember	CBST001.21
7	Explain about plain stress and plain strain condition	Remember	CBST001.21
8	Explain about Von Mises stresses.	Remember	CBST001.21
9	What are the interpolation functions.		
<b>Part-B(LongAnswerQuestions)</b>			
1	Derive general stiffness matrix for 4-noded quadrilateral element.		
2	Derive Jacobian matrix for 4-noded Isoparametric quadrilateral two-dimensional element.		
3	Explain Reliegh – Ritz method and obtain strain displacement relation matrix.		
4	Obtain equilibrium equation for 3D elastic medium and in 2D elastic medium.		
5	Explain about steps involved in finite element analysis with examples.		
6	Obtain the formulation of finite element method using principle of virtual work.		
7	What are the isoparametric elements, derive the B matrix for the 4-noded Isoparametric quadrilateral element.		
8	a) Write down finite element analysis procedure and engineering applications. b) Explain about plane stress triangle.		
<b>Part-C(ProblemSolvingandCriticalThinkingQuestions)</b>			
1	Obtain strain displacement relation for the non-zero strains and explain about various Axisymmetric elements.	Apply	
2	Obtain the stiffness matrix for three 3-noded triangular element with nodes shown   for plane strain saturation (Cartesian co-ordinates) $E = 1.2 \times 10^6 \text{ Kg/cm}^2$	Apply	
3	Determine the shape functions $N_1, N_2$ and $N_3$ for the triangular element with coordinates (2,3), (5,7) and (4,5) all the units are in meters. Hence determine the value of $N_1, N_2, N_3$ at centroid of the element.	Apply	
4	An axisymmetric solid subjected to axisymmetric loading. Obtain stress strain displacement matrix and potential energy of two-dimensional region defined by the revolving area is divided into triangular element.	Apply	
UNIT-V			
APPLICATION TO SOLID MECHANICS & COMPUTER IMPLEMENTATION			
<b>Part-A(ShortAnswerQuestions)</b>			

1	Explain about various axisymetryc element	Remember	CBST001.22
2	What are the isoparametric elements and advantages.	Remember	CBST001.22
3	DiscussaboutMaterialandGeometricnonlinearity.	Remember	CBST001.22
4	Give two examples of geometric nonlinear problems.	Remember	CBST001.22
5	Explain about super isoparametric and sub isoparametric element.	Remember	CBST001.22
6	Explain about interpolation and shape function.	Remember	CBST001.22
7	Explain about Von mises stress and Volume Co-ordinates.	Remember	CBST001.22

#### **Part-B(LongAnswerQuestions)**

1	Generate stiffness matrix for 4 noded tetrahedral element(3D problem).	Understand	CBST001.22
2	Obtain Generate stiffness matrix for 4 noded Qudrilateral element.	Understand	CBST001.22
3	Derive element stiffnesss matrices for an axisymmetric triangular ring element.	Understand	CBST001.22
4	Obtain Generate stiffness matrix for 3 noded triangular element for a plane strain saturation condition(Cartition co-ordinates) $E=1.2 \times 10^6 \text{ Kg/cm}^2$ and $\mu=0.15$	Understand	CBST001.22
5	a)What are the various applications of finite element methods n various engineering fields.List out any four softwares related to finite element method	Understand	CBST001.22
6	Obtain: a)strain-displacement relation in 2d problem. b)plain strain 2D problem	Understand	CBST001.22

#### **Part-C(ProblemSolvingandCriticalThinkingQuestions)**

1	A quadrilateral finite element has nodes at 1(1,1), 2(4,0), 3(5,4) and 4(0,3)determine $\frac{\partial N_1}{\partial x}$ and $\frac{\partial N_2}{\partial Y}$ at location $\xi=0, \eta=0$ where $\xi$ and $\eta$ are natural co ordinates	Apply	CBST001.23
2	Derive B matrix (strain –displacement) for a CST(Constant Strain Triangle) element	Apply	CBST001.23
3	What are the isoparametric element derive the B-matrics for 4 nodeed Isoparametric quadrilateral element	Understand	CBST001.23
4	What are the steps involved and uses of general purpose Finite element analysis software	Understand	CBST001.23
5	a)describe properties of shape function b)write short notes on: i)Lagrangian Elements ii)Serendipity Elements	Understand	CBST001.23
6	a)Explain various axi-symmetric elements. b)Differentiate between isoparametric,super parametric and sub parametric elements.	Understand	CBST001.23
7	a)Explain briefly non-linear analysis,with one application. b)What are the different methods to solve a non-linear problem.	Understand	CBST001.23