## INSTITUTEOFAERONAUTICALENGINEERING

(Autonomous)
Dundigal, Hyderabad - 500043

## AERONAUTICAL ENGINEERING

TUTORIAL QUESTION BANK

| Course Name | $:$ | Complex Analysis and Probability Distribution |
| :--- | :---: | :--- |
| Course Code | $:$ | AHS004 |
| Class | $:$ | II B. Tech II Semester |
| Branch | $:$ | AERO |
| Academic Year | $:$ | $2018-2019$ |
| CourseCoordinator | $:$ | Ms. C Rachana, Assistant Professor |
| Course Faculty | $:$ | Mr. Ch Soma shekhar, Assistant Professor <br> Mr. J Suresh Goud, Assistant Professor <br> Ms. P Rajani, Assistant Professor |

COURSE OBJECTIVES (COs):
The course should enable the students to:

| I | Understand the basic theory of complex functions to express the power series. |
| :---: | :--- |
| II | Evaluate the contour integration using Cauchy residue theorem. |
| III | Enrich the knowledge of probability on single random variables and probability <br> distributions. |

COURSE LEARNING OUTCOMES (CLOs):
Students, who complete the course, will have demonstrated the ability to do the following:

| CAHS004.01 | Define continuity, differentiability, analyticity of a function using limits. |
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| CAHS004.02 | Understand the conditions for a complex variable to be analytic and/or entire <br> function. |
| CAHS004.03 | Understand the concepts of Cauchy-Riemann relations and harmonic functions. |
| CAHS004.04 | Understand the concept of complex differentiation to the real-world problems of <br> signals modulated by electromagnetic waves. |
| CAHS004.05 | Evaluate the area under a curve using the concepts of indefinite integration |
| CAHS004.06 | Understand the concepts of the Cauchy's integral formula and the generalized <br> Cauchy's integral formula. |
| CAHS004.07 | Evaluate complex functions as power series and radius of convergence of power <br> series. |
| CAHS004.08 | Understand the concept of complex integration to the real-world problems of flow <br> with circulation around a cylinder. |
| CAHS004.09 | Solve the Taylor's and Laurent series expansion of complex functions |
| CAHS004.10 | Understand the concept of different types of singularities for analytic function. |


| CAHS004.11 | Evaluate poles, residues and solve integrals using Cauchy's residue theorem. |
| :--- | :--- |
| CAHS004.12 | Evaluate bilinear transformation by cross ratio property. |
| CAHS004.13 | Identify the conditions of fixed and critical point of Bilinear Transformation. |
| CAHS004.14 | Understand the concept of Cauchy's residue theorem to the real-world problems <br> of Quantum Mechanical scattering and Quantum theory of atomic collisions. |
| CAHS004.15 | Demonstrate an understanding of the basic concepts of probability and random <br> variables. |
| CAHS004.16 | Classify the types of random variables and calculate mean, variance. |
| CAHS004.17 | Finding moment about origin, central moments, moment generating function of <br> probability distribution. |
| CAHS004.18 | Understand the concept of random variables to the real-world problems like graph <br> theory, machine learning and natural language processing |
| CAHS004.19 | Recognize where the binomial distribution and poisson distribution could be <br> appropriate model and find mean, variance of the distributions. |
| CAHS004.20 | Apply the inferential methods relating to the means of normal distributions. |
| CAHS004.21 | Understand binomial distribution to the phenomena of real-world problem like <br> sick versus healthy. |
| CAHS004.22 | Understand the mapping of normal distribution in real-world problem to analyze <br> the stock market. |
| CAHS004.23 | Use poission distribution in real-world problem to predict soccer scores. |
| CAHS004.24 | Possess the knowledge and skills for employability and to succeed in national and <br> international level competitive examinations. |

## TUTORIAL QUESTION BANK

| UNIT-I |  |  |  |
| :---: | :---: | :---: | :---: |
| COMPLEX FUNCTIONS AND DIFFERENTIATION |  |  |  |
| Part - A(Short Answer Questions) |  |  |  |
| $\begin{aligned} & \text { S } \\ & \text { No } \end{aligned}$ | QUESTIONS | Blooms Taxonomy Level | Course <br> Learning <br> Outcomes (CLOs) |
| 1 | Define the term Analyticity of a complex variable function f (z). | Remember | CAHS004.1 |
| 2 | Define the term Continuity of a complex variable function $f(z)$. | Remember | CAHS004.1 |
| 3 | Define the term Differentiability of a complex variable function $\mathrm{f}(\mathrm{z})$. | Remember | CAHS004.1 |
| 4 | If $w=f(z)=z^{2}+z$. Find its real and imaginary parts. | Remember | CAHS004.1 |
| 5 | Examine the complex variable function $\mathrm{f}(\mathrm{z})=z^{3}$ to analyticity for all values of z in Cartesian form. | Understand | CAHS004.2 |
| 6 | Verify whether the function $v=x^{3} y-x y^{3}+x y+x+y$ can be imaginary part of an analytic function $\mathrm{f}(\mathrm{z})$ where $\mathrm{z}=\mathrm{x}+\mathrm{iy}$. | Understand | CAHS004.2 |
| 7 | Show that the function $\mathrm{f}(\mathrm{z})=\|z\|^{2}$ does not satisfy Cauchy-Riemann equations in Cartesian form. | Understand | CAHS004.3 |
| 8 | Examine the complex variable function $\mathrm{f}(\mathrm{z})=\frac{x-i y}{x^{2}+y^{2}}$ for analyticity in Cartesian form. | Understand | CAHS004.2 |
| 9 | Interpret whether the function $\mathrm{f}(\mathrm{z})=\sin \mathrm{x}$ siny $-\mathrm{i} \cos \mathrm{x}$ cosy is an analytic function or not in Cartesian form. | Understand | CAHS004.2 |
| 10 | Calculate the value of k such that $\mathrm{f}(\mathrm{x}, \mathrm{y})=x^{3}+3 k x y^{2}$ may be harmonic function. | Understand | CAHS004.3 |
| 11 | Determine the most general analytic function $\mathrm{f}(\mathrm{z})$ whose real part of the analytic function is $\mathrm{u}=x^{2}-y^{2}-x$. | Understand | CAHS004.2 |
| 12 | Obtain an analytic function $\mathrm{f}(\mathrm{z})$ whose imaginary part of the analytic function is $\mathrm{v}=e^{x}(x \sin y+y \cos y)$. | Understand | CAHS004.2 |
| 13 | Show that the real part of an analytic function $\mathrm{f}(\mathrm{z})$ where $\mathrm{u}=2 \log \left(x^{2}+y^{2}\right)$ is harmonic. | Understand | CAHS004.3 |
| 14 | Determine the conjugate harmonic function if the real part of an analytic function $f(z)$ is $u=y^{2}-3 x^{2} y$ is harmonic function. | Understand | CAHS004.3 |
| 15 | Estimate the values of w which correspond to $\mathrm{z}=1+3 \mathrm{i}$ when $\mathrm{w}=\mathrm{f}(\mathrm{z})=\mathrm{z}^{2}$. | Understand | CAHS004.3 |
| 16 | Show that the function $\mathrm{f}(\mathrm{z})=\|z\|^{2}$ is continuous at all points of z but not differentiable at any $z \neq 0$. | Understand | CAHS004.1 |
| 17 | Calculate all the values of k such that $\mathrm{f}(\mathrm{z})=e^{x}(\cos k y+i \sin k y)$ is an analytic function. | Understand | CAHS004.2 |
| 18 | Determine the values of $a, b, c$ such that $f(z)=x+a y-i(a x+b y)$ is differentiable function at every point. | Understand | CAHS004.1 |
| 19 | Verify whether $\mathrm{u}=x^{2}-y^{2}-y$ of an analytic function can be harmonic function of an analytic function $\mathrm{f}(\mathrm{z})$ in the whole complex plane. | Understand | CAHS004.3 |
| 20 | Justify whether every differentiable function is continuous or not. Give a valid example. | Remember | CAHS004.1 |
| Part - B (Long Answer Questions) |  |  |  |
| 1 | Show that the real part of an analytic function $f(z)$ where | Understand | CAHS004.3 |


|  | $\mathrm{u}=e^{-2 x y} \sin \left(x^{2}-y^{2}\right)$ is a harmonic function. Hence find its harmonic conjugate. |  |  |
| :---: | :---: | :---: | :---: |
| 2 | Prove that the real part of analytic function $\mathrm{f}(\mathrm{z})$ where $\mathrm{u}=\log \|z\|^{2}$ is harmonic function. If so find the analytic function by Milne Thompson method. | Understand | CAHS004.3 |
| 3 | Determine the imaginary part of an analytic function $\mathrm{f}(\mathrm{z})$ whose real part of an analytic function is $e^{x}(x \cos y-y \sin y)$. | Understand | CAHS004.2 |
| 4 | Obtain the regular function $\mathrm{f}(\mathrm{z})$ whose imaginary part of an analytic function is $\frac{x-y}{x^{2}+y^{2}}$. | Understand | CAHS004.2 |
| 5 | If $f(z)=u+i v$ is an analytic function of $z$, then calculate $f(z)$ if $2 u+v=e^{2 x}[(2 x+y) \cos 2 y+(x-2 y) \sin 2 y]$. | Understand | CAHS004.2 |
| 6 | Prove that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)\|\operatorname{Real} f(z)\|^{2}=2\left\|f^{\prime}(z)\right\|^{2} \quad$ where $\mathrm{w}=\mathrm{f}(\mathrm{z})$ is an analytic function. | Understand | CAHS004.2 |
| 7 | Find an analytic function $f(z)$ whose real part of an analytic function is $\mathrm{u}=\frac{\sin 2 x}{\cosh 2 y-\cos 2 x}$ by Milne-Thompson method. | Understand | CAHS004.3 |
| 8 | If $\mathrm{f}(\mathrm{z})$ is a regular function of z , then prove that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)\|f(z)\|^{2}=$ $4\left\|f^{\prime}(z)\right\|^{2}$. | Understand | CAHS004.2 |
| 9 | Show that the function defined by $\mathrm{f}(\mathrm{z})= \begin{cases}\frac{x y^{2}(x+i y)}{x^{2}+y^{4}}, & z \neq 0 \\ 0 & , z=0\end{cases}$ is not analytic function even though Cauchy Riemann equations are satisfied at origin. | Understand | CAHS004.3 |
| 10 | Show that real part $\mathrm{u}=x^{3}-3 x y^{2}$ of an analytic function $\mathrm{f}(\mathrm{z})$ is harmonic. Hence find the conjugate harmonic function and the analytic function. | Understand | CAHS004.3 |
| 11 | Find an analytic function $\mathrm{f}(\mathrm{z})=\mathrm{u}+\mathrm{iv}$ if the real part of an analytic function is $u=a(1+\cos \theta)$ using Cauchy-Riemann equations in polar form. | Understand | CAHS004.3 |
| 12 | Derive Cauchy-Riemann equations in polar form of an analytic function f (z). | Remember | CAHS004.3 |
| 13 | Prove that the real and imaginary parts of an analytic function $\mathrm{f}(\mathrm{z})$ are harmonic. | Remember | CAHS004.3 |
| 14 | Find the analytic function $\mathrm{f}(\mathrm{z})$ whose imaginary part of an analytic function is $r^{2} \cos 2 \theta+r \sin \theta$ by Cauchy Riemann equations in polar form. | Understand | CAHS004.3 |
| 15 | Prove that the function $\mathrm{f}(\mathrm{z})=\|z\|$ is continuous every where but nowhere differentiable. | Remember | CAHS004.1 |
| 16 | Show that the real part of an analytic function $\mathrm{f}(\mathrm{z})$ where $\mathrm{u}=e^{-x}(x \sin y-y \cos y)$ is a harmonic function. | Understand | CAHS004.3 |
| 17 | Prove that an analytic function $\mathrm{f}(\mathrm{z})$ with constant real part is always constant. | Remember | CAHS004.2 |
| 18 | Prove that an analytic function $\mathrm{f}(\mathrm{z})$ with constant modulus is always constant. | Remember | CAHS004.2 |
| 19 | Verify Cauchy-Riemann equation to the function $\mathrm{f}(\mathrm{z})=z e^{-z}$ in Cartesian form. | Understand | CAHS004.3 |
| 20 | If $u$ and $v$ are conjugate harmonic functions then show that $u v$ is also a harmonic function. | Remember | CAHS004.3 |
| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |
| 1 | If $\mathrm{f}(\mathrm{z})$ is an analytic function of z such that $\mathrm{u}+\mathrm{v}=\frac{\sin 2 x}{\cosh 2 y-\cos 2 x}$ then determine the analytic function $\mathrm{f}(\mathrm{z})$ in terms of z . | Understand | CAHS004.1 |
| 2 | If u is a harmonic, show that $w=u^{2}$ is not a harmonic function unless u is a | Remember | CAHS004.3 |


|  | constant. |  |  |
| :---: | :---: | :---: | :---: |
| 3 | Prove that if $u=x^{2}-y^{2}, v=-\frac{y}{x^{2}+y^{2}}$ both $u$ and $v$ satisfy Laplace's equation, but $\mathrm{u}+\mathrm{iv}$ is not a regular (analytic ) function of z . | Understand | CAHS004.3 |
| 4 | If $f(z)$ is an analytic function and $u-v=\frac{\cos x+\sin x-e^{-y}}{2 \cos x-e^{y}-e^{-y}}$ then determine the analytic function $\mathrm{f}(\mathrm{z})$ subjected to the condition $\mathrm{f}\left(\frac{\pi}{2}\right)=0$. | Understand | CAHS004.2 |
| 5 | Find an analytic function $\mathrm{f}(\mathrm{z})$ whose real part of it is $\left.\mathrm{u}=e^{x}\left[\left(x^{2}-y^{2}\right) \cos y-2 x y \sin y\right)\right]$. | Understand | CAHS004.2 |
| 6 | Prove that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right) \log \left\|f^{\prime}(z)\right\|=0$ where $\mathrm{w}=\mathrm{f}(\mathrm{z})$ is an analytic function. | Understand | CAHS004.2 |
| 7 | Find the analytic function $f(z)=u(r, \theta)+i v(r, \theta)$ such that $\mathrm{v}(r, \theta)=$ $\left(r-\frac{1}{r}\right) \sin \theta, \mathrm{r} \neq 0$ using Cauchy-Riemann equations in polar form. | Understand | CAHS004.3 |
| 8 | Find an analytic function $\mathrm{f}(\mathrm{z})$ such that $\operatorname{Re}\left[f^{\prime}(z)\right]=3 x^{2}-4 y-3 y^{2}$ and $f(1+i)=0$. | Understand | CAHS004.2 |
| 9 | Show that the function $\mathrm{f}(\mathrm{z})=\sqrt{\|x y\|}$ is not analytic at the origin although Cauchy -Riemann equations are satisfied at origin. | Remember | CAHS004.3 |
| 10 | If $w=\varnothing+i \varphi$ represents the complex potential for an electric field where $\varphi=x^{2}-y^{2}+\frac{x}{x^{2}+y^{2}}$ then determine the function $\varphi$. | Understand | CAHS004.3 |
| UNIT-II |  |  |  |
| COMPLEX INTEGRATION |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |
| 1 | Write the Cauchy's integral formula. | Remember | CAHS004.6 |
| 2 | Write the Cauchy's General integral formula. | Remember | CAHS004.6 |
| 3 | Define the term Radius of convergence. | Remember | CAHS004.7 |
| 4 | Define the term Power series expansions of complex functions. | Remember | CAHS004.7 |
| 5 | Define the term Line Integral of complex variable function $\mathrm{w}=\mathrm{f}(\mathrm{z})$. | Remember | CAHS004.5 |
| 6 | Define the term Contour Integration of a given curve in complex function. | Remember | CAHS004.5 |
| 7 | State Cauchy's integral theorem for multiple connected region. | Remember | CAHS004.6 |
| 8 | Estimate the value of $\int_{0}^{1+i} z^{2} d z$. | Understand | CAHS004.5 |
| 9 | Estimate the value of $\int_{C} \frac{3 z^{2}+7 z+1}{(z+1)} d z$ with $\mathrm{C}:\|z+i\|=1$ by Cauchy integral formulae. | Understand | CAHS004.6 |
| 10 | Determine the value of line integral to $\int_{0}^{2+i} z^{2} d z$ along the real axis to 2 and then vertically to $(2+i)$. | Understand | CAHS004.5 |
| 11 | Determine the value of line integral to $\int_{0}^{3+i} z^{2} d z$ along the straight line $\mathrm{y}=$ x/3. | Understand | CAHS004.5 |
| 12 | Examine the value of $\int_{C} e^{-z} d z$ with $\mathrm{C}:\|z-1\|=1$ by Cauchy integral formulae. | Understand | CAHS004.6 |
| 13 | Determine the value of line integral to $\int_{0}^{2+i}\left(x-y^{2}+i x^{3}\right) d z$ along the real axis from $\mathrm{z}=0$ to $\mathrm{z}=1$. | Understand | CAHS004.5 |
| 14 | Determine the value of the line integral $\int_{C}^{\bar{z}} z d z$ from $\mathrm{z}=\mathrm{o}$ to 2 i and then from 2 i to $\mathrm{z}=4+2 \mathrm{i}$. | Understand | CAHS004.5 |
| 15 | Estimate the radius of convergence of an infinite series $\mathrm{f}(\mathrm{z})=$ sinz. | Understand | CAHS004.7 |


| 16 | Estimate the radius of convergence of an infinite series $f(z)=\frac{1}{1-z}$. | Understand | CAHS004.7 |
| :---: | :---: | :---: | :---: |
| 17 | Estimate the radius of convergence of an infinite series $1+2^{2} z+3^{2} z^{2}+4^{2} z^{3}+\ldots \ldots \ldots$ | Understand | CAHS004.7 |
| 18 | Evaluate $\int_{0}^{1+i}\left(x^{2}-i y\right) d z$ along the path $y=x$. | Remember | CAHS004.5 |
| 19 | Estimate the value of $\int_{c} \frac{1}{z-2} d z$ around the circle $\|z-1\|=5$ by Cauchy integral formulae. | Understand | CAHS004.6 |
| 20 | Prove that by using line integral, $\int_{C} \frac{1}{(z-a)} d z=2 \pi i$ where c is the curve $\|z-a\|=r$. | Remember | CAHS004.5 |
| Part - B (Long Answer Questions) |  |  |  |
| 1 | Estimate the value of line integral to $\int_{c} \frac{z^{3}-\sin 3 z}{(z-\pi / 2)^{3}} \mathrm{dz}$ where c is the circle $\|z\|=2$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 2 | Verify Cauchy's theorem for the integral of $z^{3}$ taken over the boundary of the rectangle formed with the vertices $-1,1,1+\mathrm{i},-1+\mathrm{i}$. | Understand | CAHS004.6 |
| 3 | Determine the value of line integral to $\int_{c} \frac{e^{2 z}}{(z-1)(z-2)} \mathrm{dz}$ where c is the circle $\|z\|=3$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 4 | Determine the value of line integral to $\int_{c} \frac{z^{3} e^{-z}}{(z-1)^{3}} \mathrm{dz}$ where c is $\|z-1\|=\frac{1}{2}$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 5 | Determine the value of line integral to $\int_{c} \frac{5 z^{2}-3 z+2}{(z-1)^{3}} d z$ where c is any simple closed curve enclosing $\|z\|=1$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 6 | Estimate the value of line integral to $\int_{z=0}^{z=1+i}\left[x^{2}+2 x y+i\left(y^{2}-z\right)\right] d z$ along the curve $y=x^{2}$. | Understand | CAHS004.5 |
| 7 | Evaluate $\int_{c}\left(3 z^{2}+2 z-4\right) d z$ around the square with vertices at $(0,0),(1,0)$, $(1,1)$ and $(0,1)$. | Remember | CAHS004.5 |
| 8 | Verify Cauchy's theorem for the function $f(z)=5 \sin 2 z$ if c is the square with vertices at $1 \pm i$ and $-1 \pm i$. | Understand | CAHS004.6 |
| 9 | Determine the value of line integral to $\int_{C} \frac{(\sin z)^{6}}{\left(z-\frac{\pi}{6}\right)^{3}} d z$ around the unit circle using Cauchy's integral formula. | Understand | CAHS004.6 |
| 10 | Determine the value of to $\int_{c} \frac{e^{2 z}}{(z+1)^{4}} \mathrm{dz}$ where c is $\|z-1\|=3$ using Cauchy's general integral formulae. | Understand | CAHS004.6 |


| 11 | Evaluate using cauchy's integral formula $\int_{c} \frac{z+1}{z^{2}+2 z+4} d z$ Where $c:\|z+1+i\|=2$. | Understand | CAHS004.6 |
| :---: | :---: | :---: | :---: |
| 12 | Determine the value of line integral to $\int_{C}\left(y^{2}+z^{2}\right) d x+\left(z^{2}+x^{2}\right) d y+\left(x^{2}+y^{2}\right) d z$ from $(0,0,0)$ to $(1,1,1)$ where C is the curve $\mathrm{x}=\mathrm{t}, \mathrm{y}=\mathrm{t}^{2}, \mathrm{z}=\mathrm{t}^{3}$ in the parametric form. | Understand | CAHS004.5 |
| 13 | Estimate the value of $\int_{c} \frac{e^{z}}{z^{2}(z+1)^{3}} d z$ with $\mathrm{C}:\|z\|=2$ by Cauchy general integral formulae. | Understand | CAHS004.6 |
| 14 | Prove that if $\mathrm{f}(\mathrm{z})$ is analytic function then $\int_{A}^{B} f(z) d z$ is independent of path followed. | Remember | CAHS004.5 |
| 15 | Determine the value of line integral to $\int_{0}^{3+i} z^{2} d z$ along the parabola $x=3 y^{2}$. | Understand | CAHS004.5 |
| 16 | Estimate the value of $\int_{C} \frac{1}{e^{z}(z-1)^{3}} d z$ with $\mathrm{C}:\|z\|=2$ by Cauchy general integral formulae. | Understand | CAHS004.6 |
| 17 | Determine the value of $\int_{C} \frac{e^{z} \sin 2 z-1}{z^{2}(z+2)^{2}} d z$ where c is $\|z\|=\frac{1}{2}$ using Cauchy integral formulae. | Understan <br> d | CAHS004.6 |
| 18 | Evaluate $\int_{c}\left[\frac{e^{z}}{z^{3}}+\frac{z^{4}}{(z-i)^{2}}\right] d z \quad, c:\|z\|=2$ using Cauchy’s integral formulae. | Remember | CAHS004.6 |
| 19 | Determine the value of line integral to $\int_{C}\left(z^{2}+3 z\right) d z$ along the straight line from $(2,0)$ to $(2,2)$ and then from $(2,2)$ to $(0,2)$. | Understand | CAHS004.5 |
| 20 | Let C denote the boundary of a square whose sides lie along the lines $\mathrm{x}= \pm 2$ and $\mathrm{y}= \pm 2$ where C is described in positive sense. Then determine the value of line integral to $\int_{C} \frac{\cos h z}{z^{4}} d z$. | Understand | CAHS004.5 |
|  | Part - C (Problem Solving and Critical Thinking Questio |  |  |
| 1 | Determine the value of line integral to $\int_{c} \frac{z}{(z-1)(z-2)^{2}} \mathrm{dz}$ where c is the circle $\|z-2\|=1 / 2$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 2 | Estimate the value of line integral to $\int_{c} \frac{z^{4}}{(z+1)(z-i)^{2}} d z$ where c is the ellipse $9 x^{2}+4 y^{2}=36$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 3 | Estimate the value of line integral to $\int_{c} \frac{\mathrm{z}^{4}-3 \mathrm{z}^{2}+6}{(\mathrm{z}+1)^{3}} \mathrm{dz}$ where c is the circle $\|z\|=2$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 4 | Estimate the value of line integral to $\int_{c} \frac{z^{2}-2 z-2}{\left(z^{2}+1\right)^{2}} \mathrm{dz}$ where c is the circle | Understand | CAHS004.6 |


|  | $\|z-i\|=1 / 2$ using Cauchy's integral formula. |  |  |
| :---: | :---: | :---: | :---: |
| 5 | Estimate the value of line integral to $\int_{c} \frac{e^{z}}{\left(z^{2}+\pi^{2}\right)^{2}} d z$ where c is $\|z\|=4$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 6 | Estimate the value of line integral to $\int_{c} \frac{\cos \pi z^{2}}{(z-1)(z-2)^{3}} \mathrm{dz}$ where c is the circle $\|z\|=3$ using Cauchy's integral formula. | Understand | CAHS004.6 |
| 7 | Determine the value of line integral to $\int_{0}^{1+i}\left(x-y+i x^{2}\right) \mathrm{dz}$ <br> i) Along the straight line from $\mathrm{z}=0$ to $\mathrm{z}=1+\mathrm{i}$. <br> ii) Along the real axis from $\mathrm{z}=0$ to $\mathrm{z}=1$ and then along a line parallel to imaginary axis from $\mathrm{z}=1$ to $\mathrm{z}=1+\mathrm{i}$ <br> iii) Along the imaginary axis from $\mathrm{z}=0$ to $\mathrm{z}=\mathrm{i}$ and then along a line parallel to real axis $\mathrm{z}=\mathrm{i}$ to $\mathrm{z}=1+\mathrm{i}$. | Understand | CAHS004.5 |
| 8 | Verify Cauchy's theorem for the integral of $3 z^{2}+i z-4$ taken over the boundary of the square with vertices $-1+\mathrm{i},-1-\mathrm{i}, 1+\mathrm{i},-1-\mathrm{i}$. | Understand | CAHS004.6 |
| 9 | Derive the Cauchy general integral formulae of an analytic function $f(z)$ within a closed contour c . | Remember | CAHS004.6 |
| 10 | Estimate the value of line integral to $\int_{C}\left(y^{2}+2 x y\right) d x+\left(y^{2}-2 x y\right) d y$ where C is the boundary of the region $y=x^{2}$ and $x=y^{2}$. | Understand | CAHS004.5 |
| UNIT-III |  |  |  |
| POWER SERIES EXPANSION OF COMPLEX FUNCTION |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |
|  | State Taylor's theorem of complex power series. | Remember | CAHS004.9 |
| 2 | State Laurent's theorem of complex power series. | Remember | CAHS004.9 |
| 3 | Define the term pole of order $m$ of an analytic function $f(z)$. | Remember | CAHS004.11 |
| 4 | Define the terms Essential and Removable singularity of an analytic function $\mathrm{f}(\mathrm{z})$. | Remember | CAHS004.10 |
| 5 | Expand $\mathrm{f}(\mathrm{z})=\frac{1}{z^{2}}$ in powers of $\mathrm{z}+1$ as a Taylor's series. | Understand | CAHS004.9 |
| 6 | Expand $\mathrm{f}(\mathrm{z})=\mathrm{e}^{\mathrm{z}}$ as Taylor's series about $\mathrm{z}=1$. | Understand | CAHS004.9 |
| 7 | Estimate the Poles of $\frac{1}{z^{2}-1}$ | Understand | CAHS004.11 |
| 8 | Obtain the Taylor series expansion of $\mathrm{f}(\mathrm{z})=e^{z}$ about the point $\mathrm{z}=1$. | Understand | CAHS004.9 |
| 9 | Determine the Poles of the function $f(z)=\frac{z e^{z}}{(z+2)^{4}(z-1)}$ | Understand | CAHS004.11 |
| 10 | Define the Isolated singularity of an analytic function $\mathrm{f}(\mathrm{z})$. | Understand | CAHS004.10 |
| 11 | State Cauchy's Residue theorem of an analytic function $f(z)$ within and on the closed curve. | Remember | CAHS004.11 |
| 12 | Determine the Residue by Laurent's expansion to $f(z)=\frac{e^{z}}{(z-1)^{2}}$ about $\mathrm{z}=1$. | Understand | CAHS004.9 |


| 13 | Estimate the Residues of the function $f(z)=\frac{1}{(z-\sin z)}$ about $\mathrm{z}=0$ by Laurent's expansion. | Understand | CAHS004.9 |
| :---: | :---: | :---: | :---: |
| 14 | Estimate the Residues of the function $f(z)=\frac{z}{(z+1)(z+2)}$ as a Laurent's series about $\mathrm{z}=-2$. | Understand | CAHS004.9 |
| 15 | Estimate the value of $\oint_{C} \frac{1-2 z}{z(z-1)(z-2)} d z$ by Cauchy's Residue theorem. | Understand | CAHS004.11 |
| 16 | Determine the Bilinear transformation whose fixed points are i,-i. | Understand | CAHS004.13 |
| 17 | Obtain the fixed points of the transformation $w=\frac{1}{z-2 i}$ | Understand | CAHS004.13 |
| 18 | Discover the Bilinear transformation which maps the points ( $0,-\mathrm{i},-1$ ) into the points (i,1,0) | Understand | CAHS004.12 |
| 19 | Discover the points at which $\mathrm{w}=$ coshz is not conformal. | Understand | CAHS004.12 |
| 20 | Discuss the fixed points of the transformation $w=\frac{2 i-6 z}{i z-3}$ | Understand | CAHS004.13 |
| Part - B (Long Answer Questions) |  |  |  |
| 1 | Expand $\mathrm{f}(\mathrm{z})=\frac{z-1}{z+1}$ in Taylor's series about the point $\mathrm{z}=1$. | Understand | CAHS004.9 |
| 2 | Expand $\mathrm{f}(\mathrm{z})=\frac{z-1}{z^{2}}$ in Taylor's series in powers of $\mathrm{z}-1$. Also determine the region of convergence about the point $\mathrm{z}=1$. | Understand | CAHS004.9 |
| 3 | Obtain Laurent's series expansion of $\mathrm{f}(\mathrm{z})=\frac{z^{2}-4}{z^{2}+5 z+4}$ valid in $1<\mathrm{z}<4$. | Understand | CAHS004.9 |
| 4 | Expand $f(z)=\frac{e^{2 z}}{(z-1)^{3}}$ about $\mathrm{z}=1$ as Laurent's series. Also find the region of convergence about $\mathrm{z}=1$. | Understand | CAHS004.9 |
| 5 | Expand $\mathrm{f}(\mathrm{z})=\frac{7 z-2}{z(z+1)(z-2)}$ about $\mathrm{z}=-1$ in the region $1<\|z+1\|<3$ as Laurent's series . | Understand | CAHS004.9 |
| 6 | Expand $f(z)=\frac{2 z^{3}+1}{z(z+1)}$ in Taylor's series about the point $\mathrm{z}=1$ | Understand | CAHS004.9 |
| 7 | Find Taylor's expansion of $f(z)=\frac{z+1}{(z-3)(z-4)}$ about the point $\mathrm{z}=2$. Determine the region of convergence. | Understand | CAHS004.9 |
| 8 | Expand $f(z)=\cos z$ in taylor's series about $z=\pi i$. | Understand | CAHS004.9 |
| 9 | Obtain the Laurent's series expansion of $f(z)=\frac{e^{z}}{z(1-3 z)}$ about $\mathrm{z}=1$. | Understand | CAHS004.9 |
| 10 | Express $f(z)=\frac{1+2 z}{z^{2}+z^{3}}$ in a series of positive and negative powers of z . | Understand | CAHS004.9 |
|  |  |  |  |
| 11 | Estimate the value of $\int_{c} \frac{2 z-1}{z(2 z+1)(z+2)} d z$ where c is the circle $\|z\|=1$. | Understand | CAHS004.11 |


| 12 | Assess the value of $\oint \tan z d z$ where c is circle $\|z\|=2$. | Understand | CAHS004.11 |
| :---: | :---: | :---: | :---: |
| 13 | Estimate the value of $\oint_{c} \frac{d z}{\left(z^{2}+4\right)^{2}}$ where c is $\|z-i\|=2$. | Understand | CAHS004.11 |
| 14 | Calculate the value of $\oint_{c} \frac{\operatorname{coth} z}{z-i} d z$ where c is $\|z\|=2$. | Understand | CAHS004.11 |
| 15 | Calculate the value of $\int_{-\infty}^{\infty} \frac{x^{2} d x}{\left(x^{2}+1\right)\left(x^{2}+4\right)}$ | Understand | CAHS004.11 |
| 16 | Determine the Bi-linear transformation which carries the points from $(0,1, \infty) t o(-5,-1,3)$ | Understand | CAHS004.12 |
| 17 | Determine the Bi-linear transformation which carries the points from $(1, i,-1) t o(0,1, \infty)$. | Understand | CAHS004.12 |
| 18 | Determine the Bilinear transformation that maps the points ( $1-2 \mathrm{i}, 2+i, 2+3 i$ ) into the points ( $2+i, 1+3 i, 4$ ). | Understand | CAHS004.12 |
| 19 | Determine the Bilinear transformation that maps the points ( $1, \mathrm{i},-1$ ) into the points $(2, \mathrm{i},-2)$. | Understand | CAHS004.12 |
| 20 | Determine the Bilinear transformation that maps the points $(\infty, i, 0)$ into the points $(0, \mathrm{i}, \infty)$. | Remember | CAHS004.12 |
| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |
| 1 | Obtain the Laurent expansion of $\mathrm{f}(\mathrm{z})=\frac{1}{z^{2}-4 z+3}$ for $1<\|z\|<3$ (ii) $\|z\|<$ 1 (iii) $\|z\|>3$. | Understand | CAHS004.9 |
| 2 | Expand $\mathrm{f}(\mathrm{z})=\frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region where $(i)\|z\|<1$ (ii) $1<\|z\|<4$. | Understand | CAHS004.9 |
| 3 | Expand $\frac{1}{z^{2}(z-3)^{2}}$ as Laurent's series in the region $(i)\|z\|<1$ (ii) $\|z\|>3$. | Understand | CAHS004.9 |
| 4 | Expand $\mathrm{f}(\mathrm{z})=\frac{2}{(2 z+1)^{3}}$ in Taylor's series about $\mathrm{z}=0$ and $\mathrm{z}=2$. | Understand | CAHS004.9 |
| 5 | Expand $\mathrm{f}(\mathrm{z})=\frac{e^{z}}{z(z+1)}$ in Taylor's series about $\mathrm{z}=2$. | Understand | CAHS004.9 |
| 6 | Determine the value of $\oint_{c} \frac{z-3}{\left(z^{2}+2 z+5\right)} d z$ where c is circle $\|z\|=1$. | Understand | CAHS004.10 |
| 7 | Estimate the value of $\int_{0}^{2 \pi} \frac{d \theta}{a+b \cos \theta}$ | Remember | CAHS004.10 |
| 8 | Discover the Bilinear transformation that maps the points $(0,1, \infty)$ into the points $(-1,-2,-i)$. | Understand | CAHS004.12 |
| 9 | Obtain the fixed points of the transformation $w=\frac{3 i z+13}{z-3 i}$ | Understand | CAHS004.13 |
| 10 | Determine the Bilinear transformation that maps the points $(\infty, i, 0)$ in the z plane into the points $(0, i, \infty)$ in the w-plane. | Understand | CAHS004.12 |




|  | Find its moment generating function. |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | The density function of a random variable X is $f(x)=\left\{\begin{array}{cc}e^{-x} \quad, x \geq 0 \\ 0 & , \text { otherwise }\end{array}\right.$ Find $E(X), E\left(X^{2}\right), V(X)$. |  |  |  |  |  |  |  | Understand | CAHS004.16 |
| 19 | Compute the first four moments about the mean for the following distribution |  |  |  |  |  |  |  |  |  |
|  | Marks |  |  |  | $\begin{array}{\|l\|} \hline 30- \\ 40 \end{array}$ |  |  | $\begin{array}{\|l\|} \hline 60- \\ 70 \end{array}$ | Understan | CAHS004.17 |
|  | No. of students | value | 12 of $\beta_{1}$ | 20 |  |  |  | $5$ |  |  |
| 20 | Determine the value of A to the probability density function$\mathrm{f}(\mathrm{x})=\mathrm{Ax}^{2} \text { in } 0<\mathrm{x}<1$ |  |  |  |  |  |  |  | Understand | CAHS004.16 |
| Part - C (Problem Solving and Critical Thinking Questions) |  |  |  |  |  |  |  |  |  |  |
| 1 | Find the Mean and Variance to the following discrete distribution |  |  |  |  |  |  |  | Understand | CAHS004.16 |
|  | X | 12 | 16 | 20 |  |  |  |  |  |  |
|  | Y $1 / 8$ | 1/6 | 3/8 |  |  |  |  |  |  |  |
| 2 | A random variable X has the following probability function. |  |  |  |  |  |  |  | Understand | CAHS004.16 |
|  | X 4 | 5 | 6 | 8 |  |  |  |  |  |  |
|  | $\mathrm{P}(\mathrm{X}$ 0.1 <br> f  |  |  |  |  |  |  |  |  |  |
|  | Determine (i) Expectation (ii) variance (iii)Standard deviation. |  |  |  |  |  |  |  |  |  |
| 3 | Out of 24 mangoes, 6 mangoes are rotten. If we draw two mangoes, then obtain probability distribution of number of rotten mangoes that can be drawn. |  |  |  |  |  |  |  | Remember | CAHS004.15 |
| 4 | If X is a Continuous random variable whose density function is $f(x)=\left\{\begin{array}{rr} x & \text { if } 0<x<1 \\ 2-x & \text { if } 1 \leq x<2 \\ 0 & \text { elsewhere } \end{array}\right.$ <br> Find $E\left(25 X^{2}+30 X-5\right)$. |  |  |  |  |  |  |  | Understand | CAHS004.16 |
| 5 | The probability density function of a random variable X is $f(x)=\frac{K}{x^{2}+1},-\infty<x<\infty$. Find K and the distribution function $\mathrm{F}(\mathrm{x})$. |  |  |  |  |  |  |  | Understand | CAHS004.16 |
| 6 | If the probability density of a random variable X is given <br> by $f(x)=\left\{\begin{array}{c}k\left(1-x^{2}\right), 0<x<1 \\ 0, \text { otherwise }\end{array}\right.$ Find (i) k (ii) The cumulative distribution <br> function of X. |  |  |  |  |  |  |  | Understand | CAHS004.16 |
| 7 | The first three moments of a distribution about the value 2 of the variable are 1, 16 , and -40 . Show that the mean $=3$, the variance $=15$ and $\mu_{3}=-86$. |  |  |  |  |  |  |  | Understand | CAHS004.17 |
| 8 | Explain moments at origin of a probability distribution function. |  |  |  |  |  |  |  | Remember | CAHS004.17 |
| 9 | Explain the moment generating function of a probability distribution function. |  |  |  |  |  |  |  | Remember | CAHS004.17 |
| 10 | Explain the relation between the moments about mean in terms of moments bout arbitrary origin. |  |  |  |  |  |  |  | Remember | CAHS004.17 |
|  | UNIT-V |  |  |  |  |  |  |  |  |  |
| PROBABILITY DISTRIBUTIONS |  |  |  |  |  |  |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |  |  |  |  |  |  |
| 1 | Define the terms mean, variance of Binomial distribution. |  |  |  |  |  |  |  | Remember | CAHS004.19 |
| 2 | Draft the recurrence relation for the Binomial distribution. |  |  |  |  |  |  |  | Remember | CAHS004.19 |


| 3 | Define the term mode of a Binomial distribution. | Remember | CAHS004.19 |
| :---: | :---: | :---: | :---: |
| 4 | Determine the value of n if the mean and variance of a Binomial distribution are 3 and 9/4. | Understand | CAHS004.19 |
| 5 | Determine the Binomial distribution for which the mean is 4 and variance 3 | Understand | CAHS004.19 |
| 6 | The mean and variance of a binomial variable $X$ with parameters $n$ and $p$ are 16 and 24.Determine the value of $\mathrm{P}(\mathrm{X}=1)$. | Remember | CAHS004.19 |
| 7 | If a bank received on the average 6 bad cheques per day,Find the probability that it will receive 4 bad cheques on any given day. | Understand | CAHS004.19 |
| 8 | Define the terms Mean, Variance of Poisson distribution | Remember | CAHS004.19 |
| 9 | If X is a Poisson variate with $\mathrm{P}(\mathrm{x}=2)=2 / 3 \mathrm{P}(\mathrm{x}=1) \quad$ Compute the value of $P(x=0)$. | Understand | CAHS004.19 |
| 10 | Draft the recurrence relation for the Poisson distribution. | Remember | CAHS004.19 |
| 11 | The mean and variance of binomial distribution are 4 and $4 / 3$ respectively. Find $p(X \geq 1)$. | Remember | CAHS004.19 |
| 12 | If a bank received on the average 6 bad apples per day then estimate the probability that it will receive 4 bad cheques on any given day. | Understand | CAHS004.19 |
| 13 | If $2 \%$ of light bulbs are defective in a sample of 100 .Find at least one is defective. | Remember | CAHS004.19 |
| 14 | If a random variable has Poisson distribution such that $p(1)=p$ (2). Determine the value of $p(1<x<4)$. | Understand | CAHS004.19 |
| 15 | Define Poisson distribution. | Remember | CAHS004.19 |
| 16 | Define the term Normal Distribution. | Remember | CAHS004.20 |
| 17 | Define Binomial distribution. | Remember | CAHS004.20 |
| 18 | Define Normal curve. | Remember | CAHS004.20 |
| 19 | Draft the applications of Normal distribution. | Remember | CAHS004.20 |
| 20 | If X is Normally distributed with mean 2 and variance 0.1 , then Estimate the value of $\mathrm{P}(\|x-2\| \geq 0.01)$ | Understand | CAHS004.20 |
| Part - B (Long Answer Questions) |  |  |  |
| 1 | Derive the Variance of a Binomial Distribution. | Remember | CAHS004.19 |
| 2 | Estimate the probability that at most 5 defective components will be found in a lot of 200 . Experience shows that $2 \%$ of such components are defective. Also find the probability of more than 5 defective components. | Understand | CAHS004.19 |
| 3 | The probability that a man hitting a target is $1 / 3$. If he fires 5 times, Determine the probability that he fires <br> (i) At most 5 times (ii) At least 2 times | Understand | CAHS004.19 |
| 4 | Find the variance of a Poisson Distribution. | Remember | CAHS004.19 |
| 5 | Poisson variate has a double mode at $\mathrm{x}=2$ and $\mathrm{x}=3$, Determine the maximum probability and also find $\mathrm{p}(x \geq 2)$ | Understand | CAHS004.19 |
| 6 | Average number of accidents on any day on a national highway is 1.8 . Determine the probability that the number of accidents is (i) at least one (ii) at most one | Understand | CAHS004.19 |
| 7 | A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed to Poisson distribution with mean 1.5 . Find the proportion of days <br> (i)on which there is no demand <br> (ii)on which demand is refused. | Understand | CAHS004.19 |
| 8 | If x is a poisson variate such that $\mathrm{p}(\mathrm{x}=2)=45 \mathrm{p}(\mathrm{x}=6)-3 \mathrm{p}(\mathrm{x}=4)$. Find (i) $\mathrm{p}(x \geq 1)$ <br> (ii) $\mathrm{p}(\mathrm{x}<2)$ | Remember | CAHS004.20 |
| 9 | Derive median of the Normal distribution. | Remember | CAHS004.20 |



|  | (ii) less than 117 hours <br> (iii) will be more than 395 hours |  |  |
| :---: | :--- | :--- | :--- |
| 9 | Derive the mean of the Binomial Distribution. | Remember | CAHS004.20 |
| 10 | The marks obtained in mathematics by 1000 students are Normally <br> distributed with mean 78\% and standard deviation $11 \%$. Determine <br> (i)How many students got marks above $90 \%$ marks <br> (ii)What was the highest mark obtained by the lowest $10 \%$ of the students <br> (iii)Within what limits did the middle of $90 \%$ of the student lie. | Understand | CAHS004.20 |

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