INSTITUTEOFAERONAUTICALENGINEERING

(Autonomous)
Dundigal, Hyderabad - 500043

FRESHMAN ENGINEERING

TUTORIAL QUESTION BANK

Course Name	:	Complex Analysis and Probability Distribution
Course Code	:	AHS004
Class	:	B.Tech II Semester
Branch	:	ECE
Academic Year	:	2017 - 2018
CourseCoordinator	:	Ms. C Rachana, Assistant Professor
Course Faculty	:	Mr. Ch Kumar Swamy, Associate Professor Ms. L Indira, Associate Professor Mr. G Nagendra Kumar, Assistant Professor

COURSE OBJECTIVES (COs):

The course should enable the students to:

I	Understand the basic theory of complex functions to express the power series.
II	Evaluate the contour integration using Cauchy residue theorem.
III	Enrich the knowledge of probability on single random variables and probability distributions.

COURSE LEARNING OUTCOMES (CLOs):

Students, who complete the course, will have demonstrated the asking to do the following:

CAHS004.01	Define continuity, differentiability, analyticity of a function using limits.
CAHS004.02	Understand the conditions for a complex variable to be analytic and/or entire function.
CAHS004.03	Understand the concepts of Cauchy-Riemann relations and harmonic functions.
CAHS004.04	Understand the concept of complex differentiation to the real-world problems of signals modulated by electromagnetic waves.
CAHS004.05	Evaluate the area under a curve using the concepts of indefinite integration
CAHS004.06	Understand the concepts of the Cauchy’s integral formula and the generalized Cauchy’s integral formula.
CAHS004.07	Evaluate complex functions as power series and radius of convergence of power series.
CAHS004.08	Understand the concept of complex integration to the real-world problems of flow with circulation around a cylinder.
CAHS004.09	Solve the Taylor’s and Laurent series expansion of complex functions
CAHS004.10	Understand the concept of different types of singularities for analytic function.
CAHS004.11	Evaluate poles, residues and solve integrals using Cauchy’s residue theorem.
CAHS004.12	Evaluate bilinear transformation by cross ratio property.
CAHS004.13	Identify the conditions of fixed and critical point of Bilinear Transformation.

CAHS004.14	Understand the concept of Cauchy’s residue theorem to the real-world problems of Quantum Mechanical scattering and Quantum theory of atomic collisions.
CAHS004.15	Demonstrate an understanding of the basic concepts of probability and random variables.
CAHS004.16	Classify the types of random variables and calculate mean, variance.
CAHS004.17	Finding moment about origin, central moments, moment generating function of probability distribution.
CAHS004.18	Understand the concept of random variables to the real-world problems like graph theory, machine learning and natural language processing
CAHS004.19	Recognize where the binomial distribution and poisson distribution could be appropriate model and find mean, variance of the distributions.
CAHS004.20	Apply the inferential methods relating to the means of normal distributions.
CAHS004.21	Understand binomial distribution to the phenomena of real-world problem like sick versus healthy.
CAHS004.22	Understand the mapping of normal distribution in real-world problem to analyze the stock market.
CAHS004.23	Use poission distribution in real-world problem to predict soccer scores.
CAHS004.24	Possess the knowledge and skills for employability and to succeed in national and international level competitive examinations.

TUTORIAL QUESTION BANK

UNIT-I			
COMPLEX FUNCTIONS AND DIFFERENTIATION			
Part - A(Short Answer Questions)			
S No	**QUESTIONS**	**Blooms Taxonomy Level**	**Course Learning Outcomes (CLOs)**
1	Define the term Analyticity of a complex variable function f (z).	Remember	CHS004.1
2	Define the term Continuity of a complex variable function f (z).	Remember	CHS004.1
3	Define the term Differentiability of a complex variable function f (z).	Remember	CHS004.1
4	If $w = f(z) = z^2 + z$. Find its real and imaginary parts.	Remember	CHS004.1
5	Examine the complex variable function f (z) = z^3 to analyticity for all values of z in Cartesian form.	Understand	CHS004.2
6	Verify whether the function $v = x^3y - xy^3 + xy + x + y$ can be imaginary part of an analytic function f (z) where z= x+ iy.	Understand	CHS004.2
7	Show that the function f(z) = $\|z\|^2$ does not satisfy Cauchy-Riemann equations in Cartesian form.	Understand	CHS004.3
8	Examine the complex variable function f (z) = $\frac{x-iy}{x^2+y^2}$ for analyticity in Cartesian form.	Understand	CHS004.2
9	Interpret whether the function f (z) = sinx siny – i cosx cosy is an analytic function or not in Cartesian form.	Understand	CHS004.2
10	Calculate the value of k such that f(x, y) = $x^3 + 3kxy^2$ may be harmonic function.	Understand	CHS004.3
11	Determine the most general analytic function f (z) whose real part of the analytic function is u = $x^2 - y^2 - x$.	Understand	CHS004.2
12	Obtain an analytic function f (z) whose imaginary part of the analytic function is v = $e^x(xsiny + ycosy)$.	Understand	CHS004.2
13	Show that the real part of an analytic function f(z) where u = 2log ($x^2 + y^2$) is harmonic.	Understand	CHS004.3
14	Determine the conjugate harmonic function if the real part of an analytic function f (z) is u = $y^2 - 3x^2y$ is harmonic function.	Understand	CHS004.3
15	Estimate the values of w which correspond to z = 1+3i when w = f(z) = z^2.	Understand	CHS004.3
16	Show that the function f (z) = $\|z\|^2$ is continuous at all points of z but not differentiable at any $z \neq 0$.	Understand	CHS004.1
17	Calculate all the values of k such that f (z) = $e^x(cosky + isinky)$ is an analytic function.	Understand	CHS004.2
18	Determine the values of a, b, c such that f (z) = x+ ay- i(ax+by) is differentiable function at every point.	Understand	CHS004.1
19	Verify whether u = $x^2 - y^2 - y$ of an analytic function can be harmonic function of an analytic function f (z) in the whole complex plane.	Understand	CHS004.3
20	Justify whether every differentiable function is continuous or not. Give a valid example.	Remember	CHS004.1
Part - B (Long Answer Questions)			
1	Show that the real part of an analytic function f (z) where u = $e^{-2xy}\sin(x^2 - y^2)$ is a harmonic function. Hence find its harmonic conjugate.	Understand	CHS004.3

2	Prove that the real part of analytic function f (z) where u = $\log\lvert z\rvert^2$ is harmonic function. If so find the analytic function by Milne Thompson method.	Understand	CHS004.3
3	Determine the imaginary part of an analytic function f (z) whose real part of an analytic function is $e^x(xcosy - ysiny)$.	Understand	CHS004.2
4	Obtain the regular function f (z) whose imaginary part of an analytic function is $\frac{x-y}{x^2+y^2}$.	Understand	CHS004.2
5	If f(z) = u + iv is an analytic function of z , then calculate f(z) if 2u + v = e^{2x} [(2x+y) cos2y + (x-2y) sin2y].	Understand	CHS004.2
6	Prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) \lvert Real f(z)\rvert^2 = 2\lvert f'(z)\rvert^2$ where w = f (z) is an analytic function.	Understand	CHS004.2
7	Find an analytic function f (z) whose real part of an analytic function is u = $\frac{sin2x}{cosh2y-cos2x}$ by Milne-Thompson method.	Understand	CHS004.3
8	If f (z) is a regular function of z, then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) \lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$.	Understand	CHS004.2
9	Show that the function defined by f(z)= $\begin{cases} \frac{xy^2\ (x+iy)}{x^2+y^4} & ,z \neq 0 \\ 0 & ,z = 0 \end{cases}$ is not analytic function even though Cauchy Riemann equations are satisfied at origin.	Understand	CHS004.3
10	Show that real part u = $x^3 - 3xy^2$ of an analytic function f (z) is harmonic. Hence find the conjugate harmonic function and the analytic function.	Understand	CHS004.3
11	Find an analytic function f (z) = u +iv if the real part of an analytic function is u = a (1+cosθ) using Cauchy-Riemann equations in polar form.	Understand	CHS004.3
12	Derive Cauchy-Riemann equations in polar form of an analytic function f (z).	Remember	CHS004.3
13	Prove that the real and imaginary parts of an analytic function f (z) are harmonic.	Remember	CHS004.3
14	Find the analytic function f (z) whose imaginary part of an analytic function is $r^2cos2\theta + r\ sin\theta$ by Cauchy Riemann equations in polar form.	Understand	CHS004.3
15	Prove that the function f (z) = $\lvert z\rvert$ is continuous everywhere but nowhere differentiable.	Remember	CHS004.1
16	Show that the real part of an analytic function f (z) where u = $e^{-x}(xsiny - ycosy)$ is a harmonic function.	Understand	CHS004.3
17	Prove that an analytic function f (z) with constant real part is always constant.	Remember	CHS004.2
18	Prove that an analytic function f (z) with constant modulus is always constant.	Remember	CHS004.2
19	Verify Cauchy –Riemann equation to the function f (z) = $z\,e^{-z}$ in Cartesian form.	Understand	CHS004.3
20	If u and v are conjugate harmonic functions then show that uv is also a harmonic function.	Remember	CHS004.3
	Part - C (Problem Solving and Critical Thinking Questions)		
1	If f (z) is an analytic function of z such that u + v = $\frac{sin2x}{2cosh2y-cos2x}$ then determine the analytic function f(z) in terms of z.	Understand	CHS004.1
2	If u is a harmonic, show that $w = u^2$ is not a harmonic function unless u is a constant.	Remember	CHS004.3

3	Prove that if $u = x^2 - y^2, v = -\frac{y}{x^2+y^2}$ both u and v satisfy Laplace's equation, but u + iv is not a regular (analytic) function of z.	Understand	CHS004.3
4	If f(z) is an analytic function and u - v = $\frac{cosx+sinx-e^{-y}}{2cosx-e^{y}-e^{-y}}$ then determine the analytic function f(z) subjected to the condition f ($\frac{\pi}{2}$) = 0.	Understand	CHS004.2
5	Find an analytic function f(z) whose real part of it is u = $e^x[(x^2-y^2)cosy - 2xy\ siny)]$.	Understand	CHS004.2
6	Prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) log\lvert f'(z)\rvert = 0$ where w = f (z) is an analytic function.	Understand	CHS004.2
7	Find the analytic function $f(z) = u(r,\theta) + i\,v\,(r,\theta)$ such that $v\,(r,\theta) = \left(r-\frac{1}{r}\right)sin\theta$, r≠ 0 using Cauchy-Riemann equations in polar form.	Understand	CHS004.3
8	Find an analytic function f (z) such that $\text{Re}[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$.	Understand	CHS004.2
9	Show that the function f (z) = $\sqrt{\lvert xy\rvert}$ is not analytic at the origin although Cauchy –Riemann equations are satisfied at origin.	Remember	CHS004.3
10	If $w = \emptyset + i\varphi$ represents the complex potential for an electric field where $\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$ then determine the function φ.	Understand	CHS004.3
UNIT-II			
COMPLEX INTEGRATION			
Part - A (Short Answer Questions)			
1	Write the Cauchy's integral formula.	Remember	CHS004.6
2	Write the Cauchy's General integral formula.	Remember	CHS004.6
3	Define the term Radius of convergence.	Remember	CHS004.7
4	Define the term Power series expansions of complex functions.	Remember	CHS004.7
5	Define the term Line Integral of complex variable function w = f (z).	Remember	CHS004.5
6	Define the term Contour Integration of a given curve in complex function.	Remember	CHS004.5
7	State Cauchy's integral theorem for multiple connected region.	Remember	CHS004.6
8	Estimate the value of $\int_0^{1+i} z^2 dz$.	Understand	CHS004.5
9	Estimate the value of $\int_C \frac{3z^2+7z+1}{(z+1)} dz$ with C:$\lvert z+i\rvert = 1$ by Cauchy integral formulae.	Understand	CHS004.6
10	Determine the value of line integral to $\int_0^{2+i} z^2\, dz$ along the real axis to 2 and then vertically to (2+i).	Understand	CHS004.5
11	Determine the value of line integral to $\int_0^{3+i} z^2\, dz$ along the straight line y = x/3.	Understand	CHS004.5
12	Examine the value of $\int_C e^{-z} dz$ with C:$\lvert z-1\rvert = 1$ by Cauchy integral formulae.	Understand	CHS004.6
13	Determine the value of line integral to $\int_0^{2+i} (x - y^2 + ix^3)\, dz$ along the real axis from z=0 to z=1.	Understand	CHS004.5
14	Determine the value of the line integral $\int_C \bar{z} dz$ from z = o to 2i and then from 2i to z = 4+2i.	Understand	CHS004.5
15	Estimate the radius of convergence of an infinite series f (z) = sinz.	Understand	CHS004.7
16	Estimate the radius of convergence of an infinite series f (z) =$\frac{1}{1-z}$.	Understand	CHS004.7

17	Estimate the radius of convergence of an infinite series $1+2^2z+3^2z^2+4^2z^3+........$	Understand	CHS004.7
18	Evaluate $\int_0^{1+i}(x^2-iy)dz$ along the path $y=x$.	Remember	CHS004.5
19	Estimate the value of $\int_C \frac{1}{z-2}dz$ around the circle $\lvert z-1\rvert=5$ by Cauchy integral formulae.	Understand	CHS004.6
20	Prove that by using line integral, $\int_C \frac{1}{(z-a)}dz=2\pi i$ where c is the curve $\lvert z-a\rvert=r$.	Remember	CHS004.5
Part - B (Long Answer Questions)			
1	Estimate the value of line integral to $\int_c \frac{z^3-\sin 3z}{(z-\pi/2)^3}$ dz where c is the circle $\lvert z\rvert$=2 using Cauchy's integral formula.	Understand	CHS004.6
2	Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle formed with the vertices -1 ,1,1+i ,-1+i.	Understand	CHS004.6
3	Determine the value of line integral to $\int_c \frac{e^{2z}}{(z-1)(z-2)}$ dz where c is the circle $\lvert z\rvert$=3 using Cauchy's integral formula.	Understand	CHS004.6
4	Determine the value of line integral to $\int_c \frac{z^3e^{-z}}{(z-1)^3}$ dz where c is $\lvert z-1\rvert=\frac{1}{2}$ using Cauchy's integral formula.	Understand	CHS004.6
5	Determine the value of line integral to $\int_c \frac{5z^2-3z+2}{(z-1)^3}dz$ where c is any simple closed curve enclosing $\lvert z\rvert$=1 using Cauchy's integral formula.	Understand	CHS004.6
6	Estimate the value of line integral to $\int_{z=0}^{z=1+i}[x^2+2xy+i(y^2-z)]\ dz$ along the curve $y=x^2$.	Understand	CHS004.5
7	Evaluate $\int_c (3z^2+2z-4)dz$ around the square with vertices at (0,0), (1,0), (1,1) and (0,1).	Remember	CHS004.5
8	Verify Cauchy's theorem for the function $f(z)=5\ sin2z$ if c is the square with vertices at $1\pm i\ and -1\pm i$.	Understand	CHS004.6
9	Determine the value of line integral to $\int_C \frac{(\sin z)^6}{\left(z-\frac{\pi}{6}\right)^3}dz$ around the unit circle using Cauchy's integral formula.	Understand	CHS004.6
10	Determine the value of to $\int_c \frac{e^{2z}}{(z+1)^4}$ dz where c is $\lvert z-1\rvert=3$ using Cauchy's general integral formulae.	Understand	CHS004.6

11	Evaluate using cauchy's integral formula $\int_c \frac{z+1}{z^2+2z+4}dz$ Where $c:\|z+1+i\|=2$.	Understand	CHS004.6
12	Determine the value of line integral to $\int_C \left(y^2+z^2\right)dx+\left(z^2+x^2\right)dy+\left(x^2+y^2\right)dz$ from (0,0,0) to (1,1,1) where C is the curve x= t, y = t^2, z = t^3 in the parametric form.	Understand	CHS004.5
13	Estimate the value of $\int_c \frac{e^z}{z^2(z+1)^3}dz$ with C:$\|z\| = 2$ by Cauchy general integral formulae.	Understand	CHS004.6
14	Prove that if f(z) is analytic function then $\int_A^B f(z)dz$ is independent of path followed.	Remember	CHS004.5
15	Determine the value of line integral to $\int_0^{3+i} z^2\, dz$ along the parabola x=3y^2.	Understand	CHS004.5
16	Estimate the value of $\int_C \frac{1}{e^z(z-1)^3}dz$ with C:$\|z\| = 2$ by Cauchy general integral formulae.	Understand	CHS004.6
17	Determine the value of $\int_C \frac{e^z \sin 2z - 1}{z^2(z+2)^2}dz$ where c is $\|z\| = \frac{1}{2}$ using Cauchy integral formulae.	Understand	CHS004.6
18	Evaluate $\int_c \left[\frac{e^z}{z^3}+\frac{z^4}{(z-i)^2}\right]dz \quad ,c:\|z\|=2$ using Cauchy's integral formulae.	Remember	CHS004.6
19	Determine the value of line integral to $\int_C (z^2+3z)dz$ along the straight line from (2,0) to (2,2) and then from (2,2) to (0,2).	Understand	CHS004.5
20	Let C denote the boundary of a square whose sides lie along the lines x= $\pm$ 2 and y= $\pm$ 2 where C is described in positive sense. Then determine the value of line integral to $\int_C \frac{\cos hz}{z^4}dz$.	Understand	CHS004.5
Part - C (Problem Solving and Critical Thinking Questions)			
1	Determine the value of line integral to $\int_c \frac{z}{(z-1)(z-2)^2}$ dz where c is the circle $\|z-2\|$=1/2 using Cauchy's integral formula.	Understand	CHS004.6
2	Estimate the value of line integral to $\int_c \frac{z^4}{(z+1)(z-i)^2}$ dz where c is the ellipse 9x^2+4y^2=36 using Cauchy's integral formula.	Understand	CHS004.6
3	Estimate the value of line integral to $\int_c \frac{z^4-3z^2+6}{(z+1)^3}$dz where c is the circle $\|z\|=2$ using Cauchy's integral formula.	Understand	CHS004.6
4	Estimate the value of line integral to $\int_c \frac{z^2-2z-2}{(z^2+1)^2}$ dz where c is the circle	Understand	CHS004.6

	$\lvert z-i \rvert$ = 1/2 using Cauchy's integral formula.		
5	Estimate the value of line integral to $\int_c \frac{e^z}{(z^2+\pi^2)^2}dz$ where c is $\lvert z \rvert$ =4 using Cauchy's integral formula.	Understand	CHS004.6
6	Estimate the value of line integral to $\int_c \frac{\cos \pi z^2}{(z-1)(z-2)^3}$ dz where c is the circle $\lvert z \rvert$ =3 using Cauchy's integral formula.	Understand	CHS004.6
7	Determine the value of line integral to $\int_0^{1+i}(x-y+ix^2)$ dz i) Along the straight line from z = 0 to z = 1+i. ii) Along the real axis from z = 0 to z = 1 and then along a line parallel to imaginary axis from z = 1 to z = 1+i iii) Along the imaginary axis from z = 0 to z = i and then along a line parallel to real axis z = i to z = 1+ i.	Understand	CHS004.5
8	Verify Cauchy's theorem for the integral of $3z^2+iz-4$ taken over the boundary of the square with vertices -1+i ,-1-i,1+i ,-1-i.	Understand	CHS004.6
9	Derive the Cauchy general integral formulae of an analytic function f(z) within a closed contour c.	Remember	CHS004.6
10	Estimate the value of line integral to $\int_C (y^2+2xy)dx+(y^2-2xy)dy$ where C is the boundary of the region $y=x^2$ and $x=y^2$.	Understand	CHS004.5
UNIT-III			
POWER SERIES EXPANSION OF COMPLEX FUNCTION			
Part - A (Short Answer Questions)			
1	State Taylor's theorem of complex power series.	Remember	CHS004.9
2	State Laurent's theorem of complex power series.	Remember	CHS004.9
3	Define the term pole of order m of an analytic function f(z).	Remember	CHS004.11
4	Define the terms Essential and Removable singularity of an analytic function f(z).	Remember	CHS004.10
5	Expand f(z) = $\frac{1}{z^2}$ in powers of z+1 as a Taylor's series.	Understand	CHS004.9
6	Expand f(z) = e^z as Taylor's series about z = 1.	Understand	CHS004.9
7	Estimate the Poles of $\frac{1}{z^2-1}$.	Understand	CHS004.11
8	Obtain the Taylor series expansion of f(z) = e^z about the point z = 1.	Understand	CHS004.9
9	Determine the Poles of the function $f(z)=\frac{ze^z}{(z+2)^4(z-1)}$.	Understand	CHS004.11
10	Define the Isolated singularity of an analytic function f(z).	Understand	CHS004.10
11	State Cauchy's Residue theorem of an analytic function f(z) within and on the closed curve.	Remember	CHS004.11
12	Determine the Residue by Laurent's expansion to $f(z)=\frac{e^z}{(z-1)^2}$ about z=1.	Understand	CHS004.9

13	Estimate the Residues of the function $f(z)=\dfrac{1}{(z-\sin z)}$ about z = 0 by Laurent's expansion.	Understand	CHS004.9
14	Estimate the Residues of the function $f(z)=\dfrac{z}{(z+1)(z+2)}$ as a Laurent's series about z = -2.	Understand	CHS004.9
15	Estimate the value of $\oint_C \frac{1-2z}{z(z-1)(z-2)}dz$ by Cauchy's Residue theorem.	Understand	CHS004.11
16	Determine the Bilinear transformation whose fixed points are i,-i.	Understand	CHS004.13
17	Obtain the fixed points of the transformation $w=\dfrac{1}{z-2i}$	Understand	CHS004.13
18	Discover the Bilinear transformation which maps the points (0,-i,-1) into the points (i,1, 0)	Understand	CHS004.12
19	Discover the points at which w = coshz is not conformal.	Understand	CHS004.12
20	Discuss the fixed points of the transformation $w=\dfrac{2i-6z}{iz-3}$	Understand	CHS004.13
Part - B (Long Answer Questions)			
1	Expand f(z) = $\dfrac{z-1}{z+1}$ in Taylor's series about the point $z=1$.	Understand	CHS004.9
2	Expand f(z) = $\dfrac{z-1}{z^2}$ in Taylor's series in powers of z -1. Also determine the region of convergence about the point z = 1.	Understand	CHS004.9
3	Obtain Laurent's series expansion of f(z) = $\dfrac{z^2-4}{z^2+5z+4}$ valid in $1<z<4$.	Understand	CHS004.9
4	Expand $f(z)=\dfrac{e^{2z}}{(z-1)^3}$ about z = 1 as Laurent's series. Also find the region of convergence about z = 1.	Understand	CHS004.9
5	Expand f(z) = $\dfrac{7z-2}{z(z+1)(z-2)}$ about z=-1 in the region $1<\|z+1\|<3$ as Laurent's series .	Understand	CHS004.9
6	Expand $f(z)=\dfrac{2z^3+1}{z(z+1)}$ in Taylor's series about the point $z=1$	Understand	CHS004.9
7	Find Taylor's expansion of $f(z)=\dfrac{z+1}{(z-3)(z-4)}$ about the point z=2.Determine the region of convergence.	Understand	CHS004.9
8	Expand $f(z)=\cos z$ in taylor's series about $z=\pi i$.	Understand	CHS004.9
9	Obtain the Laurent's series expansion of $f(z)=\frac{e^z}{z(1-3z)}$ about z = 1.	Understand	CHS004.9
10	Express $f(z)=\frac{1+2z}{z^2+z^3}$ in a series of positive and negative powers of z.	Understand	CHS004.9
11	Estimate the value of $\displaystyle\int_c \frac{2z-1}{z(2z+1)(z+2)}dz$ where c is the circle $\|z\|=1$.	Understand	CHS004.11

12	Assess the value of $\oint_c \tan z dz$ where c is circle $\lvert z\rvert = 2$.	Understand	CHS004.11
13	Estimate the value of $\oint_c \frac{dz}{(z^2+4)^2}$ where c is $\lvert z-i\rvert = 2$.	Understand	CHS004.11
14	Calculate the value of $\oint_c \frac{\coth z}{z-i} dz$ where c is $\lvert z\rvert = 2$.	Understand	CHS004.11
15	Calculate the value of $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$	Understand	CHS004.11
16	Determine the Bi-linear transformation which carries the points from $(0,1,\infty) to (-5,-1,3)$.	Understand	CHS004.12
17	Determine the Bi-linear transformation which carries the points from $(1,i,-1) to (0,1,\infty)$.	Understand	CHS004.12
18	Determine the Bilinear transformation that maps the points (1-2i, 2+i,2+3i) into the points (2+i,1+3i,4).	Understand	CHS004.12
19	Determine the Bilinear transformation that maps the points (1, i,-1) into the points (2,i,-2).	Understand	CHS004.12
20	Determine the Bilinear transformation that maps the points $(\infty, i, 0)$ into the points (0,i,∞).	Remember	CHS004.12
Part - C (Problem Solving and Critical Thinking Questions)			
1	Obtain the Laurent expansion of f(z) = $\frac{1}{z^2-4z+3}$ for 1 < $\lvert z\rvert < 3$ (ii) $\lvert z\rvert <$ 1 (iii) $\lvert z\rvert > 3$.	Understand	CHS004.9
2	Expand f(z) = $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region where $(i)\lvert z\rvert < 1$ (ii)1< $\lvert z\rvert < 4$.	Understand	CHS004.9
3	Expand $\frac{1}{z^2(z-3)^2}$ as Laurent's series in the region $(i)\lvert z\rvert < 1$ $(ii)\lvert z\rvert > 3$.	Understand	CHS004.9
4	Expand f(z) = $\frac{2}{(2z+1)^3}$ in Taylor's series about z=0 and z=2.	Understand	CHS004.9
5	Expand f(z) = $\frac{e^z}{z(z+1)}$ in Taylor's series about z=2.	Understand	CHS004.9
6	Determine the value of $\oint_c \frac{z-3}{(z^2+2z+5)} dz$ where c is circle $\lvert z\rvert = 1$.	Understand	CHS004.10
7	Estimate the value of $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$	Remember	CHS004.10
8	Discover the Bilinear transformation that maps the points (0,1,∞) into the points (-1,-2,-i).	Understand	CHS004.12
9	Obtain the fixed points of the transformation $w = \frac{3iz+13}{z-3i}$	Understand	CHS004.13
10	Determine the Bilinear transformation that maps the points $(\infty, i, 0)$ in the z-plane into the points $(0, i, \infty)$ in the w-plane.	Understand	CHS004.12
UNIT-IV			

	SINGLE RANDOM VARIABLES		
	Part - A (Short Answer Questions)		
1	Define the discrete and continuous random variables with a suitable example.	Remember	CHS004.15
2	List the important Properties of probability density function.	Remember	CHS004.15
3	Obtain the probability distribution of getting number tails if we toss three coins.	Remember	CHS004.15
4	Define the term mathematical expectation of a probability distribution function.	Remember	CHS004.16
5	If X is discrete random variable then Prove that Variance of (aX + b) = a^2 Variance of (X).	Remember	CHS004.16
6	Define the term probability mass function of a probability distribution.	Remember	CHS004.15
7	If X denote random variable, prove that E[X-K} = E{X] - K where 'K' is a constant.	Remember	CHS004.16
7	If X denote random variable, Prove that E[X+K] = E[X] + K where 'K' is a constant.	Remember	CHS004.16
8	List the important properties of probability mass function.	Remember	CHS004.15
9	Explain the term Moment generating function of a probability distribution.	Remember	CHS004.17
10	Express the relation between the probability density and cumulative density function of a random variable.	Remember	CHS004.15
11	Define the term Mean and Variance of a probability mass function.	Remember	CHS004.16
12	Define the term Mean and Variance of a probability density function.	Remember	CHS004.16
13	Define the term probability density function of a probability distribution.	Understand	CHS004.15
14	Define the moments for distribution.	Understand	CHS004.17
15	Obtain the first 4 moments for the set of numbers 2, 4, 6 and 8.	Understand	CHS004.17
16	A die is thrown at random. What is the expectation of a number on it.	Understand	CHS004.15
17	The probability density function of a random variable x is $f(x) = \frac{1}{2}\exp\left[-\frac{x}{2}\right], \quad x > 0$. Estimate the value of the probability of 1 < x < 2.	Understand	CHS004.16
18	Probability density function of a random variable X is $f(x) = \begin{cases} \frac{sinx}{2} & ,0 \le x \le \pi \\ 0, elsewhere & \end{cases}$. Find the mean of the random variable X.	Understand	CHS004.16
19	A continuous random variable has the probability density function $f(x) = \begin{cases} kxe^{-\lambda x}, for\ x \ge 0, \lambda > 0 \\ 0, otherwise \end{cases}$ Determine the value of k.	Remember	CHS004.16
20	Obtain the value of $P(0 < x < 2)$ to the Probability density function of a random variable X where $f(x) = \begin{cases} \frac{sinx}{2} & ,0 \le x \le \pi \\ 0, elsewhere & \end{cases}$.	Understand	CHS004.16
	Part - B (Long Answer Questions)		
1	A random variable x has the following probability function: $\begin{array}{|c|c|c|c|c|c|c|c|} \hline X & 0 & 1 & 3 & 4 & 5 & 6 & 7 \\ \hline P(x) & 0 & k & 2k & 2k & 3k & k^2 & 7k^2+k \\ \hline \end{array}$ Find (i) k (ii) P(x<6) (iii) p(x>6)	Understand	CHS004.16
2	Let X denotes the minimum of the two numbers that appear when a pair of	Understand	CHS004.16

<table>
<tr><td></td><td>fair dice is thrown once. Find
(i)Discrete probability distribution (ii) Expectation (iii) Variance</td><td></td><td></td></tr>
<tr><td>3</td><td>A random variable X has the following probability function
<table>
<tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P(x)</td><td>0.1</td><td>K</td><td>0.2</td><td>2K</td><td>0.3</td><td>K</td></tr>
</table>
Calculate (i) k (ii) mean (iii) variance (iv) P(0 < x < 3)</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>4</td><td>A continuous random variable has the probability density function
$f(x)=\begin{cases} kxe^{-\lambda x},\ for\ x\geq 0, \lambda>0 \\ 0, otherwise \end{cases}$
Determine(i) k (ii) Mean (iii) Variance</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>5</td><td>If the Probability density function of random variable is
$f(x)=k(1-x^2), 0<x<1$ then Calculate
(i) k (ii) p(0.1 < x< 0.2) (iii) P(x > 0.5)</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>6</td><td>Two coins are simultaneously ,Let X denotes the number of heads then find expectation of X and variance of X.</td><td>Understand</td><td>CHS004.15</td></tr>
<tr><td>7</td><td>If the Probability density function of a random variable is
f(x) = $k(1+x^2), 0<x<2$ then Calculate
(i) k (ii) P(0.2 < x< 0.3) (iii) P(x > 0.7)</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>8</td><td>If a random variable X has the moment generating function is given by M(t) = $\frac{2}{2-t}$, find the variance of X.</td><td>Understand</td><td>CHS004.17</td></tr>
<tr><td>9</td><td>Let X be the random variable of the following values x=1,2,3 if f(x) =x/6.
Then find mean and variance.</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>10</td><td>Obtain the moment generating function of a random variable X having the probability density function $f(x)=\begin{cases} x, 0\leq x<1 \\ 2-x, 1\leq x<2 \\ 0, elsewhere \end{cases}$</td><td>Understand</td><td>CHS004.17</td></tr>
<tr><td>11</td><td>List the relation between moment about mean and moment about origin.</td><td>Understand</td><td>CHS004.17</td></tr>
<tr><td>12</td><td>Is the function defined by $f(x)=\begin{cases} 0, & x<2 \\ \frac{1}{18}(2x+3), & 2\leq x\leq 4 \\ 0 & ,x>4 \end{cases}$ a probability density function? Find the probability that a variate having f(x) as density function will fall in the interval $2\leq x\leq 3$.</td><td>Remember</td><td>CHS004.16</td></tr>
<tr><td>13</td><td>If E(X) = 10 ,v(x)=1 then find E[2x (x+20)].</td><td>Remember</td><td>CHS004.15</td></tr>
<tr><td>14</td><td>Find the probability distribution for sum of scores on dice if we throw two dice simultaneously.</td><td>Understand</td><td>CHS004.15</td></tr>
<tr><td>15</td><td>A discrete random variable X has the following probability distribution
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>P(X=x)</td><td>2k</td><td>4k</td><td>6k</td><td>8k</td><td>10k</td><td>12k</td><td>14k</td><td>4k</td></tr>
</table>
Find (i) k (ii) p(X<3) (iii) $p(X\geq 5)$</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>16</td><td>Let X be a random variable which can take on the values 1, 2 and 3with probabilities 1/3, 1/6 and 1/2. Calculate the third moment about mean.</td><td>Understand</td><td>CHS004.17</td></tr>
<tr><td>17</td><td>A random variable has the probability density function $f(x)=x^2, 1\leq x\leq 2$
Find its moment generating function.</td><td>Understand</td><td>CHS004.17</td></tr>
</table>

<table>
<tr><td>18</td><td>The density function of a random variable X is $f(x)=\begin{cases} e^{-x} & ,x\ge 0 \\ 0 & ,otherwise\end{cases}$
Find E(X), $E(X^2)$, V(X).</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>19</td><td>Compute the first four moments about the mean for the following distribution
<table><tr><td>Marks</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td></tr><tr><td>No. of students</td><td>8</td><td>12</td><td>20</td><td>30</td><td>15</td><td>10</td><td>5</td></tr></table>Also find the values of β_1 and β_2.</td><td>Understand</td><td>CHS004.17</td></tr>
<tr><td>20</td><td>Determine the value of A to the probability density function
$f(x) = Ax^2$ in $0 < x < 1$</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td colspan="4">Part - C (Problem Solving and Critical Thinking Questions)</td></tr>
<tr><td>1</td><td>Find the Mean and Variance to the following discrete distribution
<table><tr><td>X</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td></tr><tr><td>Y</td><td>1/8</td><td>1/6</td><td>3/8</td><td>1/4</td><td>1/2</td></tr></table></td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>2</td><td>A random variable X has the following probability function.
<table><tr><td>X</td><td>4</td><td>5</td><td>6</td><td>8</td></tr><tr><td>P(X)</td><td>0.1</td><td>0.3</td><td>0.4</td><td>0.2</td></tr></table>Determine (i) Expectation (ii) variance (iii)Standard deviation.</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>3</td><td>Out of 24 mangoes, 6 mangoes are rotten. If we draw two mangoes, then obtain probability distribution of number of rotten mangoes that can be drawn.</td><td>Remember</td><td>CHS004.15</td></tr>
<tr><td>4</td><td>If X is a Continuous random variable whose density function is
$$f(x)=\begin{cases} x & if\ 0 < x < 1 \\ 2-x & if\ 1 \le x < 2 \\ 0 & elsewhere\end{cases}$$Find $E(25X^2+30X-5)$.</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>5</td><td>The probability density function of a random variable X is
$f(x)=\frac{K}{x^2+1}$, $-\infty < x < \infty$. Find K and the distribution function F(x).</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>6</td><td>If the probability density of a random variable X is given by $f(x)=\begin{cases} k(1-x^2), 0 < x < 1 \\ 0, otherwise\end{cases}$ Find (i) k (ii) The cumulative distribution function of X.</td><td>Understand</td><td>CHS004.16</td></tr>
<tr><td>7</td><td>The first three moments of a distribution about the value 2 of the variable are1, 16, and -40. Show that the mean = 3, the variance = 15 and μ_3 = - 86.</td><td>Understand</td><td>CHS004.17</td></tr>
<tr><td>8</td><td>Explain moments at origin of a probability distribution function.</td><td>Remember</td><td>CHS004.17</td></tr>
<tr><td>9</td><td>Explain the moment generating function of a probability distribution function.</td><td>Remember</td><td>CHS004.17</td></tr>
<tr><td>10</td><td>Explain the relation between the moments about mean in terms of moments bout arbitrary origin.</td><td>Remember</td><td>CHS004.17</td></tr>
<tr><td colspan="4">UNIT-V</td></tr>
<tr><td colspan="4">PROBABILITY DISTRIBUTIONS</td></tr>
<tr><td colspan="4">Part - A (Short Answer Questions)</td></tr>
<tr><td>1</td><td>Define the terms mean, variance of Binomial distribution.</td><td>Remember</td><td>CHS004.19</td></tr>
<tr><td>2</td><td>Draft the recurrence relation for the Binomial distribution.</td><td>Remember</td><td>CHS004.19</td></tr>
<tr><td>3</td><td>Define the term mode of a Binomial distribution.</td><td>Remember</td><td>CHS004.19</td></tr>
</table>

4	Determine the value of n if the mean and variance of a Binomial distribution are 3 and 9/4.	Understand	CHS004.19
5	Determine the Binomial distribution for which the mean is 4 and variance 3	Understand	CHS004.19
6	The mean and variance of a binomial variable X with parameters n and p are 16 and 24.Determine the value of P(X=1).	Remember	CHS004.19
7	If a bank received on the average 6 bad cheques per day,Find the probability that it will receive 4 bad cheques on any given day.	Understand	CHS004.19
8	Define the terms Mean, Variance of Poisson distribution	Remember	CHS004.19
9	If X is a Poisson variate with P(x=2) = 2/3P(x = 1) Compute the value of P(x=0).	Understand	CHS004.19
10	Draft the recurrence relation for the Poisson distribution.	Remember	CHS004.19
11	The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $p(X \geq 1)$.	Remember	CHS004.19
12	If a bank received on the average 6 bad apples per day then estimate the probability that it will receive 4 bad cheques on any given day.	Understand	CHS004.19
13	If 2% of light bulbs are defective in a sample of 100.Find at least one is defective.	Remember	CHS004.19
14	If a random variable has Poisson distribution such that p (1) = p (2). Determine the value of p (1< x < 4).	Understand	CHS004.19
15	Define Poisson distribution.	Remember	CHS004.19
16	Define the term Normal Distribution.	Remember	CHS004.20
17	Define Binomial distribution.	Remember	CHS004.20
18	Define Normal curve.	Remember	CHS004.20
19	Draft the applications of Normal distribution.	Remember	CHS004.20
20	If X is Normally distributed with mean 2 and variance 0.1, then Estimate the value of P($\lvert x-2 \rvert \geq 0.01$)	Understand	CHS004.20
Part - B (Long Answer Questions)			
1	Derive the Variance of a Binomial Distribution.	Remember	CHS004.19
2	Estimate the probability that at most 5 defective components will be found in a lot of 200. Experience shows that 2% of such components are defective. Also find the probability of more than 5 defective components.	Understand	CHS004.19
3	The probability that a man hitting a target is 1/3. If he fires 5 times , Determine the probability that he fires (i) At most 5 times (ii) At least 2 times	Understand	CHS004.19
4	Find the variance of a Poisson Distribution.	Remember	CHS004.19
5	Poisson variate has a double mode at x=2 and x=3, Determine the maximum probability and also find p($x \geq 2$)	Understand	CHS004.19
6	Average number of accidents on any day on a national highway is 1.8. Determine the probability that the number of accidents is (i) at least one (ii) at most one	Understand	CHS004.19
7	A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed to Poisson distribution with mean 1.5. Find the proportion of days (i)on which there is no demand (ii)on which demand is refused.	Understand	CHS004.19
8	If x is a poisson variate such that p(x=2)=45p(x=6)-3p(x=4).Find (i)p($x \geq 1$) (ii) p(x<2)	Remember	CHS004.20
9	Derive median of the Normal distribution.	Remember	CHS004.20
10	Explain the variance of a Normal Distribution.	Remember	CHS004.20

<table>
<tr><td>11</td><td>Explain the mode of Normal distribution.</td><td>Remember</td><td>CHS004.20</td></tr>
<tr><td>12</td><td>Prove that mean deviation from the mean for Normal distribution is $\frac{4\sigma}{5}$ approximately.</td><td>Remember</td><td>CHS004.20</td></tr>
<tr><td>13</td><td>Prove that poisson distribution is limiting case of binomial distribution .</td><td>Remember</td><td>CHS004.20</td></tr>
<tr><td>14</td><td>If the masses of 300 students are normally distributed with mean 68 kg and standard deviation3 kg how many number of students have masses: greater than 72 kg (ii) less than or equal to 64 kg (iii) between 65 and 71 kg inclusive</td><td>Understand</td><td>CHS004.20</td></tr>
<tr><td>15</td><td>In a Normal distribution, 7% of the item are under 35 and 89% are under 63. Compute the mean and standard deviation of the distribution</td><td>Understand</td><td>CHS004.20</td></tr>
<tr><td>16</td><td>It has been found that 2% of the tools produced by a certain machine are defective. Estimate the probability that in a shipment of 400 such tools, i) 3% or more ii) 2% are less will prove defective.</td><td>Understand</td><td>CHS004.20</td></tr>
<tr><td>17</td><td>In a Normal distribution, 31% of the items are under 45 and 8% are over 64. Estimate the mean and variance of the distribution.</td><td>Understand</td><td>CHS004.20</td></tr>
<tr><td>18</td><td>If X is a Normal variate then determine the area A.
i) to the left of z = -1.78 ii) to the right of z = -1.45 iii) corresponding to $-0.8 \leq z \leq 1.53$ iv) to the left of z = -2.52 and the right of 1.83. Show the above by graphs.</td><td>Understand</td><td>CHS004.20</td></tr>
<tr><td>19</td><td>1000 students have written an examination with the mean of test is 35 and standard deviation is 5. Assuming the distribution to be normal find i) How many students marks like between 25 and 40? ii) How many students get more than 40? iii) How many students get below 20? iv) How many students get more than 50.</td><td>Understand</td><td>CHS004.20</td></tr>
<tr><td>20</td><td>The mean height of students in a college is 155cm and standard deviation is 15. Estimate the probability that mean height of 36 students is less than 157cm.</td><td>Understand</td><td>CHS004.20</td></tr>
<tr><td colspan="4">Part - C (Problem Solving and Critical Thinking Questions)</td></tr>
<tr><td>1</td><td>Fit a Binomial Distribution to the following data
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>f</td><td>305</td><td>365</td><td>210</td><td>80</td><td>28</td><td>9</td><td>2</td><td>1</td></tr></table></td><td>Remember</td><td>CHS004.19</td></tr>
<tr><td>2</td><td>It has been claimed that in 60%of all solar heat installations that utility bill is reduced by atleast one –third .Accordingly, What are the probabilities that the utility bill will be reduced by at least one –third in (i) four or five instalations (ii) at least four of five instalations.</td><td>Understand</td><td>CHS004.19</td></tr>
<tr><td>3</td><td>Fit a Binomial Distribution to the following data
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>f</td><td>2</td><td>14</td><td>20</td><td>34</td><td>22</td><td>8</td></tr></table></td><td>Understand</td><td>CHS004.19</td></tr>
<tr><td>4</td><td>Show that the mean, mode and median are equal in poisson distribution.</td><td>Remember</td><td>CHS004.19</td></tr>
<tr><td>5</td><td>Derive mean of the Normal distribution.</td><td>Understand</td><td>CHS004.19</td></tr>
<tr><td>6</td><td>Show that the recurrence relation for the Poisson distribution is
$$P(x) = \frac{\lambda}{x}\,.\,P(x-1)$$</td><td></td><td>CHS004.19</td></tr>
<tr><td>7</td><td>The marks obtained in statistics in a certain examination found to be normally distributed. If 15% of the students greater than or equal to 60 marks ,40% less than 30 marks. Find the mean and standard detion.</td><td>Understand</td><td>CHS004.19</td></tr>
<tr><td>8</td><td>The life of electronic tubes of a certain types may be assumed to be normal distributed with mean 155 hours and standard deviation 19 hours. Determine the probability that the life of a randomly chosen tube
(i) is between 136 hours and 174 hours.
(ii) less than 117 hours</td><td>Understand</td><td>CHS004.19</td></tr>
</table>

	(iii) will be more than 395 hours		
9	Derive the mean of the Binomial Distribution.	Remember	CHS004.20
10	The marks obtained in mathematics by 1000 students are Normally distributed with mean 78% and standard deviation 11%. Determine (i)How many students got marks above 90% marks (ii)What was the highest mark obtained by the lowest 10% of the students (iii)Within what limits did the middle of 90% of the student lie.	Understand	CHS004.20

Prepared By:
Ms. C Rachana, Assistant Professor

HOD, FRESHMAN ENGINEERING