



# INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad-500043

## FRESHMAN ENGINEERING

### TUTORIAL QUESTION BANK

Course Name	:	Computational Mathematics and Integral Calculus
Course Code	:	AHS003
Class	:	B. Tech II Semester
Branch	:	Common for AE / ME / CE
Year	:	2017 - 2018
CourseCoordinator	:	Ms. P Rajani, Associate Professor
Course Faculty	:	Dr. S Jagadha, Professor Mr. Ch Kumara Swamy, Associate Professor Ms. V Subbalaxmi, Associate Professor Mr. G Nagendrakumar, Assistant Professor

#### COURSE OBJECTIVES (COs):

The course should enable the students to:

I	Enrich the knowledge of solving algebraic, transcendental and differential equation by numerical methods.
II	Apply multiple integration to evaluate mass, area and volume of the plane.
III	Analyze gradient, divergence and curl to evaluate the integration over a vector field.
IV	Understand the Bessels equation to solve them under special conditions with the help of series solutions

#### COURSE LEARNING OUTCOMES (CLOs):

Students, who complete the course, will have demonstrated the asking to do the following:

CAHS003.01	Solve the algebraic and transcendental equations using bisection method, method of false position and Newton-Raphson method.
CAHS003.02	Apply numerical methods to interpolate the functions of values for equal intervals using finite differences.
CAHS003.03	Understand the Newton rapson method to the real-world problem for a finite barrier quantum well.
CAHS003.04	Evaluate the functional value by using lagranges interpolation formula for unequal intervals.
CAHS003.05	Understand the lagranges interpolation in real-world problem for neural network learning.
CAHS003.06	Apply method of least squares to fit linear and non linear curves.
CAHS003.07	Solve differential equation using single step method- Taylor's series.
CAHS003.08	Solve differential equation using multi step methods- Euler's, Modified Euler's and Runge-Kutta methods.
CAHS003.09	Understand the multistep methods in real-world problem for real time Aircraft dynamics.
CAHS003.10	Understand the Runge-kutta method in real-world problem for embedding the sensor signals into the iterative computation
CAHS003.11	Evaluate double integral and triple integrals.
CAHS003.12	Utilize the concept of change order of integration to evaluate double integrals.

CAHS003.13	Determine the area and volume of a given curve.
CAHS003.14	Understand transformation of co-ordinate system from plane to plane.
CAHS003.15	Analyze scalar and vector fields and compute the gradient, divergence and curl.
CAHS003.16	Understand integration of vector function.
CAHS003.17	Evaluate line, surface and volume integral of vectors.
CAHS003.18	Use Vector integral theorems to facilitate vector integration.
CAHS003.19	Analyze the concept of vector calculus in real-world problem for fluid dynamics.
CAHS003.20	Solve the Differential Equations by series solutions
CAHS003.21	Understand Gamma function to evaluate improper integrals.
CAHS003.22	Analyze Bessel's function and study its properties.
CAHS003.23	Analyze Bessel's function as a Solution to Schrödinger equation in a cylindrical function of the second kind.
CAHS003.24	Understand gamma function to finds application in such diverse areas as quantum physics, astrophysics and fluid dynamics.
CAHS003.25	Possess the knowledge and skills for employability and to succeed in national and international level competitive examinations.

### TUTORIAL QUESTION BANK

UNIT - I															
ROOT FINDING TECHNIQUES AND INTERPOLATION															
Part - A (Short Answer Questions)															
S No	QUESTIONS	Blooms Taxonomy Level	Course Learning Outcomes (CLOs)												
1	Define the term Interpolation.	Remember	CAHS003.02												
2	State Newton's forward interpolation formula for equal length of intervals.	Remember	CAHS003.02												
3	State Newton's backward interpolation formula for equal length of intervals.	Remember	CAHS003.02												
4	State Gauss forward interpolation formula for equal length of intervals.	Remember	CAHS003.02												
5	Define average operator and shift operator.	Remember	CAHS003.02												
6	Prove the relationship between forward difference operator and shift operator.	Understand	CAHS003.02												
7	Prove the relationship between backward difference operator and shift operator.	Understand	CAHS003.02												
8	Prove that relationship between forward and backward difference operator.	Understand	CAHS003.02												
9	Construct a forward difference table for $f(x)=x^3+5x-7$ if $x=-1,0,1,2,3,4,5$ .	Understand	CAHS003.02												
10	For what values of $p$ the Gauss backward interpolation formula is used to interpolate?	Understand	CAHS003.02												
11	For what values of $p$ the Gauss forward interpolation formula is used to interpolate?	Understand	CAHS003.02												
12	By using Regula-Falsi method, find an approximate root of the equation $x^4 - x - 10 = 0$ that lies between 1.8 and 2. Carry out two approximations.	Understand	CAHS003.01												
13	Evaluate the forward difference of $\cos x$ .	Understand	CAHS003.02												
14	Find the value of $y$ at $x = 3$ in the following table. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>x</math></td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td><math>y</math></td> <td>1</td> <td>3</td> <td>9</td> <td>--</td> <td>81</td> </tr> </table>	$x$	0	1	2	3	4	$y$	1	3	9	--	81	Understand	CAHS003.02
$x$	0	1	2	3	4										
$y$	1	3	9	--	81										
15	Apply Newton –Raphson method to find an approximate root of the equation	Understand	CAHS003.01												

	$x^3 - 3x - 5 = 0$ , which lies near $x=2$ carry out two approximations.																
16	Find a real root of the transcendental equation $xe^x = 2$ using method of False Position carry out three approximations.	Understand	CAHS003.01														
17	Evaluate the forward difference of $\log f(x)$ .	Understand	CAHS003.01														
18	Find a real root of the transcendental equation $xe^x - \cos x = 0$ using Newton – Raphson method carry out three approximations.	Understand	CAHS003.01														
19	Find by using Bisection method the real root of the equation $xe^x - 3 = 0$ carryout three approximations.	Understand	CAHS003.01														
20	If $f(x)$ and $g(x)$ are two functions then evaluate forward difference of product of $f(x)$ and $g(x)$ .	Understand	CAHS003.02														
<b>Part - B (Long Answer Questions)</b>																	
1	Find the positive root of $x^3 - x - 1 = 0$ using Bisection method.	Remember	CAHS003.01														
2	Find a real root of the transcendental equation $e^x \sin x = 1$ by using False position method correct up to two decimals.	Remember	CAHS003.01														
3	Solve transcendental equation $2x = \cos x + 3$ by Newton-Raphson method correct up to two decimals.	Understand	CAHS003.01														
4	Find a real root of transcendental equation $x = \cos x$ using method of False position correct up to three decimals.	Remember	CAHS003.01														
5	Find a real root of transcendental equation $3x - \cos x - 1 = 0$ using Newton Raphson method correct up to three decimals.	Remember	CAHS003.01														
6	Find the real root of the equation $x^2 - \log x - 1.0 = 0$ by newton raphson method up to three decimal places.	Remember	CAHS003.01														
7	Find the real root of the equation $x^2 - \log_{10} x - 1.2 = 0$ by false position method up to three decimal places.	Remember	CAHS003.01														
8	Find a real root of the transcendental equation $x \tan x + 1 = 0$ by Newton- Raphson method correct up to three decimals.	Remember	CAHS003.01														
9	Find $y(2.8)$ for the following data using Newton's forward interpolation formula. <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td><math>x</math></td> <td>2.4</td> <td>3.2</td> <td>4.0</td> <td>4.8</td> <td>5.6</td> </tr> <tr> <td><math>f(x)</math></td> <td>22</td> <td>17.8</td> <td>14.2</td> <td>38.3</td> <td>51.7</td> </tr> </tbody> </table>	$x$	2.4	3.2	4.0	4.8	5.6	$f(x)$	22	17.8	14.2	38.3	51.7	Remember	CAHS003.02		
$x$	2.4	3.2	4.0	4.8	5.6												
$f(x)$	22	17.8	14.2	38.3	51.7												
10	Using Newton's backward formula find the polynomial of degree 3 passing through (3,6),(4,24),(5,20)and (6,120).	Remember	CAHS003.02														
11	Find $f(42)$ from the following data using Newton's Backward interpolation formula. <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td><math>x</math></td> <td>20</td> <td>25</td> <td>30</td> <td>35</td> <td>40</td> <td>45</td> </tr> <tr> <td><math>y</math></td> <td>354</td> <td>332</td> <td>291</td> <td>260</td> <td>231</td> <td>204</td> </tr> </tbody> </table>	$x$	20	25	30	35	40	45	$y$	354	332	291	260	231	204	Remember	CAHS003.02
$x$	20	25	30	35	40	45											
$y$	354	332	291	260	231	204											
12	Find $y(25)$ given that $y(20)=24$ , $y(24)=32$ , $y(28)=35$ , $y(32)=40$ using Gauss forward interpolation formula.	Remember	CAHS003.02														
13	Find by Gauss's backward interpolating formula the value of $y$ at $x = 1936$ using the following table. <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td><math>x</math></td> <td>1901</td> <td>1911</td> <td>1921</td> <td>1931</td> <td>1941</td> <td>1951</td> </tr> <tr> <td><math>y</math></td> <td>12</td> <td>15</td> <td>20</td> <td>27</td> <td>39</td> <td>52</td> </tr> </tbody> </table>	$x$	1901	1911	1921	1931	1941	1951	$y$	12	15	20	27	39	52	Remember	CAHS003.02
$x$	1901	1911	1921	1931	1941	1951											
$y$	12	15	20	27	39	52											
14	Find by Gauss's backward interpolating formula the value of $y$ at $x = 14$ using the following table. <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td><math>x</math></td> <td>0</td> <td>5</td> <td>10</td> <td>15</td> <td>20</td> <td>25</td> </tr> <tr> <td><math>y</math></td> <td>7</td> <td>11</td> <td>14</td> <td>18</td> <td>24</td> <td>32</td> </tr> </tbody> </table>	$x$	0	5	10	15	20	25	$y$	7	11	14	18	24	32	Remember	CAHS003.02
$x$	0	5	10	15	20	25											
$y$	7	11	14	18	24	32											
15	Find $f(1.6)$ using Lagrange's formula from the following table. <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td><math>x</math></td> <td>1.2</td> <td>2.0</td> <td>2.5</td> <td>3.0</td> </tr> <tr> <td><math>f(x)</math></td> <td>1.36</td> <td>0.58</td> <td>0.34</td> <td>0.20</td> </tr> </tbody> </table>	$x$	1.2	2.0	2.5	3.0	$f(x)$	1.36	0.58	0.34	0.20	Remember	CAHS003.04				
$x$	1.2	2.0	2.5	3.0													
$f(x)$	1.36	0.58	0.34	0.20													
16	Find $y(5)$ given that $y(0)=1$ , $y(1)=3$ , $y(3)=13$ and $y(8)=123$ using Lagrange's interpolation formula.	Remember	CAHS003.04														
17	Find $y(10)$ , given that $y(5)=12$ , $y(6)=13$ , $y(9)=14$ , $y(11)=16$ using Lagrange's interpolation formula.	Remember	CAHS003.04														

18	Find the real root algebraic equation $x^3 - x - 4 = 0$ by Bisection method correct up to four decimals.	Remember	CAHS003.01
19	Find the square root of 26 up to 2 decimal places by using Newton-Raphson method.	Remember	CAHS003.01
20	Obtain the interpolating polynomial passing through the points (0,1),(1,3),(2,7),and (3,13) and hence find f(2.5).	Remember	CAHS003.02

**Part - C (Problem Solving and Critical Thinking Questions)**

1	Derive a formula to find a cube root of N using Newton-Raphson method and hence find cube root of 15.	Understand	CAHS003.01														
2	Find reciprocal of real number 18 using Newton-Raphson method.	Remember	CAHS003.01														
3	Evaluate f(10) given f(x)=168,192,336 at x=1,7,15 respectively using Lagranges interpolation formula.	Understand	CAHS003.04														
4	Prove that $\Delta[x(x+1)(x+2)(x+3)]=4(x+1)(x+2)(x+3)$ by taking difference as unity.	Understand	CAHS003.02														
5	Find y(1.6) from the following data using Newton's forward interpolation formula. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>1</td> <td>1.4</td> <td>1.8</td> <td>2.2</td> </tr> <tr> <td>y</td> <td>3.49</td> <td>4.82</td> <td>5.96</td> <td>6.5</td> </tr> </table>	x	1	1.4	1.8	2.2	y	3.49	4.82	5.96	6.5	Understand	CAHS003.02				
x	1	1.4	1.8	2.2													
y	3.49	4.82	5.96	6.5													
6	Using Gauss back ward difference formula find y(24) from the following table. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>5</td> <td>10</td> <td>15</td> <td>20</td> <td>25</td> </tr> <tr> <td>y</td> <td>7</td> <td>11</td> <td>14</td> <td>18</td> <td>24</td> <td>32</td> </tr> </table>	x	0	5	10	15	20	25	y	7	11	14	18	24	32	Understand	CAHS003.02
x	0	5	10	15	20	25											
y	7	11	14	18	24	32											
7	Find a root of the equation $4\sin x = e^x$ using Bisection method correct up to four decimals.	Understand	CAHS003.01														
8	Find a root of the equation $2x - \log x = 7$ using the False Position method correct up to four decimals.	Understand	CAHS003.01														
9	Find a root of the equation $\cos x = 2x - 3$ using Newton- Raphson method in four stages.	Understand	CAHS003.01														
10	Compute f(0.3) for the data using Lagrange's interpolation formula. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>3</td> </tr> <tr> <td>y</td> <td>1</td> <td>3</td> <td>49</td> </tr> </table>	x	0	1	3	y	1	3	49	Understand	CAHS003.04						
x	0	1	3														
y	1	3	49														

**UNIT-II**

**CURVE FITTING AND NUMERICAL SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS**

**Part - A (Short Answer Questions)**

1	State the normal equations of the straight line $y = a + bx$ .	Understand	CAHS003.06								
2	State the normal equations of the second degree equation $y = a + bx + cx^2$ .	Understand	CAHS003.06								
3	State the normal equations to fit the exponential curve of the form $y = ae^{bx}$ .	Remember	CAHS003.06								
4	State the normal equations to fit the power curve of the form $y = ab^x$ .	Remember	CAHS003.06								
5	If $y = a + \frac{b}{x}$ is a curve then write normal equations to find the constants a and b.	Remember	CAHS003.06								
6	If $y = a_0 + a_1 x + a_2 x^2$ then what is the third normal equation of $\sum x_i^2 y_i$ by least squares method?	Remember	CAHS003.06								
7	If $y = a_0 + a_1 x^2$ , then what is the first normal equation of $\sum y_i$ ?	Remember	CAHS003.06								
8	If $y = ax^b$ , then what is the first normal equation of $\sum \log y_i$ ?	Remember	CAHS003.06								
9	Fit a curve of the form $y = ax^b$ by the method of least squares to the following data. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>y</td> <td>2.98</td> <td>4.26</td> <td>5.21</td> </tr> </table>	x	1	2	3	y	2.98	4.26	5.21	Remember	CAHS003.06
x	1	2	3								
y	2.98	4.26	5.21								

10	Fit a second degree parabola to the following data by the method of least squares. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>y</td> <td>1</td> <td>1.8</td> <td>1.3</td> </tr> </table>	x	0	1	2	y	1	1.8	1.3	Remember	CAHS003.06				
x	0	1	2												
y	1	1.8	1.3												
11	Fit the curve of the form $y = ae^{bx}$ by the method of least squares to the following data. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>y</td> <td>20</td> <td>30</td> <td>52</td> </tr> </table>	x	0	1	2	y	20	30	52	Remember	CAHS003.06				
x	0	1	2												
y	20	30	52												
12	Fit a straight line to the form $y = a + bx$ by the method of least squares for the following data. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>5</td> <td>10</td> </tr> <tr> <td>y</td> <td>12</td> <td>15</td> <td>17</td> </tr> </table>	x	0	5	10	y	12	15	17	Remember	CAHS003.06				
x	0	5	10												
y	12	15	17												
13	Fit a curve $y = ae^{bx}$ to the data. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>2</td> <td>4</td> </tr> <tr> <td>y</td> <td>5.1</td> <td>10</td> <td>31.1</td> </tr> </table>	x	0	2	4	y	5.1	10	31.1	Remember	CAHS003.06				
x	0	2	4												
y	5.1	10	31.1												
14	A chemical company wishing to study the effect of extraction time on the efficiency of an extraction operation obtained the data shown in the following table. Fit a straight line to the given data by the method of least squares. <table border="1" style="margin-left: 20px;"> <tr> <td>Extraction time in min(x)</td> <td>27</td> <td>45</td> <td>41</td> </tr> <tr> <td>Efficiency (y)</td> <td>57</td> <td>64</td> <td>80</td> </tr> </table>	Extraction time in min(x)	27	45	41	Efficiency (y)	57	64	80	Remember	CAHS003.06				
Extraction time in min(x)	27	45	41												
Efficiency (y)	57	64	80												
15	Fit a parabola of the form $y = a + bx + cx^2$ to the following data. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>y</td> <td>2.3</td> <td>5.2</td> <td>9.7</td> </tr> </table>	x	1	2	3	y	2.3	5.2	9.7	Remember	CAHS003.06				
x	1	2	3												
y	2.3	5.2	9.7												
16	Fit the curve of the form $y = ab^x$ by the method of least squares to the following data. <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>y</td> <td>1</td> <td>1.8</td> <td>1.3</td> </tr> </table>	x	0	1	2	y	1	1.8	1.3	Remember	CAHS003.06				
x	0	1	2												
y	1	1.8	1.3												
17	Using Taylor's series method find an approximate value of y at $x = 0.1$ given $y(0)=1$ for the differential equation $y' = 3x + y^2$ .	Remember	CAHS003.07												
18	Using Euler's method solve $y' = y^2 + x$ , $y(0)=1$ find $y(0.1)$ and $y(0.2)$ .	Remember	CAHS003.08												
19	State the modified euler formula to find the numerical solution of ordinary differential equation.	Remember	CAHS003.08												
20	State the fourth order Runge- Kutta method to find the numerical solution of ordinary differential equation.	Remember	CAHS003.08												
<b>Part - B (Long Answer Questions)</b>															
1	By the method of least squares find the straight line that best fits the following data: <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>1</td> <td>3</td> <td>5</td> <td>7</td> <td>9</td> </tr> <tr> <td>y</td> <td>1.5</td> <td>2.8</td> <td>4.0</td> <td>4.7</td> <td>6</td> </tr> </table>	x	1	3	5	7	9	y	1.5	2.8	4.0	4.7	6	Understand	CAHS003.06
x	1	3	5	7	9										
y	1.5	2.8	4.0	4.7	6										
2	By the method of least squares find the straight line that best fits the following data:	Understand	CAHS003.06												

	<table border="1"> <tbody> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>14</td> <td>27</td> <td>40</td> <td>55</td> <td>68</td> </tr> </tbody> </table>	x	1	2	3	4	5	y	14	27	40	55	68				
x	1	2	3	4	5												
y	14	27	40	55	68												
3	<p>Fit a straight line <math>y=a + bx</math> for the following data by method of least squares:</p> <table border="1"> <tbody> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>y</td> <td>1</td> <td>1.8</td> <td>3.3</td> <td>4.5</td> <td>6.3</td> </tr> </tbody> </table>	x	0	1	2	3	4	y	1	1.8	3.3	4.5	6.3	Understand	CAHS003.06		
x	0	1	2	3	4												
y	1	1.8	3.3	4.5	6.3												
4	<p>Obtain a relation of the form <math>y=ax^b</math> for the following data by the method of least squares.</p> <table border="1"> <tbody> <tr> <td>x</td> <td>5</td> <td>10</td> <td>15</td> <td>20</td> </tr> <tr> <td>y</td> <td>15</td> <td>17</td> <td>22</td> <td>24</td> </tr> </tbody> </table>	x	5	10	15	20	y	15	17	22	24	Understand	CAHS003.06				
x	5	10	15	20													
y	15	17	22	24													
5	<p>If <math>y = a_0 + a_1x</math> find the values of <math>a_0</math> &amp; <math>a_1</math> from the following table by the method of least squares.</p> <table border="1"> <tbody> <tr> <td>x</td> <td>0</td> <td>2</td> <td>5</td> <td>7</td> </tr> <tr> <td>y</td> <td>-1</td> <td>5</td> <td>12</td> <td>20</td> </tr> </tbody> </table>	x	0	2	5	7	y	-1	5	12	20	Understand	CAHS003.06				
x	0	2	5	7													
y	-1	5	12	20													
6	<p>Fit a second degree curve <math>y=a+bx+cx^2</math> for the following data by method of least square.</p> <table border="1"> <tbody> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>y</td> <td>6</td> <td>11</td> <td>18</td> <td>27</td> </tr> </tbody> </table>	x	1	2	3	4	y	6	11	18	27	Understand	CAHS003.06				
x	1	2	3	4													
y	6	11	18	27													
7	<p>Using the method of least squares find the constants a and b such that <math>y=ae^{bx}</math> fits the following data:</p> <table border="1"> <tbody> <tr> <td>x</td> <td>0</td> <td>0.5</td> <td>1</td> <td>1.5</td> <td>2</td> <td>2.5</td> </tr> <tr> <td>y</td> <td>0.10</td> <td>0.45</td> <td>2.15</td> <td>9.15</td> <td>40.35</td> <td>180.75</td> </tr> </tbody> </table>	x	0	0.5	1	1.5	2	2.5	y	0.10	0.45	2.15	9.15	40.35	180.75	Understand	CAHS003.06
x	0	0.5	1	1.5	2	2.5											
y	0.10	0.45	2.15	9.15	40.35	180.75											
8	<p>Obtain a relation of the form <math>y=ab^x</math> for the following data by the method of least squares.</p> <table border="1"> <tbody> <tr> <td>x</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>y</td> <td>8.3</td> <td>15.4</td> <td>33.1</td> <td>65.2</td> <td>127.4</td> </tr> </tbody> </table>	x	2	3	4	5	6	y	8.3	15.4	33.1	65.2	127.4	Understand	CAHS003.06		
x	2	3	4	5	6												
y	8.3	15.4	33.1	65.2	127.4												
9	<p>Using Taylor's series method find an approximate value of y at <math>x = 0.2</math> for the differential equation <math>y' - 2y = 3e^x</math>, <math>y(0)=0</math>.</p>	Understand	CAHS003.07														
10	<p>Solve by Euler's method <math>y' + y = 0</math> given <math>y(0) = 1</math> and find <math>y(0.02)</math> taking step size <math>h = 0.01</math>.</p>	Understand	CAHS003.08														
11	<p>Solve by Euler's method <math>y' = x + y</math>, <math>y(0) = 1</math> and find the value of <math>y(0.2)</math> taking step size <math>h = 0.1</math>. compare the result obtained by this method with the result obtained by analytical methods.</p>	Understand	CAHS003.08														
12	<p>Using Runge-Kutta method of fourth order, find <math>y(0.2)</math> where <math>y' = y - x</math>, <math>y(0)=2</math>, <math>h = 0.2</math>.</p>	Understand	CAHS003.08														
13	<p>Apply the 4<sup>th</sup> order Runge-Kutta method to find an approximate value of y when <math>x=1.1</math> in steps of 0.1, given that <math>y' = x^2 + y^2</math>, <math>y(1)=1.5</math>.</p>	Understand	CAHS003.08														
14	<p>Fit a second degree curve <math>y=a+bx+cx^2</math> by method of least squares.</p> <table border="1"> <tbody> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>y</td> <td>6</td> <td>11</td> <td>18</td> <td>27</td> </tr> </tbody> </table>	x	1	2	3	4	y	6	11	18	27	Understand	CAHS003.06				
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x	2	3	4	5	6												
y	8.3	15.4	33.1	65.2	127.4												

16	Solve $y' = x^2 - y$ , $y(0) = 1$ , using Taylor's series method and compute $y(0.1)$ , $y(0.2)$ , $y(0.3)$ and $y(0.4)$ .	Understand	CAHS003.07
17	Using Euler's method, solve the differential equation from $\frac{dy}{dx} = 3x^2 + 1$ , for $x = 2$ , $y(1) = 2$ , taking step size $h = 0.5$ .	Understand	CAHS003.08
18	Using modified euler's method, find the approximate value of $x$ when $x=0.1$ given differential equation $\frac{dy}{dx} = x + y$ and $y(0) = 1$ .	Understand	CAHS003.08
19	Using Runge-Kutta method of second order, find $y(2.5)$ given the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ , $y(2)=2$ , $h = 0.25$ .	Understand	CAHS003.08
20	Find $y(0.1)$ by Runge-Kutta method of 4 <sup>th</sup> order for the differential equation $y' = xy + y^2$ , $y(0)=1$ .	Understand	CAHS003.08

**Part – C (Problem Solving and Critical Thinking)**

1	Using Runge-Kutta method find $y(0.2)$ for the differential equation $\frac{dy}{dx} = y - x$ , $y(0)=1$ , take $h=0.2$ .	Understand	CAHS003.08																
2	Given the differential equation $y' + y=0$ , $y(0)=1$ using Runge-Kutta method take $h=0.1$ find $y(0.1)$ .	Understand	CAHS003.08																
3	Apply the 4 <sup>th</sup> order Runge-Kutta method to find an approximate value of $y$ when $x = 1.1$ in steps of $h= 0.1$ given the differential equation $y' = x^2 + y^2$ , $y(1) = 1.5$ .	Understand	CAHS003.08																
4	Solve the first order differential equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ , $y(0) = 1$ and estimate $y(0.1)$ using Euler's method.	Understand	CAHS003.08																
5	Using modified Euler's method find $y(0.2)$ given differential equation $y' = y + e^x$ , $y(0) = 0$ .	Understand	CAHS003.08																
6	Given the differential equation $\frac{dy}{dx} = -xy^2$ , $y(0) = 2$ . Compute $y(0.1)$ in steps of 0.1, using modified Euler's method.	Remember	CAHS003.08																
7	Solve: $\frac{dy}{dx} = \log_{10}(x+y)$ ; $y(0) = 2$ at $x = 0.4$ with $h = 0.2$ , using modified Euler's method.	Understand	CAHS003.08																
8	By the method of least squares, fit a second degree polynomial $y=a+bx+cx^2$ to the following data.	Understand	CAHS003.06																
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>y</td> <td>4.63</td> <td>2.11</td> <td>0.67</td> <td>0.09</td> <td>0.63</td> <td>2.15</td> <td>4.58</td> </tr> </table>	x	-3	-2	-1	0	1	2	3	y	4.63	2.11	0.67	0.09	0.63	2.15	4.58		
x	-3	-2	-1	0	1	2	3												
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58												
9	Find the solution of differential equation $\frac{dy}{dx} = x - y$ , $y(0)=1$ at $x = 0.1$ using modified euler's method.	Remember	CAHS003.08																
10	Find $y(0.1)$ using modified euler's formula given the differential equation $y' = x^2 - y$ , $y(0)=1$ .	Remember	CAHS003.08																

**UNIT-III**

**MULTIPLE INTEGRALS**

**Part - A (Short Answer Questions)**

1	Evaluate the double integral $\int_0^2 \int_0^x y dy dx$ .	Understand	CAHS003.11
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2	Evaluate the double integral $\int_0^\pi \int_0^{a \sin \theta} r dr d\theta$ .	Understand	CAHS003.11
3	Evaluate the double integral $\int_0^3 \int_0^1 xy(x+y) dx dy$ .	Understand	CAHS003.11
4	Find the value of double integral $\int_1^2 \int_1^3 xy^2 dx dy$ .	Understand	CAHS003.11
5	Find the value of triple integral $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx dy dz$ .	Understand	CAHS003.11
6	Evaluate the double integral $\int_0^2 \int_0^x y dy dx$ .	Understand	CAHS003.11
7	Evaluate the double integral $\int_0^{\frac{\pi}{2}} \int_{-1}^1 x^2 y^2 dx dy$ .	Understand	CAHS003.11
8	Evaluate the double integral $\int_0^\pi \int_0^{a \sin \theta} r dr d\theta$ .	Understand	CAHS003.11
9	Evaluate the double integral $\int_0^\infty \int_0^{\frac{\pi}{2}} e^{-r^2} r d\theta dr$ .	Understand	CAHS003.11
10	Evaluate the double integral $\int_0^\pi \int_0^{a(1+\cos \theta)} r dr d\theta$ .	Understand	CAHS003.11
11	State the formula to find area of the region using double integration in Cartesian form.	Remember	CAHS003.13
12	Find the volume of the tetrahedron bounded by the coordinate planes and the plane $x+y+z=1$ .	Understand	CAHS003.13
13	State the formula to find volume of the region using triple integration in Cartesian form.	Remember	CAHS003.13
14	Find area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using double integration.	Understand	CAHS003.13
15	State the formula to find area of the region using double integration in polar form.	Remember	CAHS003.13
16	Find the area of the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ .	Understand	CAHS003.13
17	Find the area of the curve $r=2a \cos \theta$ using double integration in polar coordinates.	Understand	CAHS003.13
18	Find the area enclosed between the parabola $y=x^2$ and the line $y=x$ .	Understand	CAHS003.13
19	Find the area of the curve $r=2a \sin \theta$ .	Understand	CAHS003.13
20	Find area of the circle $x^2+y^2=a^2$ .	Understand	CAHS003.13
<b>Part – B (Long Answer Questions)</b>			
1	Evaluate the triple integral $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xyz dx dy dz$ .	Understand	CAHS003.11
2	Evaluate the double integral $\int_0^\pi \int_0^{a(1+\cos \theta)} r^2 \cos \theta dr d\theta$ .	Understand	CAHS003.11
3	Evaluate the double integral $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$ .	Understand	CAHS003.11



4	Evaluate the double integral $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$ .	Understand	CAHS003.11
5	Evaluate the double integral $\int_0^1 \int_0^{\pi/2} r \sin \theta d\theta dr$ .	Understand	CAHS003.11
6	By changing the order of integration evaluate the double integral $\int_0^1 \int_{x^2}^{2-x} xy dx dy$ .	Understand	CAHS003.11
7	Evaluate the double integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) dy dx$ .	Understand	CAHS003.11
8	Evaluate the triple integral $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dx dy dz$ .	Understand	CAHS003.11
9	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$ .	Understand	CAHS003.11
10	Find the value of $\iint xy dx dy$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .	Understand	CAHS003.11
11	Evaluate the double integral using change of variables $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ .	Understand	CAHS003.11
12	Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes by triple integration.	Understand	CAHS003.11
13	By transforming into polar coordinates Evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ with $b > a$ .	Understand	CAHS003.14
14	Find the area of the region bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$ .	Understand	CAHS003.13
15	Evaluate $\int \int r^3 dr d\theta$ over the area included between the circles $r = 2 \sin \theta$ and $r = 4 \sin \theta$ .	Understand	CAHS003.13
16	Using triple integration find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ .	Understand	CAHS003.13
17	Find the area of the cardioid $r = a(1 + \cos \theta)$ .	Understand	CAHS003.13
18	Find the area of the region bounded by the curves $y = x^3$ and $y = x$ .	Understand	CAHS003.13
19	Evaluate $\iiint_v dx dy dz$ where $v$ is the finite region of space formed by the planes $x=0, y=0, z=0$ and $2x+3y+4z=12$ .	Understand	CAHS003.13
20	Find the area bounded by curves $xy=2, 4y=x^2$ and the line $y=4$ .	Understand	CAHS003.13
<b>Part - C (Problem Solving and Critical Thinking Questions)</b>			
1	Evaluate $\int_0^a \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$ by changing to polar coordinates.	Understand	CAHS003.12
2	Evaluate $\iiint_R (x + y + z) dz dy dx$ where $R$ is the region bounded by the plane	Understand	CAHS003.11

	$x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$		
3	Evaluate $\iint x^2 dx dy$ over the region bounded by hyperbola $xy = 4, y = 0, x = 1, x = 4.$	Understand	CAHS003.11
4	Find the area bounded by curves $xy=2, 4y=x^2$ and the line $y=4.$	Understand	CAHS003.13
5	Evaluate the double integral $\int_0^2 \int_0^x e^{(x+y)} dy dx.$	Understand	CAHS003.11

6	Evaluate by converting $\int_0^a \int_1^{\sqrt{a^2-x^2}} (x^2 + y^2) dy dx$ to polar co-ordinates.	Understand	CAHS003.12
7	Find the volume of tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$	Understand	CAHS003.13
8	Using double integral, find area of the cardioid $r = a(1-\cos\theta).$	Remember	CAHS003.13
9	Evaluate the area of $\iint r^3 dr d\theta$ over the region included between the circles $r = \sin\theta, r = 4\sin\theta.$	Understand	CAHS003.13
10	If R is the region bounded by the planes $x=0, y=0, z=1$ and the cylinder $x^2 + y^2 = 1,$ evaluate $\iiint_R xyz dx dy dz.$	Remember	CAHS003.11

#### UNIT-IV

#### VECTOR CALCULUS

#### Part – A (Short Answer Questions)

1	Define gradient of scalar point function.	Remember	CAHS003.15
2	Define divergence of vector point function.	Remember	CAHS003.15
3	Define curl of vector point function.	Remember	CAHS003.15
4	State Laplacian operator?	Remember	CAHS003.15
5	Find curl $\vec{f}$ where $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz).$	Remember	CAHS003.15
6	Find the angle between the normal to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3).$	Understand	CAHS003.15
7	Find a unit normal vector to the given surface $x^2y + 2xz = 4$ at the point $(2, -2, 3).$	Remember	CAHS003.15
8	If $\vec{a}$ is a vector then prove that $\text{grad}(\vec{a} \cdot \vec{r}) = \vec{a}.$	Understand	CAHS003.15
9	Define irrotational vector and solenoidal vector of vector point function.	Remember	CAHS003.15
10	Show that $\nabla \left( f(r) \right) = \frac{\vec{r}}{r} f'(r).$	Remember	CAHS003.15
11	Prove that $f = yzi + z xj + xyk$ is irrotational vector.	Remember	CAHS003.15
12	Show that $(x+3y)i + (y-2z)j + (x-2z)k$ is solenoidal.	Understand	CAHS003.15
13	Show that $\text{curl}(\text{grad}\phi) = 0$ where $\phi$ is scalar point function.	Understand	CAHS003.15
14	State Stokes theorem of transformation between line integral and surface integral.	Understand	CAHS003.15
15	Prove that $\text{div curl}\vec{f} = 0$ where $\vec{f} = f_1\vec{i} + f_2\vec{j} + f_3\vec{k}.$	Understand	CAHS003.15
16	Define line integral on vector point function.	Remember	CAHS003.15
17	Define surface integral of vector point function $\vec{F}.$	Remember	CAHS003.15
18	Define volume integral on closed surface S of volume V.	Remember	CAHS003.15
19	State Green's theorem of transformation between line integral and double integral.	Understand	CAHS003.18
20	State Gauss divergence theorem of transformation between surface integral and volume integral.	Understand	CAHS003.18

#### Part – B (Long Answer Questions)

1	Evaluate $\int_C \vec{f} \cdot d\vec{r}$ where $\vec{f} = 3xyi - y^2j$ and C is the parabola $y = 2x^2$ from points	Understand	CAHS003.17
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	(0, 0) to (1, 2).		
2	Evaluate $\iint_S \vec{F} \cdot d\vec{s}$ if $\vec{F} = yz\vec{i} + 2y^2\vec{j} + xz^2\vec{k}$ and S is the Surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between the planes $z = 0$ and $z = 2$ .	Understand	CAHS003.17
3	Find the work done in moving a particle in the force field $\vec{F} = (3x^2)\vec{i} + (2zx - y)\vec{j} + z\vec{k}$ along the straight line from (0,0,0) to (2,1,3).	Understand	CAHS003.17
4	Find the circulation of $\vec{F} = (2x - y + 2z)\vec{i} + (x + y - z)\vec{j} + (3x - 2y - 5z)\vec{k}$ along the circle $x^2 + y^2 = 4$ in the xy plane.	Remember	CAHS003.15
5	Verify Gauss divergence theorem for the vector point function $F = (x^3 - yz)\vec{i} - 2yx\vec{j} + 2z\vec{k}$ over the cube bounded by $x = y = z = 0$ and $x = y = z = a$ .	Remember	CAHS003.18
6	Verify Gauss divergence theorem for $2x^2y\vec{i} - y^2\vec{j} + 4xz^2\vec{k}$ taken over the region of first octant of the cylinder $y^2 + z^2 = 9$ and $x = 2$ .	Remember	CAHS003.18
7	Verify Green's theorem in the plane for $\int_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where C is a square with vertices (0,0),(2,0),(2,2),(0,2).	Remember	CAHS003.18
8	Applying Green's theorem evaluate $\iint_C (y - \sin x)dx + \cos x dy$ where C is the plane triangle enclosed by $y = 0$ , $y = \frac{2x}{\pi}$ , and $x = \frac{\pi}{2}$ .	Remember	CAHS003.18
9	Apply Green's Theorem in the plane for $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is a is the boundary of the area enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$ .	Remember	CAHS003.18
10	Verify Stokes theorem for $f = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection of the xy plane.	Remember	CAHS003.18
11	Verify Stokes theorem for $\vec{f} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ over the box bounded by the planes $x=0$ , $x=a$ , $y=0$ , $y=b$ .	Remember	CAHS003.18
12	Find the directional derivative of the function $\phi = xy^2 + yz^3$ at the point P(1,-2,-1) in the direction to the surface $x \log z - y^2 = -4$ at (-1,2,1).	Remember	CAHS003.16
13	If $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ evaluate $\int_S \vec{F} \cdot \vec{n} ds$ where S is the surface of the cube $x = 0$ , $x = a$ , $y = 0$ , $y = a$ , $z = 0$ , $z = a$ .	Understand	CAHS003.17
14	If $\vec{f} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ evaluate $\int_C \vec{f} \cdot d\vec{r}$ along the curve C in xy-plane $y = x^3$ from (1,1) to (2,8).	Understand	CAHS003.17
15	Evaluate the line integral $\int_C (x^2 + xy)dx + (x^2 + y^2)dy$ where C is the square formed by lines $x = \pm 1$ , $y = \pm 1$ .	Understand	CAHS003.17
16	If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ show that $\nabla r^n = nr^{n-2}\vec{r}$ .	Understand	CAHS003.15
17	Evaluate by Stokes theorem $\int_C (e^x dx + 2ydy - dz)$ where c is the curve $x^2 + y^2 = 9$ and $z = 2$ .	Understand	CAHS003.17
18	Verify Stokes theorem for the function $x^2\vec{i} + xy\vec{j}$ integrated round the square in the plane $z=0$ whose sides are along the line $x=0$ , $y=0$ , $x=a$ , $y=a$ .	Understand	CAHS003.18

19	Evaluate by Stokes theorem $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$ where C is the boundary of the triangle with vertices (0,0,0),(1,0,0),(1,1,0).	Understand	CAHS003.18
20	Verify Green's theorem in the plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is a region bounded by $y = \sqrt{x}$ and $y = x^2$ .	Understand	CAHS003.18

**Part – C (Problem Solving and Critical Thinking)**

1	Verify Gauss divergence theorem for $\vec{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by $x=0, x=a, y=0, y=b, z=0, z=c$ .	Understand	CAHS003.18
2	Find the work done in moving a particle in the force field $\vec{F} = (3x^2)\vec{i} + (2zx - y)\vec{j} + z\vec{k}$ along the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x=0$ and $x=2$ .	Understand	CAHS003.17
3	Show that the force field given by $\vec{F} = 2xyz^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{k}$ is conservative. Find the work done in moving a particle from (1,-1,2) to (3,2,-1) in this force field.	Understand	CAHS003.17
4	Show that the vector $(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational and find its scalar potential function.	Understand	CAHS003.15
5	Using Gauss divergence theorem evaluate $\iiint_S \vec{F} \cdot d\vec{s}$ , for the $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylinder region S given by $x^2 + y^2 = a^2, z = 0$ and $z = b$ .	Understand	CAHS003.18
6	Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point (1,-2,-1) in the direction of the normal to the surface $f(x, y, z) = x \log z - y^2$ at (-1,2,1).	Understand	CAHS003.16
7	Using Green's theorem in the plane evaluate $\int_C (2xy - x^2)dx + (x^2 + y^2)dy$ where C is the region bounded by $y = x^2$ and $y^2 = x$ .	Understand	CAHS003.18
8	Applying Green's theorem evaluate $\int_C (xy + y^2)dx + x^2 dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$ .	Understand	CAHS003.18
9	Verify Green's Theorem in the plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the region bounded by $x=0, y=0$ and $x + y=1$ .	Understand	CAHS003.18
10	Verify Stokes theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ where S is the surface of the cube $x=0, y=0, z=0$ and $x=2, y=2, z=2$ above the xy-plane.	Understand	CAHS003.18

**UNIT-V**

**SPECIAL FUNCTIONS**

**Part - A (Short Answer Questions)**

1	Show that the value of $\gamma(1/2) = \sqrt{\pi}$ .	Understand	CAHS003.21
2	State the value of $\gamma(-7/2)$ .	Remember	CAHS003.21
3	Compute the value of $\gamma(11/2)$ .	Remember	CAHS003.21
4	Define ordinary point of a differential equation.	Remember	CAHS003.20
5	Define regular singular point of differential equation.	Remember	CAHS003.20
6	Explain Frobenius method about zero.	Understand	CAHS003.20
7	Find the singular points and classify them (regular or irregular) $x^2 y'' - 5y' + 3x^2 y = 0$ .	Understand	CAHS003.20
8	Prove that $\frac{d}{dx}(J_0(x)) = -J_1(x)$ where $J_1(x)$ is the Bessel's function.	Understand	CAHS003.22

9	Find the singular points and classify them (regular or irregular ) $x^2 y'' + ax y' + by = 0$ .	Understand	CAHS003.20
10	State the expansion of $j_n(x)$ .	Understand	CAHS003.22
11	Prove the Bessel's recurrence relation $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ .	Remember	CAHS003.22
12	Prove the Bessel's recurrence relation $xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$ .	Remember	CAHS003.22
13	State the expansion of $j_n(-x)$ .		CAHS003.22
14	Prove the Bessel's recurrence relation $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$ .	Remember	CAHS003.22
15	Express $J_2(x)$ in terms of $J_0(x)$ and $J_1(x)$ .	Remember	CAHS003.22
16	Prove that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \dots) = 1$ .	Remember	CAHS003.22
17	Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ .	Remember	CAHS003.22
18	Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ .	Remember	CAHS003.22
19	Prove that $\left[ J_{\frac{1}{2}} \right]^2 + \left[ J_{-\frac{1}{2}} \right]^2 = \frac{2}{\pi x}$ .	Remember	CAHS003.22
20	Prove that $J_n(x) = 0$ has no repeated roots except at $x=0$ .	Remember	CAHS003.22
<b>Part - B (Long Answer Questions)</b>			
1	If $m$ and $n$ are real constants greater than $-1$ , prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n}{(m+1)^{n+1}} \Gamma(n+1)$ and evaluate $\int_0^1 x^4 (\log \frac{1}{x})^3 dx$ .	Understand	CAHS003.21
2	Show that $\int_0^x x^n J_{n-1}(x) dx = x^n J_n(x)$ where $J_n(x)$ is Bessel's function.	Understand	CAHS003.22
3	Prove that $[J_n^2 + J_{n+1}^2] = \frac{2}{x} [nJ_n^2 - (n+1)J_{n+1}^2]$ where $J_n(x)$ is Bessel's function.	Understand	CAHS003.22
4	Show that $\frac{d}{dx} [xJ_n(x)J_{n+1}(x)] = x[J_n^2(x) - J_{n+1}^2(x)]$ where $J_n(x)$ is Bessel's function.	Remember	CAHS003.22
5	Show that $\int_0^x x^{n+1} J_n(x) dx = x^{n+1} J_{n+1}(x)$ where $J_n(x)$ is Bessel's function.	Remember	CAHS003.22
6	Show that $J_n(x)$ is an even function when $n$ is even and odd function when $n$ is odd function.	Remember	CAHS003.22
7	Prove that $\int J_3(x) dx = -J_2(x) - \frac{2}{x} J_1(x)$ using Bessel's Recurrence relation.	Understand	CAHS003.22
8	State and prove orthogonality of Bessel's function	Remember	CAHS003.22
9	Using generating function show that $\cos(x \sin \theta) = J_0 + 2(J_2 \cos 2\theta + J_4 \cos 4\theta + \dots)$ .	Understand	CAHS003.22
10	Using generating function show that $\sin(x \sin \theta) = 2(J_1 \sin \theta + J_3 \sin 3\theta + J_5 \sin 5\theta + \dots)$ .	Understand	CAHS003.22
11	Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right)$ .	Understand	CAHS003.22
12	Use Frobenius method to solve $2x(1-x)y'' + (1-x)y' + 3y = 0$ .	Understand	CAHS003.20
13	Solve in series the differential equation $\frac{d^2y}{dx^2} + xy = 0$ when $x=0$ is ordinary point of the equation.	Understand	CAHS003.20
14	Solve in series the differential equation $y'' + y = 0$ when $x=0$ is ordinary point of the equation.	Understand	CAHS003.20
15	Solve in series the differential equation $2x^2 y'' + (x^2 - x)y' + y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
16	Find the power series solution of the differential equation $x(1-x)y'' - (1+3x)y' - y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20

17	Find the power series solution of the differential equation $(x - x^2)y'' + (1 - 5x)y' - 4y = 0$ Using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
18	Solve in series the differential equation $2x^2 y'' + x y' - (x + 1)y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
19	Solve in series the equation $x(1 - x)y'' - 3x y' - y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
20	Solve in series the equation $9x(1 - x) \frac{d^2y}{dx^2} - 12 \frac{dy}{dx} + 4y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
<b>Part – C (Problem Solving and Critical Thinking)</b>			
1	Solve in series the differential equation $y'' + x y' + y = 0$ about the ordinary point $x=0$ .	Remember	CAHS003.20
2	Solve in series the equation $x \frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Remember	CAHS003.20
3	Solve in series the differential equation $\frac{d^2y}{dx^2} - y = 0$ when $x=0$ is ordinary point of the equation.	Remember	CAHS003.20
4	Solve in series the differential equation $\frac{d^2y}{dx^2} - xy = 0$ when $x=0$ is ordinary point of the equation.	Remember	CAHS003.20
5	Solve in series the equation $4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
6	Prove that $J_n(-x) = (-1)^n J_n(x)$ where $n$ is a positive or negative integer.	Understand	CAHS003.22
7	Solve in series $3xy'' + (1 - x) y' - y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
8	Solve in series $xy'' + (1 + x)^2 y' + 2y = 0$ using Frobenius method when $x=0$ is regular singular point of the equation.	Understand	CAHS003.20
9	If $\alpha$ and $\beta$ are the distinct roots of $J_n(x)=0$ , then show that $\int_0^{\frac{\pi}{2}} x J_n(\alpha x) J_n(\beta x) dx = 0$ .	Understand	CAHS003.22
10	Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{1}{x} \sin x - \cos x \right)$ .	Remember	CAHS003.22

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