Hall Ticket No | | | | | | | | | | | Question Paper Code: AECB08

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

MODEL QUESTION PAPER –II

Four Year B.Tech III Semester End Examinations(Regular), November - 2019

Regulations: IARE-R18

PROBABILITY THEORY AND STOCHASTIC PROCESS

(ECE)

Time: 3 hours **Max. Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1 a) How do you explain statistically independent events using Baye's rule? [7M]

b) Suppose that a laboratory test to detect a certain disease has the following statistics. Let A= event that the tested person has the disease B= event that the test result is positive it is known that P(B/A)= 0.99 and P(B/$\bar{A}$)= 0.005 and 0.1 percent of the population actually has the disease. What is the probability that a person has the disease given that the test result is positive? [7M]

2 a) A continuous random variable X that can assume any value between x=2 and x=3 has a density function given $f(x) = k(1 + x)$. Find $P(X < 4)$. [7M]

b) A fair die is tossed. Let X denotes twice the number appearing, and let Y denotes 1 or 3 according as an odd or an even number appears. Find the distribution, expectation, variance and standard deviation of (i) X (ii) Y (iii) X+ Y. [7M]

UNIT – II

3 a) Show that the variance of a weighted sum of uncorrelated random variables equals the weighted sum of the variances of the random variables. [7M]

b) The pdf of a random variable X is given by fx(x)= x/20 ; 2 ≤x≤5, find the pdf of Y=3X-5. [7M]

4 a) Define the joint Distribution function and explain the properties of joint Distribution function? [7M]

b) Consider the bivariate r.v. (X, Y) [7M]

$$f_{XY}(x,y) = \begin{cases} k(x+y), & 0 < x < 2, 0 < y < 2 \\ 0, & otherwise \end{cases}$$

i. Find the conditional pdf's fY/X (y / x) and fX/Y,(x/y).

ii. Find P(0 < Y < 1/2, X = 1).

UNIT – III

5 a) Explain Linear Transformations of Gaussian Random Variables. [7M]

b) The pdf of a random variable X is fx(x)=(1/2)cos (x) ; $-\pi/2<x<\pi/2$, find the mean, mean of the function $g(X)=4X^2$. [7M]

6 a) Explain expected value of a function of multiple random variables and give the expression for joint moments and joint central moments. [7M]

b) Calculate the following [7M]

i) The variance of the sum of X and Y

ii) The variance of the difference of X and Y for two random variables X and Y have zero mean and variance $\sigma_X^2 = 16$ and $\sigma_Y^2 = 36$ and correlation coefficient is 0.5.

UNIT – IV

7 a) A random process is defined as $X(t)=A.\sin(\omega t+\theta)$ where A is a constant and 'θ' is a random variable, uniformly distributed over$(-\pi,\pi)$. Check X(t) for stationary. [7M]

b) Define Ergodic process. State and explain various properties of autocorrelation function [7M]

8 a) Define random process and classify the random process with example. [7M]

b) Consider a random variable process $X(t)=a\cos\omega t$, where 'ω' is a constant and a is a random variable uniformly distribution over (0,1). Find the auto correlation and covariance of X(t)? [7M]

UNIT – V

9 a) State and explain various properties power spectral density function. [7M]

b) An ergodic random process is known to have an auto correlation function of the from [7M]

$R_{XY}(\tau) = 1 - |\tau|, \ |\tau| \leq 1$

$= 0, \ |\tau|, > 1$

Show the spectral density is given by $S_{XY}(\omega) = \left[\frac{\sin \omega/2}{\omega/2}\right]^2$

10 a) Briefly explain the concept of cross power density spectrum. [7M]

b) Find the auto correlation function of the process X(t) for which the power density specrum is given by [7M]

$$S_{XX}(w) = \begin{cases} 1 + w^2, & |w| \leq 1 \\ 0, & |w| > 1 \end{cases}$$

COURSE OBJECTIVES:

The course should enable the students to:

S.No	Description
I	Understand the random experiments, sample space and event probabilities.
II	Study the random variables, density and distribution functions, moments and transformation of random variables.
III	Understand the concept of random process and sample functions (signals)
IV	Explore the temporal and spectral characteristics of random processes

COURSE OUTCOMES (COs):

CO Code	Description
CO 1	Appreciate the concept of the random experiments, event probabilities, random variables and their description, functions of random variables
CO 2	Learn and understand the Single Random Variable Transformation- Multiple Random Variables
CO 3	Understand the Operations multiple random variables and their expectations
CO 4	Understand the concept of random processes and their time domain description
CO 5	Explore the spectral characteristics of random processes, and filtered random processes

COURSE LEARNING OUTCOMES:

Students who complete the course will have demonstrated the ability to do the following

CLO Code	Description
AECB08.01	Describe the basic concepts of the random experiments, event probabilities, joint and conditional probabilities- Bayes theorem
AECB08.02	Learn and understand the concept of random variables, continuous and discrete variables, the probability density functions (pdfs), Probability Distribution Functions (PDFs), different random variables and their properties
AECB08.03	Learn and understand the functions of a random variable, standard and central moments, and their physical significance
AECB08.04	Understand the Characteristic and Moment Generating Functions; Understand and apply the transformations on continuous and discrete random variables - Expectations
AECB08.05	Learn and understanding of Vector random variables, joint, Marginal and Conditional distribution functions, joint, Marginal and Conditional density functions.
AECB08.06	Learn and understand the Conditional distribution and density functions: point and interval conditioning
AECB08.07	State and Explain the Central limit theorem ,Sum of several random variables
AECB08.08	Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions

AECB08.09	Learn and understanding of Jointly Gaussian random variables; and Transformations of multiple random variables
AECB08.10	Learn and understanding of Random Process, sample functions and time domain characteristics: Stationary, Independence and Ergodicity
AECB08.11	Contrasting of Correlation and Covariance functions, Gaussian and Poisson Random Processes
AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra
AECB08.13	Understand and Discuss the linear time invariant (LTI) systems driven by random process, Input-output Spectral relations, White and Colored noises

MAPPING OF SEMESTER END EXAMINATION TO COURSE LEARNING OUTCOMES:

SEE Question No.		CLO Code	Course learning Outcomes	CO code	Blooms Taxonomy Level
1	a	AECB08.01	Describe the basic concepts of the random experiments, event probabilities, joint and conditional probabilities- Bayes theorem	CO 1	Understand
	b	AECB08.01	Describe the basic concepts of the random experiments, event probabilities, joint and conditional probabilities- Bayes theorem	CO 1	Understand
2	a	AECB08.02	Learn and understand the concept of random variables, continuous and discrete variables, the probability density functions (pdfs), Probability Distribution Functions (PDFs), different random variables and their properties	CO 1	Understand
	b	AECB08.03	Learn and understand the functions of a random variable, standard and central moments, and their physical significance	CO 1	Understand
3	a	AECB08.04	Understand the Characteristic and Moment Generating Functions; Understand and apply the transformations on continuous and discrete random variables – Expectations	CO 2	Understand
	b	AECB08.07	State and Explain the Central limit theorem ,Sum of several random variables	CO 2	Understand
4	a	AECB08.05	Learn and understanding of Vector random variables, joint, Marginal and Conditional distribution functions, joint, Marginal and Conditional density functions.	CO 2	Understand
	b	AECB08.04	Understand the Characteristic and Moment Generating Functions; Understand and apply the transformations on continuous and discrete random variables – Expectations	CO 2	Understand
5	a	AECB08.08	Learn and understanding of Jointly Gaussian random variables; and Transformations of multiple random variables	CO 3	Understand
	b	AECB08.08	Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions	CO 3	Understand
6	a	AECB08.08	Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions	CO 3	Remember
	b	AECB08.08	Learn and understanding of functions of vector random variables, Joint standard and central moments, joint characteristic functions	CO 3	Understand

SEE Question No.		CLO Code	Course learning Outcomes	CO code	Blooms Taxonomy Level
7	a	AECB08.10	Learn and understanding of Random Process, sample functions and time domain characteristics: Stationary, Independence and Ergodicity	CO 4	Understand
	b	AECB08.10	Learn and understanding of Random Process, sample functions and time domain characteristics: Stationary, Independence and Ergodicity	CO 4	Understand
8	a	AECB08.11	Contrasting of Correlation and Covariance functions, Gaussian and Poisson Random Processes	CO 4	Understand
	b	AECB08.11	Contrasting of Correlation and Covariance functions, Gaussian and Poisson Random Processes	CO 4	Understand
9	a	AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra	CO 5	Understand
	b	AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra	CO 5	Understand
10	a	AECB08.12	Distinguish between Auto- and Cross- power density spectra, properties, relationship between Correlation functions and Power density spectra	CO 5	Understand
	b	AECB08.13	Understand and Discuss the linear time invariant (LTI) systems driven by random process, Input- output Spectral relations, White and Colored noises	CO 5	Understand

Signature of Course Coordinator **HOD, ECE**