



INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)
Dundigal, Hyderabad -500 043

FRESHMAN ENGINEERING

TUTORIAL QUESTION BANK

Course Name	:	LINEAR ALGEBRA AND ORDINARY DIFFERENTIAL EQUATIONS
Course Code	:	AHS002
Class	:	I B. Tech I Semester
Branch	:	Common for all branches
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OBJECTIVES

To meet the challenge of ensuring excellence in engineering education, the issue of quality needs to be addressed, debated and taken forward in a systematic manner. Accreditation is the principal means of quality assurance in higher education. The major emphasis of accreditation process is to measure the outcomes of the program that is being accredited.

In line with this, Faculty of Institute of Aeronautical Engineering, Hyderabad has taken a lead in incorporating philosophy of outcome based education in the process of problem solving and career development. So, all students of the institute should understand the depth and approach of course to be taught through this question bank, which will enhance learner's learning process.

S No	QUESTION	Blooms taxonomy level	Course Outcomes
UNIT - I THEORY OF MATRICES			
Part - A (Short Answer Questions)			
1	Define conjugate of a matrix.	Remember	1
2	If A is Hermitian matrix Prove that iA is skew- Hermitian matrix	Analyse	1
3	Prove that $\frac{1}{2} \begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is a unitary matrix.	Understand	1
4		Evaluate	1

	Find the value of k such that rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.		
5	Find the Skew- symmetric part of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$	Evaluate	1
6	Define Rank of a matrix .	Remember	1
7	Define Hermitian matrix.	Remember	1
8	Prove that $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ is orthogonal	Analyze	1
9	Define Unitary matrix.	Remember	1
10	Prove that the matrix $\frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is orthogonal.	Analyze	1
11	Determine the values of a,b,c when $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.	Understand	1
12	Prove that inverse of a Matrix if exists is Unique.	Analyze	1
13	Express the matrix A as sum of symmetric and skew – symmetric matrices. Where $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$	Understand	1
14	Define Skew Hermitian matrix.	Evaluate	1
15	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 0 & 5 \end{bmatrix}$	Evaluate	1
16	Explain Gauss – Jordan method.	Understand	1
17	Find the value of x such that A is singular where $A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -(1+x) \end{bmatrix}$	Evaluate	1
18	Find the value of k such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.	Analyze	1

19	Define minor of a matrix.	Remember	1
20	Define transposed conjugate of a matrix.	Remember	1
Part - B (Long Answer Questions)			
1	Express the matrix $\begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$ as the sum of Hermitian and Skew-Hermitian matrix.	Understand	1
2	Find the rank of the matrix $\begin{bmatrix} -1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$	Evaluate	1
3	Find the rank of the matrix $\begin{bmatrix} 1 & 0 & -4 & 5 \\ 2 & -1 & 3 & 0 \\ 8 & 1 & 0 & -7 \end{bmatrix}$	Evaluate	1
4	Find a and b such that rank of $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ is 3.	Evaluate	1
5	Find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$	Evaluate	1
6	Given that $A = \begin{bmatrix} 0 & 1-2i \\ -1-2i & 0 \end{bmatrix}$ show that $(I-A)(I+A)^{-1}$ is unitary matrix.	Analyze	1
7	For what value of K such that the matrix $\begin{pmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{pmatrix}$ has rank 3	Analyze	1
8	Find rank by reducing to Normal form of matrix $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$	Evaluate	1
9	Reduce the matrix A to its normal form where $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ and hence find the rank	Evaluate	1
10	Solve the system of equations $x+y+z = 1$, $3x+y-3z = 5$, $x-2y-5z = 10$ by using Method of factorisation.	Evaluate	1
11	Using LU – decomposition method solve $x + 3y+8z = 4$, $x+4y+3z = -2$, $x+3y+4z = 1$	Apply	1

12	Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary row operation.	Analyze	1
13	Solve the system of equations $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$ by LU – decomposition method	Analyze	1
14	If $A = \begin{bmatrix} 2 & 3-2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$ show that A is Hermitian and iA is a skew-Hermitian matrix.	Analyze	1
15	Find rank by reducing to Echelon form of $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$	Evaluate	1
16	Find Inverse by elementary row operations of $\begin{bmatrix} -1 & -3 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & 5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$	Analyze	1
17	Find rank by reducing to Echelon form of $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$	Evaluate	1
18	By reducing the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into normal form, find its rank.	Evaluate	1
19	Find the inverse of $\begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ using elementary row operations.	Evaluate	1
20	Solve the following equations by L-U decomposition method $2x+y-z=3$, $x-2y-2z=1$, $x+2y-3z=9$	Analyze	1
Part - C (Problem Solving and Critical Thinking Questions)			
1	Show that $A = \begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary if and only if $a^2+b^2+c^2+d^2=1$.	Analyze	1
2	Find the value of k for which matrix $A = \begin{bmatrix} 1 & 0 & -k \\ 0 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible.	Evaluate	1

3	What is rank of 4x5 matrix.	Analyze	1
4	If A is nxn matrix , rank is k and normal form is $\begin{bmatrix} I_k & 0 \\ 0 & 0 \end{bmatrix}$ then find order of null matrix below side of I_k .	Analyze	1
5	If $A-B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $A+B = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$ then find AB.	Evaluate	1
6	If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ then find A(adj A).	Evaluate	1
7	Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$.	Evaluate	1
8	What is the rank of 6x5 matrix.	Analyze	1
9	Express the matrix A as sum of symmetric and skew – symmetric matrices. Where $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$	Understand	1
10	Find the minor of matrix of order 3 if $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}_{4 \times 3}$	Evaluate	1

UNIT-II LINEAR TRANSFORMATIONS

Part – A (Short Answer Questions)

1	Find the Eigen values of the matrix $\begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$	Evaluate	2
2	State Cayley- Hamilton Theorem.	Remember	2
3	Find the Eigen values of the matrix $\begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$	Evaluate	2
4	Define modal matrix.	Remember	2
5	If 2,3,4 are the Eigen values of A then find the Eigen values of adjA.	Evaluate	2
6	Find the sum of Eigen values of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$	Evaluate	2
7	Prove that if λ is an Eigen value of a non-singular matrix A then $\frac{ A }{\lambda}$ is an Eigen	Analyze	2

	value of matrix $\text{adj}A$.		
8	Find the characteristic roots of the matrix $\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$	Evaluate	2
9	Show that the vectors $X_1=(1,1,2)$, $X_2=(1,2,5)$ and $X_3=(5,3,4)$ are linearly dependent.	Analyze	2
10	Show that the $X_1=(1,1,1)$ $X_2=(3,1,2)$ and $X_3=(2,1,4)$ are linearly independent.	Analyze	2
11	Find the characteristic equation of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	Evaluate	2
12	Show that the Eigen values of a unitary matrix are of unit modulus.	Analyze	2
13	Find the Eigen values of $A = \begin{bmatrix} \frac{1}{2}i & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2}i \end{bmatrix}$	Evaluate	2
14	Find the Eigen values of $A = \begin{bmatrix} 3i & 2+i \\ -2+i & -i \end{bmatrix}$	Evaluate	2
15	Show that the vectors $(1, 2, 3)$, $(3,-2, 1)$, $(1,-6,-5)$ form a linearly dependent set.	Analyze	2
16	Show that the Vector $X_1= (2,2,1)$, $X_2=(1,4,-1)$ and $X_3=(4,6,-3)$ are linearly independent.	Analyze	2
17	Suppose A and P are the square matrices of order n such that P is non singular. Then show that A and $P^{-1}AP$ have the same Eigen values.	Analyze	2
18	Prove that if λ is an Eigen value of an Orthogonal matrix then $\frac{1}{\lambda}$ is also an Eigen value of an Orthogonal matrix.	Analyze	2
19	Prove that if λ is an Eigen value of the matrix A then $\lambda + K$ is an Eigen value of the matrix $A+KI$.	Analyze	2
20	If 2,3,1 are the eigen values of A then find the eigen values of $\text{adj}A$.	Analyze	2
Part - B (Long Answer Questions)			
1	Diagonalize the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$ and find A^4 .	Analyze	3
2	Prove that the Eigen Values of Real symmetric matrix are Real.	Analyse	2
3	Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and find A^{-1} & A^4 .	Evaluate	4
4	Prove that the sum of the Eigen Values of a matrix is equal to its trace and Product of the Eigen Values is equal to its determinant.	Analyze	2

5	Find the Eigen values and Eigen vectors of Hermitian matrix $\begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$	Evaluate	2
6	Prove that Eigen values of a skew- Hermitian matrix are either zero or purely imaginary.	Analyse	2
7	Express $A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A, where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$	Understand	2
8	Determine the Eigen values and Eigen vectors of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	Understand	2
9	Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ verify Cayley-Hamilton theorem hence find A^{-1} and A^4 .	Apply	4
10	Diagonalize the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ by similarity transformation and hence find A^4 .	Apply	3
11	Show that $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is a skew-Hermitian matrix and also unitary. Find the Eigen values and corresponding Eigen vectors of A.	Analyse	2
12	Prove that the Eigen values of a Hermitian matrix are real.	Analyse	2
13	Show that the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation Hence find A^{-1} .	Analyse	2
14	Prove that for a real symmetric matrix, the Eigen vectors corresponding to two distinct Eigen values are orthogonal.	Analyse	2
15	Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ -1 & 2 & 2 \end{bmatrix}$	Evaluate	3
16	Diagonalize the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$	Evaluate	3
17	Determine the modal matrix P of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Verify that $P^{-1}AP$ is a diagonal matrix.	Understand	3
18	Find the Eigen values and Eigen vectors of the matrix A and its inverse, where	Evaluate	2

	$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$		
19	Show that $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^{-1} .	Analyse	4
20	verify Cayley-Hamilton theorem and find the inverse of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$	Analyse	4

Part - C (Problem Solving and Critical Thinking Questions)

1	If a=diagonal(1 -1 2) and b=diagonal (2 3 -1) then find 3a+4b.	Understand	3
2	If 1,2,3 are Eigen values of A then find Eigen values of Adj A.	Evaluate	2
3	If $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ are two orthogonal vectors of 3x3 matrix then find third eigen vector.	Apply	2
4	If $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$ then express A^3 in terms of A.	Apply	2
5	Find the trace A of the matrix if $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$	Evaluate	2
6	If A is a singular matrix then find the product of the Eigen values of A .	Analyse	2
7	If the trace of A (2x2 matrix) is 5 and the determinant is 4, then what are the Eigen values?	Analyse	2
8	If $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ the eigen values of A are (2, 2, -2) then find $P^{-1}A^3P$?	Evaluate	2
9	Find the Model matrix of $A = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$	Evaluate	3
10	Find the spectral matrix of $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$	Evaluate	3

UNIT-III

DIFFERENTIAL EQUATIONS OF FIRST ORDER AND THEIR APPLICATIONS

Part - A (Short Answer Questions)

1	Solve $(x+1)dy/dx - y = e^{3x} (x+1)^2$	Evaluate	5
2	Write the working rule to find orthogonal trajectory in Cartesian form.	Understand	6
3	Form the differential equation by eliminate c in $y = 1 + c\sqrt{1-x^2}$	Analyse	4
4	Solve $(x+y+1) dy/dx = 1$	Analyse	5
5	Prove that the system of parabolas $y^2 = 4a(x+a)$ is self orthogonal.	Analyse	4
6	Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$	Evaluate	6
7	State Newton's law of cooling.	Remember	5
8	A bacterial culture, growing exponentially, increases from 200 to 500 grams in the period from 6 a.m to 9 a.m. How many grams will be present at noon.	Analyse	5
9	Define simple harmonic motion and give its differential equation.	Remember	8

10	Define differential equation of first order and first degree and mention its classification.	Remember	5
11	Find orthogonal trajectory of $y=ax$.	Apply	6
12	Write the working rule to find orthogonal trajectory in polar form.	Understand	6
13	Solve $\sqrt{1+x^2} dy + \sqrt{1+y^2} dx = 0$.	Analyse	5
14	Find the orthogonal trajectory of the family of curves $x^2+y^2=a^2$.	Evaluate	6
15	Find the orthogonal trajectory of the family of curves $x^{2/3}+y^{2/3}=a^{2/3}$.	Evaluate	6
16	Find the orthogonal trajectory of the family of curves $r=2a(\cos\theta+\sin\theta)$.	Evaluate	6
17	State the law of Natural growth or Decay.	Remember	5
18	Solve $x^2y dx - (x^3+y^3) dy = 0$.	Analyse	5
19	Solve $(y+y^2)dx + xy dy = 0$.	Analyse	5
20	Find the orthogonal trajectory of family of curves $r = a \cos \theta$.	Evaluate	6
Part – B (Long Answer Questions)			
1	A bacterial culture, growing exponentially, increases from 200 to 500 grams in the period from 6 am to 9 am. How many grams will be present at noon?	Understand	5
2	solve $x dx + y dy = \frac{a^2(x dy - y dx)}{x^2 + y^2}$.	Analyse	5
3	solve $2xy dy - (x^2 - y^2 + 1) dx = 0$	Analyse	5
4	Find the orthogonal trajectories of the family of curves $x^2 + y^2 = a^2$	Evaluate	6
5	solve $(x+1) \frac{dy}{dx} - y = e^{3x}(x+1)^2$	Analyse	5
6	Obtain the orthogonal trajectories of the family of curves $r(1 + \cos \theta) = 2a$	Apply	6
7	A particle is executing S.H.M, with amplitude 5 meters time 4 sec. find the time required by the Particle in passing between points which are at distances 4 & 2 meters from the centre of force and are on the same side of it.	Understand	8
8	solve $x \frac{dy}{dx} + y = x^3 y^6$	Analyse	5
9	solve $x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x$	Analyse	5
10	The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hours?	Understand	5
11	A copper ball is heated to a temperature of $80^\circ C$ and time $t=0$, then it is placed in water which is maintained at $30^\circ C$. If at $t=3$ minutes, the temperature of the ball is reduced to $50^\circ C$ find the time at which the temperature of the ball is $40^\circ C$.	Understand	5
12	If the air is maintained at $25^\circ C$ and the temperature of the body cools from $100^\circ C$ to $80^\circ C$ in 10 minutes, find the temperature of the body after 20 minutes and when the temperature will be $40^\circ C$.	Understand	5
13	solve $(1+e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1-\frac{x}{y})dy = 0$	Analyse	5

14	A body weighing 10kgs is hung from a spring pull of 20kgs will stretch the spring to 10 cms. The body is pulled down to 20 cms below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position at time 't' seconds, the maximum Velocity and the period of oscillation.	Understand	5
15	Find the orthogonal trajectories of the family of circles $x^2+y^2+2gx+c=0$, Where g is the parameter.	Evaluate	6
16	Solve $\frac{dy}{dx} + y \tan x = y^2 \sec x$	Analyse	5
17	Solve $\frac{dy}{dx} (x^2 y^3 + xy) = 1$	Analyse	5
18	solve $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$	Analyse	5
19	Solve $\frac{dy}{dx} + (y-1) \cos x = e^{-\sin x} \cos^2 x$	Analyse	5
20	Solve $x \frac{dy}{dx} + y = \log x$	Analyse	5

Part – C (Problem Solving and Critical Thinking)

1	Find the order and degree of $\frac{d^2 y}{dx^2} = \left[y + \left(\frac{dy}{dx} \right)^6 \right]^{\frac{1}{4}}$	Understand	5
2	Find the order and degree of $y = x \frac{dy}{dx} + \sqrt{1 + \left(\frac{dy}{dx} \right)^2}$	Understand	5
3	If $x \frac{dy}{dx} = x^2 + y - 2$, $y(1) = 1$ then find $y(2)$.	Apply	5
4	When the differential equation is said to be homogeneous.	Remember	5
5	What is the order of the differential equation $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} - 3y = x$	Remember	5
6	What is the general solution of linear differential equation.	Remember	5
7	What is the orthogonal form of the functions $f(x, y, \frac{dx}{dy})$, $f(r, \theta, \frac{dr}{d\theta})$	Remember	6
8	When the Bernoulli's differential equation becomes linear differential equation.	Remember	5
9	What is the general solution of linear differential equation $\frac{dy}{dx} = e^{x+y}$	Analyse	5
10	What is the general solution of $p^2 - 5p - 6 = 0$ where $p = \frac{dy}{dx}$	Analyse	5

UNIT-IV

HIEHER ORDER LINEAR DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS

Part – A (Short Answer Questions)

1	Solve $\frac{d^3 y}{dx^3} - 3 \frac{dy}{dx} + 2y = 0$	Analyse	7
2	Solve $(4D^2 - 4D + 1)y = 100$.	Analyse	7

3	Write the differential equation of L.R. and L.C.R circuits.	Understand	7
4	Solve the differential equation $(D^2+5D+6)y=e^x$	Analyse	7
5	Solve $y^{11}-4y^1+3y=4e^{3x}$	Analyse	7
6	Solve $y^{11}+4y^1+4y=4\cos x+3\sin x$.	Analyse	7
7	Solve $(D^2+9)y=\cos 3x$.	Analyse	7
8	solve $(D^4-2D^3-3D^2+4D+4)y=0$	Analyse	7
9	Find the particular integral of $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{2x}$	Evaluate	7
10	Find the particular integral of $(D^2-3D+2)y=\cos 3x$	Evaluate	7
11	solve $(D^2+D+1)y=x^3$	Analyse	7
12	What is the Integrating factor of $\frac{dy}{dx}+p(x)y=q(x)$.	Analyse	7
13	What is the integrating factor of $(1-x^2)y+xy=ax$.	Remember	7
14	Define S.H.M. and give its differential equation.	Remember	8
15	Solve $\frac{d^3y}{dx^3}-9\frac{d^2y}{dx^2}+23\frac{dy}{dx}-15y=0$	Analyse	7
16	Solve $y^{11}+y^1-2y=0$, $y(0)=4$, $y^1(0)=1$.	Analyse	7
17	Solve the Differential equation $(D^2+4)y=\tan 2x$.	Analyse	7
18	Solve the Differential equation $(D^2+4)y=\sec 2x$.	Analyse	7
19	Solve the Differential equation $(D^2+4D+3)y=e^{e^x}$	Analyse	7
20	Find the particular integral of $(D^3-3D-2)y=x^2$	Evaluate	7
Part – B (Long Answer Questions)			
1	solve $(D^2+3D+2)y=2\cos(2x+3)+2e^x+x^2$	Analyse	7
2	solve $D^2(D^2+4)y=96x^2+\sin 2x-k$	Analyse	7
3	By using method of variation of parameters solve $(D^2+4)y=\sec 2x$	Analyse	7
4	solve $(D^4+2D^2+1)y=x^2\cos^2 x$	Analyse	7
5	solve $(D^3-6D^2+11D-6)y=e^{-2x}+e^{-3x}$	Analyse	7
6	solve $(D^2+1)y=\sin x\sin 2x+e^x x^2$	Analyse	7
7	Solve $(D^3+1)y=3+5e^x$.	Analyse	7
8	solve $(D^2-3D+2)y=\cos hx$	Analyse	7
9	solve $(D^2-4)y=2\cos^2 x$	Analyse	7
10	solve $(D^2+1)y=\sin x\sin 2x$	Analyse	7
11	solve $(D^2+9)y=\cos 3x+\sin 2x$.	Analyse	7
12	Solve $(D^2+3D+2)y=2\cos(2x+3)+2e^x+x^2$	Analyse	7
13	solve $(D^2+5D-6)y=\sin 4x\sin x$	Analyse	7

14	solve $(D^2 - 2D + 1)y = x^2 e^{3x} - \sin 2x + 3$.	Analyse	7
15	Using the of method of variation Parameters, solve $\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x$.	Apply	7
16	If a voltage of $20 \cos 5t$ is applied to a series circuit consisting of 10 ohm resister and 2 henry inductor, determine the current at any time t.	Apply	8
17	Solve $(D^3 - 4D^2 - D + 4)y = e^{3x} \cos 2x$.	Analyse	7
18	Solve $(D^2 + 9)y = \cos 3x$.	Analyse	7
19	Solve $(D^2 - 4D + 4)y = x^2 \sin x + e^{2x} + 3$.	Analyse	7
20	Solve $(D^3 - 7D^2 + 14D - 8)y = e^x \cos 2x$.	Analyse	7
Part – C (Problem Solving and Critical Thinking)			
1	Mention two applications of higher order differential equations.	Create	8
2	Find the complementary function for $(D^2 + 6D + 9)y = 0$.	Apply	7
3	solve $y^{11} + 6y^1 + 9y = 0, y(0) = -4, y^1(0) = 14$.	Analyse	7
4	Find the particular integral of $(D^3 - 2D + 1)y = \cosh x$.	Evaluate	7
5	Find the complementary function of $(D - 1)^2 y = \sin 2x$.	Evaluate	7
6	Find $\frac{1}{D + 2}(x + e^x)$	Evaluate	7
7	Find $\frac{1}{D - 2} \sin x$	Evaluate	7
8	Find the particular integral of $(D^2 - 5D + 6)y = e^{2x}$.	Evaluate	7
9	Find $\frac{1}{D^2 + D + 1} \sin x$	Evaluate	7
10	Find the particular integral of $(D^2 + a^2)y = \cos a x$.	Evaluate	7
UNIT-V			
FUNCTIONS OF SINGLE AND SEVERAL VARIABLES			
Part - A (Short Answer Questions)			
1	Verify Rolle's theorem for $f(x) = (x+2)^3(x-3)^4$ in $[-2, 3]$.	Analyse	9
2	What is the value of c in cauchy's mean value theorem for the function $f(x) = x^2, g(x) = x^3$ in $(1, 2)$.	Analyse	9
3	Define Rolle's Theorem.	Remember	9
4	Define Lagrange's mean value Theorem.	Remember	9
5	Define Cauchy's Mean Value Theorem.	Remember	9
6	Define functional dependence.	Remember	9
7	If $x + y^2 = u, y + z^2 = v, z + x^2 = w$ find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.	Evaluate	9
8	Show that the functions $u = x + y + z, v = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$ and $w = x^3 + y^3 + z^3 - 3xyz$ are functionally related.	Analyse	9

9	If $x + y + z = u$, $y + z = uv$, $z = uvw$ then evaluate $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.	Evaluate	9
10	If $u = x^2 - y^2$, $v = 2xy$ where $x = r \cos \theta$, $y = r \sin \theta$ then Show that $\frac{\partial(u, v)}{\partial(x, y)} = 4r^3$	Evaluate	9
11	Find the maximum & minimum of the function $f(x) = x^5 - 3x^4 + 5$.	Evaluate	10
12	Show that the functions $u = e^x \sin y$, $v = e^x \cos y$ are not functionally related.	Analyse	9
13	Find the max & min values of the function $f(x) = x^5 - 3x^4 + 5$.	Evaluate	9
14	If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$, then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$	Evaluate	9
15	Verify mean value Theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in $[a, b]$.	Apply	9
16	Find the maximum and minimum values of $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$	Evaluate	10
17	Prove that $u = x + y + z$, $v = xy + yz + zx$, $w = x^2 + y^2 + z^2$ are functionally dependent.	Analyse	9
18	If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$	Analyse	9
19	If $u = \tan^{-1} \left[\frac{2xy}{x^2 - y^2} \right]$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$	Analyse	9
20	Define extreme values and stationary points.	Remember	10
Part - B (Long Answer Questions)			
1	Verify Rolle's theorem for the function $f(x) = e^{-x} \sin x$ in $[0, \pi]$.	Apply	9
2	Verify Rolle's theorem for the functions $\log \left(\frac{x^2 + ab}{x(a+b)} \right)$ in $[a, b]$, $a > 0$, $b > 0$.	Apply	9
3	Verify whether Rolle's Theorem can be applied to the following functions in the intervals. i) $f(x) = \tan x$ in $[0, \pi]$ and ii) $f(x) = 1/x^2$ in $[-1, 1]$.	Apply	9
4	Using Rolle's Theorem, show that $g(x) = 8x^3 - 6x^2 - 2x + 1$ has a zero between 0 and 1.	Apply	9
5	Verify Lagrange's Mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in $[0, 4]$.	Apply	9
6	If $a < b$, P.T $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ using Lagrange's Mean value theorem. Deduce the following. (i) $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$ ii) $\frac{5\pi + 4}{20} < \tan^{-1} 2 < \frac{\pi + 2}{4}$	Apply	9
7	Show that for any $x > 0$, $1 + x < e^x < 1 + xe^x$.	Apply	9
8	Using Mean value theorem prove that $\tan x > x$ in $0 < x < \pi/2$.	Apply	9

9	Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x} \text{ \& } g(x) = \frac{1}{\sqrt{x}} \text{ in } [a,b] \text{ where } 0 < a < b$	Apply	9
10	Verify Cauchy's Mean value theorem for $f(x) = e^x$ & $g(x) = e^{-x}$ in $[3,7]$ and find the value of c.	Apply	9
11	Verify Cauchy's theorem for , $f(x) = \log x$, $g(x) = (1/x)$ on $[1, e]$.	Apply	9
12	If $x = \frac{u^2}{v}$, $y = \frac{v^2}{u}$ find $\frac{\partial(u,v)}{\partial(x,y)}$	Evaluate	9
13	If $x = e^r \sec \theta$, $y = e^r \tan \theta$ Prove that $\frac{\partial(x,y)}{\partial(r,\theta)} \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.	Evaluate	9
14	Calculate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ if $x = \sqrt{vw}$, $y = \sqrt{uw}$, $z = \sqrt{uv}$ and $u = r \sin \theta \cos \phi$, $v = r \sin \theta \sin \phi$, $w = r \cos \theta$.	Evaluate	9
15	Find the stationary points & examine their nature of the following functions $u = x^4 + y^4 - 2x^2 + 4xy - 2y^2$, $(x > 0, y > 0)$	Evaluate	10
16	Find the maxima & minima of the function $f(x) = x^3 y^2 (1-x-y)$.	Analyse	10
17	Find the maxima & minima of the function $f(x) = 2(x^2 - y^2) - x^4 + y^4$.	Analyse	10
18	Find the maximum value of xyz when $x + y + z = a$.	Analyse	10
19	Find three positive numbers whose sum is 100 and whose product is maximum.	Analyse	10
20	Verify Cauchy's mean value theorem for $f(x) = \log x$, $g(x) = 1/x$ on $[1,e]$.	apply	9
Part – C (Problem Solving and Critical Thinking)			
1	Find the maxima and minima of the function $f(x) = 2(x^2 - y^2) - x^4 + y^4$.	Analyse	10
2	Verify Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$ where m,n are positive integers in $[a,b]$.	Apply	9
3	Verify Rolle's theorem for $f(x) = x^2 - 2x - 3$ in the interval $(1,-3)$.	Evaluate	9
4	Explain the method of finding stationary values using Lagrange's method.	Remember	9
5	Explain the working rule to find maximum and minimum values of $f(x, y)$.	Remember	10
6	Define jacobian of (x,y) to (u,v) .	Remember	9
7	Find the value of 'c' of Rolle's Mean value theorem for $f(x) = \frac{x^3}{3} - 3x \text{ in } [0, 3].$	Evaluate	9
8	If $x = u(1-v)$, $y = uv$ Prove that $JJ' = 1$.	Analyse	9
9	What is the value of c of Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ defined on $[a,b]$, $0 < a < b$.	Apply	9
10	If $x = r \cos \theta$, $y = r \sin \theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.	Evaluate	9

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