INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad -500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

TUTORIAL QUESTION BANK

Course Name	:	**Engineering Mathematics –III**
Course Code	:	A30007
Class	:	II-I B. Tech
Branch	:	ECE,EEE
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Course Faculty	:	K JAGAN MOHAN RAO, CH. SOMA SHEKAR, V SUBBA LAXMI , C RACHANA.

OBJECTIVES

To meet the challenge of ensuring excellence in engineering education, the issue of quality needs to be addressed, debated and taken forward in a systematic manner. Accreditation is the principal means of quality assurance in higher education. The major emphasis of accreditation process is to measure the outcomes of the program that is being accredited.

In line with this, Faculty of Institute of Aeronautical Engineering, Hyderabad has taken a lead in incorporating philosophy of outcome based education in the process of problem solving and career development. So, all students of the institute should understand the depth and approach of course to be taught through this question bank, which will enhance learner's learning process.

1. Group - A (Short Answer Questions)

S.No	Question	Blooms Taxonomy Level	Course Outcome
	UNIT-I **Linear ODE with variable coefficients and series solution (second order only)**		
	Group - A (Short Answer Questions)		
1	Solve in series the equation $\frac{d^2y}{dx^2} - y = 0$ about x=0.	Evaluate	c
2	Solve in series the equation $y'' + y = 0$ about x=0.	Evaluate	c
3	Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0$.	Evaluate	c
4	Solve in series the equation $y'' + x^2y = 0$ about x=0.	Evaluate	c
5	Solve in series the equation$2x^2y'' + (x^2 - x)\, y' + y = 0$.	Evaluate	c
6	Solve in series the equation$x(1-x)y'' - (1+3x)\ y' - y = 0$.	Evaluate	c
7	Solve in series the equation $(x - x^2)y'' + (1-5x)y' - 4y = 0$.	Evaluate	c
8	Solve$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12logx}{x^2}$.	Evaluate	c
9	Solve$\left(x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4\right)y = x^4$.	Evaluate	c
10	Find the power series solution of the equation $y'' + (x-3)y' + y = 0$in powers of (x-2) (i.e,about x=2).	Analyse	c
	Group - B (Long Answer Questions)		

S.No	Question	Blooms Taxonomy Level	Course Outcome
1	Solve$\left(\boldsymbol{x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y}\right)=(\boldsymbol{1+x})^{\boldsymbol{2}}.$	Evaluate	a
2	Solve$\left(\boldsymbol{x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8}\right)\boldsymbol{y=65cos(logx)}.$	Understand	a
3	Solve$(\boldsymbol{x+1})^2\frac{\boldsymbol{d^2y}}{\boldsymbol{dx^2}}+(\boldsymbol{x+1})\frac{\boldsymbol{dy}}{\boldsymbol{dx}}=$ (2x+1)(2x+4).	Evaluate	a
4	$\boldsymbol{Solve(x+1)^2\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}+y=sin2(log(1+x))}.$	Evaluate	a
5	Find the power series solution oftheequation $\boldsymbol{y''+\ (x-3\ )y'+y=0}$in powers of (x-2) (i.e,about x=2).	Evaluate	c
6	Solve in series the equation $\boldsymbol{2x(1-x)\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+3y=0}.$	Analyse	c
7	Solve in series the equation $\boldsymbol{(x-x^2)y''+\ (1-x)y'-y=0}.$	Understand	c
8	Solve in series the equation $\boldsymbol{x(1-x)y''-(1+3x)\ y'-y=0}.$	Evaluate	c
9	Solve $\boldsymbol{(x^2D^2-4xD+6\ )y=\ (logx)^2}.$	Analyse	b
10	Solve $\boldsymbol{(x+a)^2\frac{d^2y}{dx^2}-4(x+a)\frac{dy}{dx}+6y=x}.$	Evaluate	b
Group –C (Analytical Questions)			
1	Find the singular points and classify them (regular or irregular) $x^2y''+\ ax\ y'+by=0$.	Analyse	b
2	Find the singular points and classify them (regular or irregular) $x^2y''+x\ y'+(x^2-n^2)y=0.$	Evaluate	b
3	Find the singular points and classify them (regular or irregular) $(1-x^2)y''-2xy'+n(n+1)y=0.$	Understand	b
4	Define Ordinary and Regular singular point.	Analyse	b
5	Explain Frobenius method about regular singular points.	Evaluate	b
6	Explain the method of solving Legendre's differential equation.	Analyse	a
7	Explain the method of solving Cauchy's differential equation.	Analyse	a
8	Find the singular points and classify them (regular or irregular) $x^2y''-5y'+3x^2y=0.$	Evaluate	b
9	Find the singular points and classify them (regular or irregular) $x^2y''+(x+x^2)y'-y=0.$	Analyse	b
10	Find the singular points and classify them (regular or irregular) $x^3(x-2)\ y''+\ x^3y'+6y=0.$	Understand	b
UNIT-II **Special functions**			
Group - A (Short Answer Questions)			
1	Show that $\int_0^x x^nJ_{n-1}(x)dx\ =\ x^nJ_n\ (x).$	Analyse	d
2	Showthat $\int_0^x x^{n+1}J_n(x)dx\ =\ x^{n+1}J_{n+1}\ (x).$	Analyse	d
3	Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.	Analyse	d
4	Show that $J_0(x)\ =\ \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta)d\theta$satisfies Bessel's equation of order zero.	Analyse	d
5	Express $J_2(x)\ in\ terms\ of\ J_0(x)\ and\ J_1(x).$	Apply	d
6	Show that $J_3(x)+3J_0'(x)+4J_0'''(x)=0$.	Analyse	d
7	Prove that $\frac{d}{dx}[x^nJ_n(x)\ ]=x^nJ_{n-1}(x).$	Analyse	d
8	Prove that $\int_0^r x\,J_0(ax)=\ \frac{r}{a}J_1(ar).$	Analyse	d

S.No	Question	Blooms Taxonomy Level	Course Outcome
9	Show that $J_n(x)$ is an even function if ‘n’ is even and odd function when ‘n’ is odd.	Remember	d
10	Prove that $\left[J_{\frac{1}{2}}\right]^2 + \left[J_{-\frac{1}{2}}\right]^2 = \frac{2}{\pi x}$.	Analyse	d
	Group - B (Long Answer Questions)		
1	State and prove Rodrigue’s formula.	Evaluate	d
2	Show that $x^4 = \frac{8}{35}P_4(x) + \frac{4}{7}P_2(x) + \frac{1}{5}P_0(x)$.	Understand	d
3	Express P(x) = $x^4 + 2x^3 + 2x^2 - x - 3$in terms of Legendre Polynomials.	Evaluate	d
4	Using Rodrigue’s formula prove that $\int_{-1}^{1} x^m p_n(x)dx = 0$if m<n.	Evaluate	d
5	State and prove orthogonality of Legendre polynomials.	Analyse	d
6	If f(x) = 0 if -1 < x < 0 =1 if 0<x<1 then show that f(x) = $\frac{1}{2}P_0(x) + \frac{3}{4}P_1(x) - \frac{7}{16}P_3(x) + \cdots$	Evaluate	d
7	Show that P_n(x) is the coefficient of t^nin the expansion of $(1 - 2xt + t^2)^{\frac{-1}{2}}$.	Remember	d
8	Prove (2n+1) x$P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$.	Understand	d
9	Prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = \begin{cases} 0, if \alpha \neq \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2 \ if \ \alpha = \beta \end{cases}$.	Evaluate	d
10	Show that $a) J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\theta - xsin\theta)d\theta$. b) $J_0(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta)d\theta = \frac{1}{\pi}\int_0^{\pi} cos(xcos\theta)d\theta$.	Remember	d
	Group - C (Analytical Questions)		
1	Find the value of $J_{\frac{1}{2}}(x)$.	Evaluate	d
2	Find the generating function forJ_n(x) .	Apply	
3	Write the integral form of Bessel’s function .	Analyse	d
4	Find the value of $J_{\frac{5}{2}}(x)$.	Analyse	d
5	Express $J_5(x)\ in\ terms\ of\ J_0(x)\ and\ J_1(x)$.	Evaluate	d
6	Prove that $2J_0''(x) = J_2(x) - J_0(x)$.	Remember	d
7	Show that $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$.	Analyse	d
8	Show that $J_0(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta)d\theta$satisfies Bessel’s equation of order zero.	Analyse	d
9	Prove that $J_n(-x) = (-1)^n J_n(x)$ where n is a positive or negative integer .	Evaluate	d
10	Find the value of $\int_{-1}^{1} P_3(x)P_4(x)dx$.	Analyse	d
	UNIT-III **Complex functions-differentiation and integration**		
	Group - A (Short Answer Questions)		
1	Let w = f(z) = z^2 find the values of w which correspond to (i) z = 2+i (ii) z = 1+3i	Analyse	e

S.No	Question	Blooms Taxonomy Level	Course Outcome
2	Show that f(z) = $\lvert z\rvert^2$ is a function which is continuous at all z but not differentiable at any $z \neq 0$.	Understand	e
3	Find all values of k such that f(x) = $e^x(cosky + isinky)$ is analytic.	Understand	e
4	Show that $u = e^{-x}(xsiny - ycosy)$ is harmonic.	Understand	e
5	Verify that u = $x^2 - y^2 - y$ is harmonic in the whole complex plane and find a conjugate harmonicfunction v of u .	Understand	e
6	Find k such that f(x,y) = $x^3 + 3kxy^2$ may be harmonic and find its conjugate.	Analyse	f
7	Find the most general analytic function whose real part is u = $x^2 - y^2 - x$.	Analyse	f
8	Find an analytic function whose imaginary part is v = $e^x(xsiny + ycosy)$.	Understand	f
9	If f(z) is an analytic function of z and if u - v = $\frac{cosx+sinx-e^{-y}}{2cosx-e^{y}-e^{-y}}$,find f(z) subject to the condition f($\frac{\pi}{2}$) = 0.	Analyse	f
10	If f(z) is an analytic function of z and if u + v = $\frac{sin2x}{2cosh2y-cos2x}$find f(z) in terms of z.	Remember	f
	Group - B (Long Answer Questions)		
1	Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \lvert Realf(z)\rvert^2 = 2\lvert f'(z)\rvert^2$ where w = f(z) is analytic.	Apply	e
2	Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) log\lvert f'(z)\rvert$ where w = f(z) is analytic.	Apply	e
3	If f(z) is a regular function of z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \lvert f(z)\rvert^2 = \lvert f'(z)\rvert^2$.	Apply	e
4	Show that the function defined by f(z)= $\begin{cases} \frac{xy^2\ (x+iy}{x^2+y^4} & ,z \neq 0 \\ 0 & ,z = 0 \end{cases}$ is not analytic although Cauchy Riemann equations are satisfied at the origin.	Apply	e
5	Show that u = $x^3 - 3xy^2$is harmonic and find its harmonic conjugate and the corresponding analytic function f(z) in terms of z.	Analyse	f
6	Evaluate $\int_0^{1+i}(x - y + ix^2)\,dz$ (i) along the straight from z = 0 to z = 1+i . (ii) along the real axis from z = 0 to z = 1 and then along a line parallel to real axis from z = 1 to z = 1+i along the imaginary axis from z = 0 to z = I and then along a line parallel to real axis z = i to z = 1+ i.	Apply	e
7	Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices -1 ,1,1+i ,-1+i.	Apply	e
8	Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)}$ dz where c is the circle $\lvert z\rvert$ =3.	Apply	e
9	Evaluate $\int_c \frac{z^3 e^{-z}}{(z-1)^3}$ dz where c is $\lvert z-1\rvert = \frac{1}{2}$ using Cauchy's integral	evaluate	e

S.No	Question	Blooms Taxonomy Level	Course Outcome
	formula.		
Group - C (Analytical Questions)			
1	Show that f(z) = z^3 is analytic for all z.	Understand	e
2	Find whether f(z) = sinxsiny – icosxcosy is analytic or not.	Understand	e
3	Show that both the real and imaginary parts of an analytic function are harmonic.	Analyse	e
4	Show that the function u = $2\log(x^2+y^2)$ *is harmonic and find its* harmonic conjugate.	Analyse	e
5	Find an analytic function whose real part is u = $e^x[(x^2-y^2)cosy - 2xysiny)]$.	Analyse	e
6	Find an analytic function whose real part is u = $\frac{sin2x}{cosh2y-cos2x}$.	Evaluate	e
7	If f(z) is an analytic function of z and if u - v = $\frac{cosx+sinx-e^{-y}}{2cosx-e^{y}-e^{-y}}$find f(z) subject to the condition f($\frac{\pi}{2}$) = 0.	Analyse	e
8	Find whether f(z) = sinxsiny – icosxcosy is analytic or not.	Evaluate	e
9	Show that f(z) = x +iy is everywhere continous but is not analytic.	Understand	f
10	Show that u = $e^{-x}(xsiny - ycosy)$ is harmonic.	Analyse	f
UNIT-IV **Power series expansions of complex functions and contour integration**			
Group - A (Short Answer Questions)			
1	Find the poles and residues of $\frac{1}{z^2-1}$.	Analyse	g
2	Find zeros and poles of $\left(\frac{z+1}{z^2+1}\right)^2$.	Analyse	g
3	Find the poles of the function $f(z)=\frac{1}{(z+1)(z+3)}$ and residues at these poles.	Analyse	g
4	Find the residue of the function $f(z)=\frac{z^3}{(z^2-1)}$ *at* $z=\infty$.	Evaluate	g
6	Find the residue of $\frac{z^2}{(z-a)(z-b)(z-c)}$ *at* $z=\infty$.	Evaluate	g
7	Determine the poles and the residue of the function $f(z)=\frac{ze^z}{(z+2)^4(z-1)}$.	Remember	g
8	Find the region in the w-plane in which the rectangle bounded by the lines x= 0 ,y = 0 x=2 and y=1 is mapped under the transformation w = z+(2+3i).	Analyse	g
9	Obtain the Taylor series expansion of f(z) = $\frac{1}{z}$ about the point z = 1.	Analyse	k
10	Obtain the Taylor series expansion of f(z) = e^z about the point z = 1.	Evaluate	k
Group - B (Long Answer Questions)			

S.No	Question	Blooms Taxonomy Level	Course Outcome
1	Expand f(z) = $\frac{z-1}{z+1}$ in Taylor's series about the point (i) $z=0$ (ii) z = 1 .	Apply	k
2	Expand f(z) = $\frac{z-1}{z^2}$ in Taylor's series in powers of z -1 and determine the region of convergence.	Evaluate	k
3	Obtain Laurent's series expansion of f(z) = $\frac{z^2-4}{z^2+5z+4}$ valid in 1< z < 2.	Analyse	k
4	Expand $f(z)=\frac{e^{2z}}{(z-1)^3}$ about z = 1 as Laurent's series and also find the region of convergence.	Evaluate	k
5	Expand f(z) = $\frac{7z-2}{z(z+1)(z-2)}$ about z=-1 in the region $1<\lvert z+1\rvert<3$ as Laurent's series .	Evaluate	k
6	Determine the poles and the residue of the function $f(z)=\frac{ze^z}{(z+2)^4(z-1)}$.	Evaluate	h
7	Evaluate $\oint_c \frac{4-3z}{(z-2)(z-1)z}$ dzwhere c is the circle $\lvert z\rvert = 1.5$ using residue theorem.	Apply	e
8	Show that $\int_0^{2\pi} \frac{1+4\cos\theta}{17+8\cos\theta}d\theta=0$.	Apply	i
9	Evaluate $\int_0^{\infty} \frac{dx}{x^6+1}$.	Apply	i
10	Show that $\int_0^{2\pi} \frac{d\theta}{4\cos^2\theta+\sin^2\theta}=\pi$.	Analyse	i
	Group -C (Analytical Questions)		
1	Expand f(z) = $\frac{z-1}{z+1}$ in Taylor's series about the point (i) $z=0$ (ii) z = 1 .	Apply	k
2	Expand f(z) = $\frac{z-1}{z^2}$ in Taylor's series in powers of z -1 and determine the region of convergence.	Evaluate	k
3	Obtain Laurent's series expansion of f(z) = $\frac{z^2-4}{z^2+5z+4}$ valid in 1< z < 2.	Analyse	k
4	Expand $f(z)=\frac{e^{2z}}{(z-1)^3}$ about z = 1 as Laurent's series and also find the region of convergence.	Evaluate	k

S.No	Question	Blooms Taxonomy Level	Course Outcome
5	Expand f(z) = $\frac{7z-2}{z(z+1)(z-2)}$ about z=-1 in the region $1<\lvert z+1\rvert<3$ as Laurent's series .	Evaluate	k
6	Determine the poles and the residue of the function $f(z)=\frac{ze^{z}}{(z+2)^{4}(z-1)}$.	Evaluate	h
7	Evaluate $\oint_{c}\frac{4-3z}{(z-2)(z-1)z}$ dzwhere c is the circle $\lvert z\rvert$ = 1.5 using residue theorem.	Apply	e
8	Show that $\int_{0}^{2\pi}\frac{1+4\cos\theta}{17+8\cos\theta}d\theta=0$.	Apply	i
9	Evaluate $\int_{0}^{\infty}\frac{dx}{x^{6}+1}$.	Apply	i
10	Show that $\int_{0}^{2\pi}\frac{d\theta}{4\cos^{2}\theta+\sin^{2}\theta}=\pi$.	Analyse	i
UNIT-V **Conformal mapping**			
Group - A (Short Answer Questions)			
1	Determine the bilinear transformation whose fixed points are 1,-1.	Remember	i
2	Determine the bilinear transformation whose fixed points are i,-i.	Analyse	i
3	Find the fixed points of the transformation $w=\frac{2i-6z}{iz-3}$.	Remember	i
4	Evaluate $\int_{c}\frac{2z-1}{z(2z+1)(z+2)}dz$ where c is the circle $\lvert z\rvert$ = 1.	Evaluate	i
5	Evaluate $\int_{c}\frac{12z-7}{(2z+3)(z-1)^{2}}dz$ where c is the circle $x^2+y^2=$ 4.	Understand	m
6	Evaluate $\int_{c}\frac{e^{z}}{(z-3)z}dz$ where c is the circle $\lvert z\rvert$ = 2 using Residue theorem.	Understand	m
7	Evaluate $\int_{c}\frac{3z-4}{(z-1)z}dz$ where c is the circle $\lvert z\rvert$ = 2 using Residue theorem.	Understand	m
8	Show that $\int_{0}^{2\pi}\frac{d\theta}{4\cos^{2}\theta+\sin^{2}\theta}=\pi$.	Understand	m

S.No	Question	Blooms Taxonomy Level	Course Outcome
9	Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$.	Understand	m
10	Evaluate $\int_{0}^{\infty} \frac{x \sin mx}{(x^4+16)} dx$.	Understand	m
	Group - B (Long Answer Questions)		
1	Find the Bi-linear transformation which carries the points from $(0,1,\infty) to (-5,-1,3)$.	Evaluate	m
2	Find the image of the triangle with vertices 1,1+I,1-i in the z-plane under the transformation w=3z+4-2i.	Evaluate	m
3	Find the Bi-linear transformation which carries the points from (-i,0,i) to (-1,i,1).	Remember	m
4	Sketch the transformation $w = e^z$.	Understand	m
5	Sketch the transformation $w = \log z$.	Understand	m
6	Find the Bi-linear transformation which carries the points from $(1,i,-1) to (0,1,\infty)$	Apply	m
7	Show that transformation w = z^2 maps the circle $\|z-1\| = 1$ into the cardioid r = 2(1+cosθ) where $w = re^{i\theta}$ in the w-plane.	Evaluate	m
8	Determine the bilinear transformation that maps the points (1-2i,2+i,2+3i) into the points (2+i,1+3i,4).	Apply	m
9	Under the transformation $w = \frac{z-i}{1-iz}$, find the image of the circle (i) $\|w\| = 1$ $(ii) \|z\|=1$ in the w-plane.	Apply	m
10	Find the image of the region in the z-plane between the lines y = 0 and y = $\frac{\pi}{2}$ under the transformation w = e^z.	Evaluate	m
	Group - C (Analytical Questions)		
1	Show that the function $w = \frac{1}{z}$ transforms the straight line x=c in the z-plane into a circle in the w-plane.	Analyse	m
2	Find the fixed points of the transformation $w = \frac{2i-6z}{iz-3}$.	Analyse	m
3	Find the invariant points of the tranformation $w = \frac{z-1}{z+1}$.	Understand	m
4	Find the critical points of w = $\frac{6z-9}{z}$.	Understand	m
5	Show that the transformation $w = \frac{2z+3}{z-4}$ changes the circle	Analyse	m

S.No	Question	Blooms Taxonomy Level	Course Outcome
	$x^2 + y^2 - 4x = 0$ into the straight line 4u+3=0.		
6	Find the image of the triangle with vertices at i,1+i,1-i in the z-plane under the transformation w = 3z+4-2i.	Understand	m
7	Find the image of the domain in the z-plane to the left of the line x=-3 under the transformation w = z^2.	Understand	m
8	Show that the function w = $\frac{4}{z}$ transforms the straight line x = c in the z-plane into a circle in the w- plane.	Understand	m
9	Define Translation ,Rotation and magnification of the transform.	Evaluate	m
10	Define and sketch Joukowski's transformation.	Apply	m

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