# INSTITUTE OF AERONAUTICAL ENGINEERING <br> (Autonomous) <br> Dundigal, Hyderabad -500 043 

## COMPUTER SCIENCE AND ENGINEERING

TUTORIAL QUESTION BANK

| Course Name | PROBABILITY AND STATISTICS |
| :--- | :--- |
| Course Code | A30008 |
| Class | II-I B. Tech |
| Branch | Computer Science Engineering |
| Year | $2016-2017$ |
| Course Faculty | Mr. J Suresh Goud, Associate Professor, Freshman Engineering <br> Ms. L Indira, Associate Professor, Freshman Engineering <br> Ms. P Srilatha, Assistant Professor, Freshman Engineering |

## OBJECTIVES

To meet the challenge of ensuring excellence in engineering education, the issue of quality needs to be addressed, debated and taken forward in a systematic manner. Accreditation is the principal means of quality assurance in higher education. The major emphasis of accreditation process is to measure the outcomes of the program that is being accredited.

In line with this, Faculty of Institute of Aeronautical Engineering, Hyderabad has taken a lead in incorporating philosophy of outcome based education in the process of problem solving and career development. So, all students of the institute should understand the depth and approach of course to be taught through this question bank, which will enhance learner's learning process.

## UNIT-I

SINGLE RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS
Part - A (Short Answer Questions)

| S. No | Question | Blooms Taxonomy <br> Level | Course <br> Outcome |
| :---: | :--- | :---: | :---: |
| 1 | If X is Poisson variate such that $\mathrm{p}(\mathrm{x}=1)=24 \mathrm{p}(\mathrm{x}=3)$. Find the mean | Evaluate | 4 |
| 2 | Find the probability distribution for sum of scores on dice if we <br> throw two dice | Evaluate | 4 |
| 3 | Out of 24 mangoes, 6 mangoes are rotten . If we draw two <br> mangoes . Obtain probability distribution of number of rotten <br> mangoes that can be drawn. | Analyze | 4 |
| 4 | Determine the binomial distribution for which the mean is 4 and <br> variance 3 | Understand | 4 |
| 5 | If X is normally distributed with mean 2 and variance 0.1, then find <br> $\mathrm{P}(\|x-2\| \geq 0.01) ?$ | Evaluate | 4 |
| 6 | If X \& Y is a random variable then Prove $\mathrm{E}[\mathrm{X}+\mathrm{K}]=\mathrm{E}[\mathrm{X}]+\mathrm{K}$, where ' $\mathrm{K} '$ <br> constant | Understand | 2 |
| 7 | Prove that $\sigma^{2}=E\left(X^{2}\right)-\mu^{2}$ | Understand | 2 |
| 8 | Explain probability distribution for discrete and continuous | Analyze | 3 |


| 9 | If X is Discrete Random variable then Prove that Var $(\mathrm{a} \mathrm{X} \mathrm{+b})=\mathrm{a}^{2} \operatorname{var}(\mathrm{X})$ | Understand | 3 |
| :---: | :--- | :---: | :---: |
| 10 | Write the properties of the Normal Distribution | Analyze | 5 |
| 11 | Write the importance and applications of Normal Distribution | Apply | 5 |
| 12 | Define different types of random variables with example | Remember | 3 |
| 13 | Derive variance of binomial distribution | Evaluate | 4 |
| 14 | Derive mean of Poisson distribution | Evaluate | 4 |
| 15 | Explain about Moment generating function | Analyze | 5 |
|  |  |  |  |

Part - B (Long Answer Questions)

| 1 | A random variable x has the following probability function: $\begin{array}{ccccccccccc} \mathrm{x} & 0 & 1 & 3 & 4 & 5 & 6 & 7 \\ \mathrm{P}(\mathrm{x}) & 0 & \mathrm{k} & 2 \mathrm{k} & 2 \mathrm{k} & 3 \mathrm{k} & k^{2} & 7 k^{2}+\mathrm{k} \end{array}$ <br> Find the value of k (ii) evaluate $\mathrm{p}(\mathrm{x}<6), \mathrm{p}(\mathrm{x}>6)$ |  |  |  |  |  | Evaluate | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | Let X denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once. Determine the (i) Discrete probability distribution (ii) Expectation (iii) Variance |  |  |  |  |  | Understand \& Evaluate | 3 |
| 3 | A random va <br> Then find (i) | $\begin{gathered} \text { able } \\ \hline-1 \\ \hline \mathrm{~K} \\ \hline \end{gathered}$ | $\begin{gathered} \hline \text { has the } \\ \hline 0 \\ \hline 0.2 \\ \hline \text { an (iii) } \end{gathered}$ | $\begin{gathered} \hline \text { follow } \\ \hline 1 \\ \hline 2 \mathrm{~K} \\ \hline \text { varian } \end{gathered}$ | $\begin{gathered} \hline \text { ing prc } \\ \hline 2 \\ \hline 0.3 \\ \hline e \text { (iv) } \end{gathered}$ | 3ability function: <br> K <br> $(0<x<3)$ | Evaluate | 3 |
| 4 | A continuous random variable has the probability density function$\begin{aligned} & f(x)=\left\{\begin{array}{l} k x e^{-\lambda x}, \text { for } x \geq 0, \lambda>0 \\ 0, \text { otherwise } \end{array} \quad\right. \text { Determine (i) k (ii) Mean (iii) } \\ & \text { Variance } \end{aligned}$ |  |  |  |  |  | Evaluate | 3 |
| 5 | If the PDF of Random variable $\mathrm{f}(\mathrm{x})=k\left(1-x^{2}\right), 0<x<1$ then find (i) k (ii) $\mathrm{p}[0.1<\mathrm{x}<0.2]$ (iii) $\mathrm{P}[\mathrm{x}>0.5]$ |  |  |  |  |  | Evaluate | 3 |
| 6 | If the masses of 300 students are normally distributed with mean 68 kg and standard deviation 3 kg how many students have masses: greater than 72 kg (ii) less than or equal to 64 kg (iii) between 65 and 71 kg inclusive |  |  |  |  |  | Analyze | 5 |
| 7 | Out of 800 families with 5 children each, how many would you expect to have (i)3 boys (ii) 5 girls (iii)either 2 or 3 boys? Assume equal probabilities for boys and girls. |  |  |  |  |  | Understand \& Evaluate | 4 |
| 8 | If a Poisson distribution is such that $P(X=1) \cdot \frac{3}{2} P(X=3)$, find (i) $P(X \geq 1)_{\text {(ii) }} P(X \leq 3)_{\text {(iii) }} P(2 \leq X \leq 5)$. |  |  |  |  |  | Evaluate | 4 |
| 9 | Average number of accidents on any day on a national highway is 1.8. Determine the probability that the number of accidents is (i) at least one |  |  |  |  |  | Analyze \& Evaluate | 4 |


|  | (ii) at most one |  |  |
| :---: | :--- | :--- | :---: |
| 10 | In a Normal distribution, $7 \%$ of the item are under 35 and $89 \%$ are <br> under 63. Find the mean and standard deviation of the distribution. | Evaluate | 5 |

## Part - C (Problem Solving and Critical Thinking Questions)

| 1 | When the classical definition of probability fails. | Analyze | 2 |
| :---: | :--- | :---: | :---: |
| 2 | The function $\mathrm{f}(\mathrm{x})=\mathrm{Ax}^{2}$ In $0<\mathrm{x} 0<1$ is valid probability density function <br> then find the value of A. | Evaluate | 3 |
| 3 | Define Normal distribution | Understand | 5 |
| 4 | Explain about Moments | Analyze | 6 |
| 5 | Derive mean deviation from the mean for Normal Distribution | Analyze | 5 |
| 6 | What is the area under the whole normal curve? | Analyze | 5 |
| 7 | In which distribution the mean, mode and median are equal. | Evaluate | 4 |
| 8 | The mean and variance of a binomial variable $X$ with parameters n and p <br> are 16 and Find $P(X \geq 1)$ | Analyze | 5 |
| 9 | Where the traits of normal distribution lies. | Understand | 2 |
| 10 | Write the properties of continuous random variable |  |  |

## UNIT-II

MULTIPLE RANDOM VARIABLES, CORRELATION \&REGRESSION

## Part - A (Short Answer Questions)

| 1 | State the properties of joint distribution function of two random variable | Analyze | 5 |
| :---: | :--- | :---: | :---: |
| 2 | The equations of two regression lines obtained in a correlation <br> analysis are $3 x+12 y=19,3 y+9 x=46$. Find means of x and y | Evaluate | 5 |
| 3 | Given $\mathrm{n}=10, \sigma_{x}=5.4, \sigma_{y}=6.2$ and sum of the product of <br> deviation from the mean of X and Y is 66 find the correlation co- <br> efficient | Evaluate | 5 |
| 4 | From the following data calculate (i) correlation c coefficient (ii) <br> standard deviation of Y <br> bxy=0.85, byx=0.89, $\sigma_{x}=3$ | Evaluate | 6 |
| 5 | If $r_{12}=0.77, r_{13}=0.72, r_{23}=0.52$ Find the multiple correlation <br> coefficient. | Evaluate | 5 |
| 6 | Determine the probability of getting at least 60 heads when 100 <br> coins are tossed. |  |  |
| 7 | Expaluate | Analyze | 6 |
| 8 | If a random variable W=X+Y where X and Y are two independent <br> random variables what is the density function of W | Analyze | 6 |
| 9 | Explain types of correlations | Remember | 7 |
| 10 | Write the properties of rank correlation coefficient | Analyze | 7 |
| 11 | Write the properties of regression lines | Analyze | 7 |
| 12 | Write the difference between correlation and regression | Remember | 7 |
| 13 | The rank correlation coefficient between the marks in two subjects is <br> $0.8 . t h e ~ s u m ~ o f ~ t h e ~ s q u a r e s ~ o f ~ t h e ~ d i f f e r e n c e ~ b e t w e e n ~ t h e ~ r a n k s ~ i s ~ 33 . f i n d ~$ <br> the number of students | Evaluate | 7 |
| 14 | Find the angle between the regression lines if S.D of Y is twice the S.D of | Evaluate | 7 |


|  | X and $\mathrm{r}=0.25$ |  |  |
| :---: | :--- | :---: | :---: |
| 15 | Derive the angle between the two regression lines | Evaluate | 7 |

Part - B (Long Answer Questions)

| 1 | Consider the joint probability density function $f(x, y)=x y, 0<x<1,0<y$ <2. Find marginal density function |  |  |  |  |  |  |  | Evaluate | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | Two independent variable X and Y have means 5 and 10 and variances 4 and 9 respectively. Find the coefficient of correlation between $U$ and $V$ where $U=3 x+4 y, V=3 x-y$ |  |  |  |  |  |  |  | Understand \& Evaluate | 7 |
| 3 | The probability density function of a random variable x is $f(x)=\frac{1}{2} \exp \left[-\frac{x}{2}\right], \quad x>0$. Find the probability of $1<\mathrm{x}<2$. |  |  |  |  |  |  |  | Evaluate | 6 |
| 4 | Let X and Y random variables have the joint density function $f(x, y)=2,0<x<y<1$ then find marginal density function |  |  |  |  |  |  |  | Evaluate | 6 |
| 5 | Find the rank correlation coefficient for the following ranks of 16 students$\begin{aligned} & (1,1),(2,10),(3,3),(4,4),(5,5),(6,7),(7,2),(8,6),(9,8),(10,11),(11,15),(12,9),(13 \\ & , 14),(14,12),(15,16)(16,13) \end{aligned}$ |  |  |  |  |  |  |  | Apply | 7 |
| 6 | Calculate the coefficient of correlation between age of cars and annual maintain cost and comment: |  |  |  |  |  |  |  | Apply | 7 |
| 7 | If $\sigma_{\mathrm{x}=}=\sigma_{\mathrm{y}=}=\sigma$ and the angle between the regression lines is Tan ${ }^{-1}(4 / 3)$. Find r . |  |  |  |  |  |  |  | Apply | 7 |
| 8 | For 20 army personal the regression of weight of kidneys (Y) on weight of heart $(\mathrm{X})$ is $\quad \mathrm{Y}=3.99 \mathrm{X}+6.394$ and the regression of weight of heart on weight of kidneys is $\mathrm{X}=1.212 \mathrm{Y}+2.461$. Find the correlation coefficient between the two variable and also their means |  |  |  |  |  |  |  | Apply | 7 |
| 9 | From 10 observations on price X and supply Y the following data was obtained <br> Find coefficient of correlation, line of regression of Yon $X$ and $X$ on $Y$ |  |  |  |  |  |  |  | Apply | 7 |
| 10 | If the variance of X is 9 . The two regression equations are $8 \mathrm{X}-10 \mathrm{Y}+66=0$ and 40X-18Y-214=0. Find correlation coefficient between X and Y and standard deviation of Y |  |  |  |  |  |  |  | Apply | 7 |

Part - C (Problem Solving and Critical Thinking Questions)

| 1 | Derive the angle between the two regression lines | Evaluate | 7 |
| :---: | :--- | :---: | :---: |
| 2 | If $\theta$ is the angle between two regression lines then show that $\sin \theta \leq 1-\mathrm{r}^{2}$ | Apply | 7 |
| 3 | What is the marginal distributions of X and Y. | Analyze | 6 |
| 4 | Write the normal equations of straight line | Analyze | 7 |
| 5 | Find mean value of the variables X and Y and coefficient of correlation <br> from the following regression equations $2 Y-X-50=0,3 Y-2 X-10=0$ | Evaluate | 7 |
| 6 | Define regression and give its uses | Remember | 7 |
| 7 | What are normal equations for regression lines? | Analyze | 7 |
| 8 | When the Regression coefficient is independent | Analyze | 7 |


| 9 | Find correlation coefficient if bxy=085y, byx $=089 \mathrm{x} \sigma_{x}=3$ | Evaluate | 7 |
| :---: | :--- | :---: | :---: |
| 10 | When the coefficient of correlation is maximum | Analyze | 7 |

UNIT-III
SAMPLING DISTRIBUTIONS AND TESTING OF HYPOTHESIS

Part - A (Short Answer Questions)

| 1 | Explain different Types and Classification of sampling | Analyze | 8 |
| :---: | :--- | :---: | :---: |
| 2 | Write about Point Estimation, Interval Estimation | understand | 9 |
| 3 | What is the maximum error one can expect to make with <br> probability 0.9 when using mean of a random sample of size n=64 <br> to estimate the means of a population with $\sigma^{2}=256$ | understand | 9 |
| 4 | A random sample of 500 apples was taken from a large <br> consignment and 60 were found to be bad, find the standard error. | Evaluate | 9 |
| 5 | Three masses are measured as 62.34,20, 48, 35. 97 kgs with S.D <br> $0.54,0.21,0.46$ kgs. Find the mean and S.D of the sum of masses. | Evaluate | 9 |
| 6 | What is the value of correction factor if n=5 and N=200. | Apply | 9 |
| 7 | Find the value of finite population correction factor for n=10 and <br> N=100. | Evaluate | 9 |
| 8 | Write a short note on Hypothesis, Null and Alternative with suitable <br> examples | understand | 9 |
| 9 | Write a short Note on Type I \& Type II error in sampling theory | understand | 9 |
| 10 | Prove that Sample Variance is not an Unbiased Estimation of Population <br> Variance | understand | 9 |
| 11 | Write Properties of t-distribution | Analyze | 10 |
| 12 | Explain about Chi-Square | Analyze | 10 |
| 13 | Write a short note on Distinguish between t, F, Chi square test | understand | 10 |
| 14 | Explain about Bayesian estimation | Analyze | 9 |
| 15 | Compare Large Samples and Small sample tests | Create | 10 |

Part - B (Long Answer Questions)

| 1 | The mean of a random sample is an unbiased estimate of the mean of the <br> population 3,6, 9,15,27. (i) List of all possible samples of size 3 that can <br> be taken without replacement from the finite population. (ii) Calculate the <br> mean of the each of the samples listed in (iii) And assigning each sample <br> a probability of $1 / 10$. | Apply | 8 |
| :---: | :--- | :--- | :--- |
| 2 | An ambulance service claims that it takes on the average 8.9 minutes to <br> reach its destination In emergency calls. To check on this claim the <br> agency which issues license to Ambulance service has then timed on fifty <br> emergency calls getting a mean of 9.2 minutes with 1.6 minutes. What <br> can they conclude at 5\% level of significance? | Apply | 9 |
| 3 | A sample of 400 items is taken from a population whose standard deviation <br> is 10.The mean of sample is 40.Test whether the sample has come from a <br> population with mean 38 also calculate 95\% confidence interval for the <br> population | Apply | 9 |
| 4 | The means of two large samples of sizes 1000 and 2000 members are 67.5 <br> inches and 68.0 inches respectively. Can the samples be regarded as <br> drawn from the same population of S.D 2.5 inches | Apply | 9 |
| 5 | Experience had shown that $20 \%$ of a manufactured product is of the top <br> quality. In one day's production of 400 articles only 50 are of top quality <br> .Test the hypothesis at 0.05 levels. | Analyze \& Evaluate | 9 |


| 6 | A sample of 26 bulbs gives a mean life of 990 hrs. With S.D. of 20 hours. The manufacture claims that the mean life bulb is 1000 hrs . is the sample not up to the standard |  |  |  |  |  |  |  | Apply | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | In a one sample of 10 observations the sum of squares of deviations from mean was 90 and other sample of 12 observations it was 108 .test whether the difference is significant at $5 \%$ level of significance. |  |  |  |  |  |  |  | Apply | 10 |
| 8 | The no. of automobile accidents per week in a certain area as follows: $12,8,20,2,14,10,15,6,9,4$ are these frequencies in agreement with the belief that accidents were same in the during last 10 weeks. |  |  |  |  |  |  |  | Apply | 10 |
| 9 | Two independent samples of 7 items respectively had the following values |  |  |  |  |  |  |  | evaluate | 10 |
|  | Sample I 11 11 <br> Sample <br> II 9 11 | 13 | $\begin{array}{l\|l}  & 11 \\ \hline & 13 \end{array}$ | $\begin{aligned} & \hline 12 \\ & \hline 9 \end{aligned}$ | $\begin{aligned} & \hline 9 \\ & \hline 8 \end{aligned}$ | $\begin{aligned} & \frac{12}{10} \\ & \hline \end{aligned}$ | - 14 |  |  |  |
| 10 | A die is thrown 264 times with the following results .show that the die is unbiased |  |  |  |  |  |  |  | Understand | 10 |
|  | $\begin{aligned} & \text { No appeared } \\ & \text { on die } \end{aligned}$ | 1 | 2 | 3 | 4 |  | 5 | $6$ |  |  |
|  | Frequency | 40 | 32 | 28 | 58 |  | 54 | 52 |  |  |

## Part - C (Problem Solving and Critical Thinking Questions)

| 1 | Which error is called producer's risk? | Understand | 9 |
| :---: | :--- | :---: | :---: |
| 2 | Which error is called consumer's risk. | Understand | 9 |
| 3 | When the single tailed test is used. | Analyze | 9 |
| 4 | What is test statistics for testing single mean? | Analyze | 9 |
| 5 | How to calculate limit for true mean. | Analyze | 9 |
| 6 | If $\mathrm{p}=0.15 \mathrm{q}=0.85 \mathrm{n}=10$ find confidence limits | Evaluate | 9 |
| 7 | What must be sample size to apply t test. | Evaluze | 8 |
| 8 | If $\bar{x}=47.5, \mu=42.1, s=8.4, n=24$ find t . What is shape of t | 10 |  |
| 9 | What is the range of F distribution? | Understand | 10 |
| 10 | Which distribution is used to test the equality of population means? | Analyze | 10 |

## UNIT-IV <br> QUEUING THEORY

Part - A (Short Answer Questions)

| 1 | Explain queue discipline | Analyze | 11 |
| :---: | :--- | :---: | :---: |
| 2 | Define Balking. | Remember | 11 |
| 3 | Calculate traffic intensity if inter arrival time is 125 minutes and inter <br> service time is 10 minutes. | Evaluate | 11 |
| 4 | If average number of arrivals is 4 per hour and average number of <br> services is 6 per hour. What is the probability that a new arrival need not <br> wait for the service. | Understand | 11 |
| 5 | If $\lambda=8$ and $\mu=12$ per hour. Calculate the average time spent by a <br> customer in the system | Apply | 11 |
| 6 | What is the probability that there are more than or equal to 10 customers <br> in the system. | Understand | 11 |


| 7 | Explain pure birth process | Analyze | 11 |
| :---: | :--- | :---: | :---: |
| 8 | Explain pure death process | Analyze | 11 |
| 9 | Derive expected number of customers | Evaluate | 11 |
| 10 | Derive average waiting time in queue | Evaluate | 12 |
| 11 | If $\lambda=6$ and $\mu=18$ per hour. Calculate the service time. | Evaluate | 12 |
| 12 | Define transient state and steady sate | Remember | 12 |
| 13 | Explain M/M/1 model | Analyze | 12 |
| 14 | Explain M/M/1 with infinite population | Analyze | 12 |
| 15 | Derive probability of having n customers P(n ) in a queue M/M/1, having <br> Poisson arrival | Evaluate | 12 |

Part - B (Long Answer Questions)

|  | Consider a box office ticket window being managed by a single server. <br> Customer arrive to purchase ticket according to Poisson input process <br> with a mean rate of 30 per hour. The time required to serve a customer <br> has an exponential distribution with a mean of 910 sec. Determine the <br> following: a)Fraction of the time the server is busy b)The average <br> number of customers queuing for service | Apply | 11 |
| :---: | :--- | :--- | :--- |
|  | Patients arrive at a clinic in a Poisson manner at an average rate of 6 per <br> hour. The doctor on average can attend to 8 patients per hour. Assuming <br> that the service time distribution is exponential, find Average number of <br> patients waiting in the queue, Average time spent by a patient in the <br> clinic | Evaluate | Analyze |


|  | exponential with mean 5 minutes The car space in front of the window including that for the serviced can accommodate a maximum of 3 cars. Other cars can wait outside the space. i) What is the probability that an arriving customer can drive directly to the space in front of the window? Ii) What is the probability that an arriving customer will have to wait outside the indicated space? Iii) How long is an arriving customer expected to wait before starting service |  |  |
| :---: | :---: | :---: | :---: |
| 8 | A fast food restaurant has one drive window. Cars arrive according to a Poisson process. Cars arrive at the rate of 2 per 5 minutes. The service time per customer is 1.5 minutes. Determine i) The Expected number of customers waiting to be served. ii) The probability that the waiting line exceeds 10iii) Average waiting time until a customer reaches the window to place an order. iv) The probability that the facility is idle | Apply | 12 |
| 9 | At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the various number of trains in the system. Find also the average waiting time of a new train coming into the yard | Apply | 12 |
| 10 | Consider a single server queuing system with Poisson input and exponential service time. Suppose the mean rate is 3 calling units per hour with the expected service time as 0.25 hours and the maximum permissible number of calling units in the system is two. Obtain the steady state probability distribution of the number of calling units in the system and then calculate the expected number in the system | Apply | 12 |

Part - C (Problem Solving and Critical Thinking Questions)

| 1 | What is probability of arrivals during the service time of any given <br> customer? | Analyze | 11 |
| :---: | :--- | :--- | :---: |
| 2 | What is FIFO means? | Remember | 11 |
| 3 | Define Jack eying. | Understand | 11 |
| 4 | Define reneging. | Understand | 11 |
| 5 | Define m/m/1:FIFO | Understand | 11 |
| 6 | Model of queuing system. | Understand | 10 |
| 7 | Define balking. | Analyze | 11 |
| 8 | What is the pattern according to which customers are served? | Analyze | 11 |
| 9 | What is variance of queue length? | Evaluate | 10 |
| 10 | How to calculate the idle time of the server according to queue theory | 11 |  |

UNIT-V STOCHASTIC PROCESSES

Part - A (Short Answer Questions)

| 1 | Define stochastic process | Remember | 13 |
| :---: | :---: | :---: | :---: |
| 2 | Define a regular Markov chain | Remember | 13 |
| 3 | Find whether the matrix $\left[\begin{array}{ccc}0.75 & 0.25 & 0 \\ 0 & 0.5 & 0.5 \\ 0.6 & 0.4 & 0\end{array}\right]$ is a regular transition matrix or not. | Evaluate | 13 |
| 4 | Find periodic and aperiodic states in each of following transition probability matrices. <br> (i) $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$ (ii) $\left[\begin{array}{ll}\frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2}\end{array}\right]$ | Evaluate | 13 |
| 5 | Define reducible and non-reducible states. | Remember | 13 |
| 6 | Consider the Markov chain with transition probability matrix $\left[\begin{array}{cccc}0 & 0 & 1 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1\end{array}\right]$ is | Analyze | 13 |
| 7 | Explain different types of stochastic process | Analyze | 13 |
| 8 | Give examples of stochastic process | Create | 13 |
| 9 | Find the expected duration of the game for double stakes | Evaluate | 13 |
| 10 | Define Markov's chain | Understand | 13 |
| 11 | Explain Markov's property | Understand | 13 |
| 12 | Explain transition probabilities | Understand | 13 |
| 13 | Explain stationary distribution | Understand | 13 |
| 14 | Explain limiting distribution | Understand | 13 |
| 15 | Explain irreducible and reducible | Understand | 13 |

## Part - B (Long Answer Questions)

| 1 | The transition probability matrix is given by $P=\left[\begin{array}{lll}0.1 & 0.4 & 0.5 \\ 0.2 & 0.2 & 0.6 \\ 0.7 & 0.2 & 0.1\end{array}\right]$ and $P_{0}=\left[\begin{array}{lll}0.4 & 0.4 & 0.2\end{array}\right]$ (a) Find the distribution after three transitions. (b) Find the limiting probabilities. | Evaluate | 13 |
| :---: | :---: | :---: | :---: |
| 2 | If the transition probability matrix of market shares of three brands $\mathrm{A}, \mathrm{B}$, and C is $\left[\begin{array}{ccc}0.4 & 0.3 & 0.3 \\ 0.8 & 0.1 & 0.1 \\ 0.35 & 0.25 & 0.4\end{array}\right]$ and the initial market shares are $50 \%, 25 \%$ and $25 \%$, Find (a) The market shares in second and third periods (b) The limiting probabilities. | Evaluate | 13 |
| 3 | Define the stochastic matrixes which of the following stochastic matrices are regular. (a) $\left[\begin{array}{ccc}1 / 2 & 1 / 4 & 1 / 4 \\ 0 & 1 & 0 \\ 1 / 2 & 0 & 1 / 2\end{array}\right]$ (b) $\left[\begin{array}{ccc}2 & 1 / 2 & 0 \\ 1 / 2 & 1 / 2 & 0 \\ 1 / 4 & 1 / 4 & 1 / 2\end{array}\right]$ | Remember \& Evaluate | 13 |
| 4 | Three boys A, B, C are throwing a ball to each other. A always throws the ball to B ; B always throws the ball to C ; but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. Do all the states are ergodic | Understand \& Apply | 13 |
| 5 | A gambler has Rs.2. He bets Rs. 1 at a time and wins Rs. 1 with probability 0.5 . He stops Playing if he loses Rs. 2 or wins Rs.4.i)What is the Transition probability matrix of the related markov chain? (b) What is the probability that he has lost his money at the end of 5 plays | Understand \& Apply | 14 |
| 6 | Check whether the following markov chain is regular and $\text { ergodic? }\left[\begin{array}{cccc} 1 & 1 / 2 & 1 / 2 & 0 \\ 1 / 2 & 0 & 0 & 1 / 2 \\ \mathbf{1} / \mathbf{2} & 0 & 0 & 1 / 2 \\ 0 & 1 / 2 & 1 / 2 & 1 / 2 \end{array}\right]$ | Apply | 13 |
| 7 | The transition probability matrix of a marker chain is given by $\left[\begin{array}{ccc}0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8\end{array}\right]$ irreducibleor not? | Evaluate | 13 |
| 8 | Which of the following matrices are Stochastic <br> i) $\left[\begin{array}{cc}1 / 2 & 0 \\ 0 & 1\end{array}\right]$ <br> ii) $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$ <br> iii) $\left[\begin{array}{ccc}1 / 2 & 1 / 4 & 1 / 4 \\ 1 & 1 & 0 \\ 1 / 2 & 1 / 2 & 0\end{array}\right]$ | Apply | 13 |
| 9 | Which of the following Matrices are Regular $\quad$ i) $\left[\begin{array}{cc}\mathbf{1} / \mathbf{2} & \mathbf{1} / \mathbf{2} \\ \mathbf{0} & \mathbf{1}\end{array}\right]$ <br> ii) $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right] \quad$ iii) $\left[\begin{array}{ccc}1 / 2 & 1 / 4 & 1 / 4 \\ 0 & 1 & 0 \\ 1 / 2 & 1 / 2 & 0\end{array}\right]$ | Apply | 13 |
| 10 | a) Is the Matrix $\left[\begin{array}{cccc}\mathbf{0 . 4} & \mathbf{0 . 6} & \mathbf{0} & \mathbf{0} \\ \mathbf{0 . 3} & \mathbf{0 . 7} & \mathbf{0} & \mathbf{0} \\ \mathbf{0 . 2} & \mathbf{0 . 4} & \mathbf{0 . 1} & \mathbf{0 . 3} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1}\end{array}\right]$ irreducible? <br> (b) Is the Matrix $\mathrm{p}=\left[\begin{array}{ccc}\mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} / \mathbf{2} & \mathbf{1 / 6} & \mathbf{1 / 3} \\ \mathbf{1 / 3} & \mathbf{2 / 3} & \mathbf{0}\end{array}\right]$ Stochastic? | Evaluate | 13 |

## Part - C (Problem Solving and Critical Thinking Questions)

| 1 | What do you call the random variable in stochastic process? | Analyze | 13 |
| :---: | :--- | :--- | :---: |
| 2 | When the state is said to be Ergodic. | Analyze | 13 |
| 3 | What is null persistent state? | Understand | 13 |
| 4 | What is Markov process? | Understand | 13 |
| 5 | Give an example of discrete parameter Markov chain. | Understand | 13 |
| 6 | When a matrix is said to be regular. | Understand | 13 |
| 7 | What is the use of Markov process? | Understand | 13 |
| 8 | When the state is said to be commute with each other. |  |  |
| 9 | Let $p=\frac{1}{2}, q=\frac{1}{2}, z=500, a=1000$ then find the expected duration of <br> the game | Apply | 14 |
| 10 | If the stakes are doubled while the initial capital remain unchanged the <br> probability ruin decreases for the player whose probability of success is <br> P<1/2 and increases for the adversary | 14 |  |

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