INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

TUTORIAL QUESTION BANK

Course Name	:	**PROBABILITY THEORY AND STOCHASTIC PROCESSES**
Course Code	:	**AEC003**
Class	:	**B. Tech III Semester**
Branch	:	**ECE**
Academic Year	:	**2018 – 2019**
Course Coordinator	:	**Mrs. G Ajitha, Assistant Professor, Department of ECE**
Course Faculty	:	**Dr. M V Krishna Rao, Professor, Department of ECE** **Mr. N Nagaraju, Assistant Professor, Department of ECE** **Mr. G Anil kumar reddy, Assistant Professor, Department of ECE**

I. COURSE OBJECTIVES:

The course should enable the students to:

S.No	Description
I	Know the theoretical formulation of probability random variables and stochastic processes
II	Be familiar with the basic concepts of the theory of random variables in continuous and discrete time domains and analyze various analytical properties such as statistical averages.
III	Understand the concept of stationary in random processes and study various properties such as auto -correlation, cross -correlation and apply them for signal analysis.
IV	Relate time domain and frequency domain representations of random processes and model different scenarios of random environment in signal processing and applications.

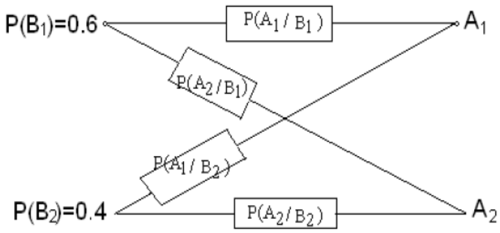
II. COURSE LEARNING OUTCOMES:

Students who complete the course will have demonstrated the ability to do the following

CAEC003.01	Understand probabilities and be able to solve using an appropriate sample space
CAEC003.02	Remember different random variables and their properties
CAEC003.03	Discuss various operations like expectations from probability density functions (pdfs) and probability distribution functions
CAEC003.04	Remember Transformations of random variables
CAEC003.05	Perform Likelihood ratio tests from pdfs for statistical engineering problems.
CAEC003.06	Understand Operations on multiple random variables like moments
CAEC003.07	Calculate Mean and covariance functions for simple random variables.
CAEC003.08	Understand the Ergodic processes
CAEC003.09	Understand Auto-correlation and cross correlation properties between two random variables.
CAEC003.10	Explain the concept of random process, differentiate between stochastic, stationary and ergodic processes.
CAEC003.11	Explain the concept of power spectral density and power density spectrum of a random process.
CAEC003.12	Apply the power density spectrum of a random process in system concepts.
CAEC003.13	Remember the Autocorrelation to stochastic process
CAEC003.14	Understand the Cross correlation to stochastic process
CAEC003.15	Apply the Gaussian Noise to stochastic process
CAEC003.16	Apply the concept of probability theory and random process to understand and analyze real time applications
CAEC003.17	Acquire the knowledge and develop capability to succeed national and international level competitive examinations.

UNIT-1 PROBABILITY AND RANDOM VARIABLE			
PART – A (SHORT ANSWER QUESTIONS)			
S. No	**Question**	**Blooms Taxonomy Level**	**Course Learning Outcomes**
1	Define probability.	Remember	CAEC003.01
2	Discuss probability with axioms.	Understand	CAEC003.01
3	Write short note on conditional probability.	Remember	CAEC003.01
4	Define joint probability.	Remember	CAEC003.01
5	Discuss total probability theorem.	Understand	CAEC003.01
6	State bayes theorem.	Remember	CAEC003.01
7	Discuss how probability can be considered as relative frequency	Understand	CAEC003.01
8	Describe random variable concept.	Understand	CAEC003.01
9	Identify a sample space for an event A.	Understand	CAEC003.01
10	State multiplication theorem .	Understand	CAEC003.01
11	Define Demorgans' law.	Remember	CAEC003.01
12	Define sample space and classify the types of sample space.	Remember	CAEC003.01
13	Discuss the conditions for a function to be a random variable.	Understand	CAEC003.01
14	Calculate What is the probability of face having 3 dots or 6 dots to appear In the experiment of tossing a dice.	Understand	CAEC003.01
15	Describe the classifications of Random variable.	Understand	CAEC003.02
PART – B (LONG ANSWER QUESTIONS)			
1	Define probability .State and prove total probability theorem and Bayes' theorem.	Remember	CAEC003.01
2	A computer manufacturer uses chips from three sources. Chips from sources A, B, and C are defective with probabilities 0.001, 0.005 and 0.01 respectively. If a randomly selected chip is found to be defective, find the probability that the manufacturer was A; that manufacturer was C.	Remember	CAEC003.01
3	A man wins in a gambling game if he gets two heads in five flips of a biased coin. The probability of getting a head with the coin is 0.7. i) Find the probability the man will win. Should he play this game. ii) What is the probability of winning if he wins by getting at least four heads in five flips. Should he play this new game.	Understand	CAEC003.01
4	In the experiment of throwing two fair dice, let A be the event that the first die is odd, B be the event that the second die is odd, and C is the event that the sum is odd. Prove that events A, B and C are pair wise independent, but A, B and C are not independent.	Understand	CAEC003.01
5	Define the following with example (i) Relative frequency definition of probability (ii)Conditional probability and (iii)Total probability.	Understand	CAEC003.01
6	A certain large city averages three murders per week and their occurrences follows a Poisson distribution 1. What is the probability that there will be five or more murders in a given week. 2. On the average, how many weeks a year can this city expect to have no murders. 3. How many weeks per year (average) can the city expect the number of murders per week to equal or exceed the average number per week.	Remember	CAEC003.01

7	Sketch a sample space showing possible outcomes for this experiment and illustrate how the points map onto the real line x that defines the values of the random variable X=”dollars won on a trial”. Show a second mapping for a random variable Y=”dollars won by the friend on a trial”. For A man matches coin flips with a friend. He wins 2 Rs if coins match and loses 2 Rs if they do not match.	Understand	CAEC003.01
8	Define following types of events. 1) Simple events 2) Compound events 3) Independent events 4)Joint events 5) Conditional events with examples	Remember	CAEC003.01
9	In a box there are 500 color red balls 75 black 150 green 175 red 70 white and 30 blue what are the probabilities of selecting a ball of each color.	Understand	CAEC003.01
10	Two cards are drawn from a 52 Cards i. Given the first card is a queen, what is the probability that the second is also a queen. ii. Repeat part a) for the first card a queen and the second card a 7 iii. What is the probability that both cards will be a queen.	Understand	CAEC003.01
11	An experiment consists of rolling a single die. Two events are defined as A = { a 6 shows up}: and B= {a 2 or a 5 shows up} i. Find P(A) and P(B) ii. Define a third event C so that P(C) = 1-P(A)-P(B) iii. The six sides of a fair die are numbered from 1 to 6. The die is rolled 4 times. How many sequences of the four resulting numbers are possible.	Remember	CAEC003.01
12	In a bolt factory there are four machines A, B, C, D manufacturing 20%, 15%, 25%, 40% of the total production of these 5%, 4%, 3%, 2% are found to be defective. If a bolt is drawn at random and was found to be defective what is the probability that it was manufactured by A or D.	Remember	CAEC003.01
PART – C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)			
1	Find the probability that at least one diode is defective. If a box contains 75 good diodes and 25 defective diodes and 12 diodes are selected at random.	Remember	CAEC003.01
2	Given that two events $A_1$and A_2 are statistically independent show that $A_1$is independent of A_2^C show that A_1^C is independent of A_2 show that A_1^C is independent of A_2^C	Remember	CAEC003.01
3	A pack contains 4 white and 2 green pencils another contains 3 white and 5 green pencils if 1 pencil is drawn from each pack find the probability that 1) Both are white. 2) One is white and another is green	Remember	CAEC003.01
4	Find the Probability that the transistor came from company X. If We are given a box containing 5000 transistors, 1000 of which are manufactured by company X and the rest by company Y. 10% of the transistors made by company X are defective and 5% of the transistors made by company Y are defective. If a randomly chosen transistor is found to be defective, probability that it came from company X.	Understand	CAEC003.01
5	What is the probability that exactly 5 of the items tested are defective if A batch of 50 items contains 10 defective items. Suppose 10 items are selected at random and tested.	Understand	CAEC003.01

<table>
<tr><td>6</td><td>Calculate the probabilities of system error and correct system transmission of symbols for an elementary binary communication system shown in below figure consisting of a transmitter that sends one of two possible symbols (a 1 or a 0) over a channel to a receiver. The channel occasionally causes errors to occur so that a “1“ show up at the receiver as a “0. And vice versa. Assume the symbols „1“ and „0“ are selected for a transmission as 0.6 and 0.4 respectively.

</td><td>Understand</td><td>CAEC003.01</td></tr>
<tr><td>7</td><td>Find the individual, joint and conditional probabilities. For a given problem as shown below. In a box there are 100 resistors having resistance and tolerance values given in table. Let a resistor be selected from the box and assume that each resistor has the same likelihood of being chosen. Event A: Draw a 47Ω resistor, Event B: Draw a resistor with 5% tolerance, Event C: Draw a 100Ω resistor.
<table>
<tr><td rowspan="2">Resistance (Ω)</td><td colspan="2">Tolerance</td><td rowspan="2">Total</td></tr>
<tr><td>5%</td><td>10%</td></tr>
<tr><td>22</td><td>10</td><td>14</td><td>24</td></tr>
<tr><td>47</td><td>28</td><td>16</td><td>44</td></tr>
<tr><td>100</td><td>24</td><td>8</td><td>32</td></tr>
<tr><td>Total</td><td>62</td><td>38</td><td>100</td></tr>
</table></td><td>Understand</td><td>CAEC003.01</td></tr>
<tr><td>8</td><td>Suppose a coin is tossed thrice let the event A be “getting 3 heads” And B be the event of “getting a head on the first toss” show that A and B are not independent events</td><td>Remember</td><td>CAEC003.01</td></tr>
<tr><td>9</td><td>Two boxes are selected randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball</td><td>Remember</td><td>CAEC003.01</td></tr>
<tr><td>10</td><td>A shipment of components consists of 3 identical boxes. one box contains 2000 components of which 25%are defective , the second box has 5000 components of which 20%are defective third box contains 2000 components of which 55% are defective. A box is selected at random and a component is removed at random from the box what is the probability that this component is defective, what is the probability by that it came from the second box.</td><td>Remember</td><td>CAEC003.01</td></tr>
<tr><td>11</td><td>An ordinary 52 card deck is thoroughly shuffled. When four cards are drawn then What is the probability that all four cards are seven?</td><td>Remember</td><td>CAEC003.01</td></tr>
<tr><td>12</td><td>If A,B and C are events such that P(A) = 1/3 , P(B) = 1/4 and P(A ∪ B) = 1/ 2 , find
i. P (B/A) ii. P (B/A^C)</td><td>Remember</td><td>CAEC003.01</td></tr>
<tr><td colspan="4">UNIT-II
DISTRIBUTION AND DENSITY FUNCTIONS</td></tr>
<tr><td colspan="4">PART – A (SHORT ANSWER QUESTIONS)</td></tr>
<tr><td>1</td><td>Define probability density function.</td><td>Remember</td><td>CAEC003.02</td></tr>
<tr><td>2</td><td>Discuss probability distribution function.</td><td>Understand</td><td>CAEC003.02</td></tr>
<tr><td>3</td><td>List any two properties of density function.</td><td>Remember</td><td>CAEC003.02</td></tr>
<tr><td>4</td><td>List any two properties of distribution function.</td><td>Remember</td><td>CAEC003.02</td></tr>
</table>

5	Define uniform density function.	Remember	CAEC003.02
6	Discuss uniform distribution function.	Understand	CAEC003.02
7	Plot Gaussian density function.	Understand	CAEC003.02
8	Define Gaussian distribution function.	Remember	CAEC003.02
9	State Poisson distribution function.	Remember	CAEC003.02
10	Define mean and mean square values.	Understand	CAEC003.02
11	Calculate the expected value of the sum of number of points on Two dice. In an experiment when two dice are thrown simultaneously.	Understand	CAEC003.02
12	Find the expression for distribution function of uniform Random variable.	Remember	CAEC003.02
13	Estimate the maximum value of Gaussian density function.	Understand	CAEC003.02
PART – B (LONG ANSWER QUESTIONS)			
1	Derive expressions for mean and variance for uniform random variable.	Understand	CAEC003.02
2	Define conditional distribution and density function with properties.	Remember	CAEC003.02
3	State density function with four properties. Calculate E[X] when X is exponential random variable	Understand	CAEC003.03
4	State and prove properties of distribution function. Discuss the method of defining a conditioning event.	Remember	CAEC003.03
5	A random variable X can have values -4, -1, 2, 3 and 4 each with a probability 1/5. Find mean and variance of the random variable $y=3x^3$	Understand	CAEC003.02
6	Derive expressions for mean and variance for Poisson random variable.	Remember	CAEC003.02
7	State expressions for mean and variance for binomial random variable.	Understand	CAEC003.02
8	Derive expressions for mean and variance for exponential random variable.	Remember	CAEC003.02
9	State and prove three properties of moment generating function and calculate moment generating function of exponentially distributed random variable.	Understand	CAEC003.02
10	Find Moment Generating Function (MGF) of the random variable with probability law $(X = a) = q^{x-1}\beta$, X=1,2,……. Also find mean and variance.	Remember	CAEC003.02
11	A random variable is known to have a distribution function $F_X(x) = u(x)[1 - e^{-\frac{x^2}{b}}]$ where b > 0 is a constant. Find its density function.	Remember	CAEC003.02
12	If X has the probability density function $f_X(x) = 1/2\, e^{-\lvert x\rvert}$, $-\infty < x < \infty$, show that the characteristic function of X is given by $\phi_x(\omega) = 1/1+\omega^2$. Hence find the mean and variance of X	Understand	CAEC003.02
PART-C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)			
1	Find the value of C, P(1/2<x<3/4) for a random variable X has a probability density function fx(x)= Cx(1-x) ; 0<x<1 = 0 ; elsewhere.	Understand	CAEC003.02
2	The characteristic function for a Gaussian random variable X, having a mean value of 0, is $\Phi_X(\omega) = EXP(-w^2/\sigma^2)$ Find all the moments of X using $\Phi_X(\omega)$.	Understand	CAEC003.02
3	Find the density function of Y=(12-x).for The probability density function of a random variable X is given by $fx(x) = X^2/81$ for -3<x<6 and equal to zero otherwise.	Understand	CAEC003.02

4	Find the value of K and also find $P\{2 \le X \le 5\}$ Let X be a Continuous random variable with density function fx(x)=x/9+k ; 0<x<6 =0 ; otherwise	Understand	CAEC003.03
5	Evaluate the value of C, P(0.3<x<0.6) for A random variable X has a probability density function $f_x(x)=Cx(1-x)$; 0<x<1 =0 ; elsewhere.	Understand	CAEC003.02
6	a) Write short notes on Gaussian distribution and density function. b) Consider that a fair coin is tossed 3 times, Let X be a random variable, defined as X= number of tails appeared, find the expected value of X.	Remember	CAEC003.02
7	Calculate the Moment generating function of Y If X is a discrete random variable with a Moment generating function of $M_x(v)$, i) Y=aX+b ii) Y=KX	Understand	CAEC003.03
8	A random variable has a probability density function $f_X(x)= (5/4)(1-x^4)$; 0<x<1 = 0 ; other wise Find a) E[X] b)E[4X+2] and $E[X^2]$	Understand	CAEC003.03
9	The exponential density function given by $f_X(x)=(1/b)e^{-(x-a)/b}$ for x>a = 0;otherwise Find variance and co-efficient of skewness.	Understand	CAEC003.03
10	Calculate the density function of the random variable Y=2X+3 Where X is a uniform random variable over (-1, 2).	Remember	CAEC003.03
11	A random variable X has the density function $e^{(-x)}$, x>0 Show that Chebyshev's inequality gives P[|X-1|>2] <1/4 and show that actual probability is e^{-3}.	Understand	CAEC003.03
12	If X is a random variable uniformly distributed in (0, 1), Find the PDF of Y=SinX. Also find the mean and variance of Y.	Remember	CAEC003.03

UNIT III
MULTIPLE RANDOM VARIABLES AND OPERATIONS

PART – A (SHORT ANSWER QUESTIONS)

1	Define joint probability density function for two random variables.	Remember	CAEC003.06
2	Define joint probability distribution function for two random variables.	Remember	CAEC003.06
3	Write properties of joint probability density function.	Remember	CAEC003.06
4	Write properties of joint probability distribution function.	Understand	CAEC003.06
5	Explain marginal density function.	Understand	CAEC003.06
6	Define marginal distribution function.	Remember	CAEC003.06
7	Define central limit theorem.	Remember	CAEC003.06
8	Discuss conditional joint distribution function.	Understand	CAEC003.06
9	Define conditional joint density function.	Understand	CAEC003.09
10	Explain distribution and density function of a sum of two random variables.	Understand	CAEC003.07
11	Write joint probability density function of Gaussian random variable.	Understand	CAEC003.09
12	Differentiate point conditioning and interval conditioning	Understand	CAEC003.07

CIE II

1	Define Expected Value of a Function of Random Variables.	Remember	CAEC003.06
2	Define Joint Moments about the Origin.	Remember	CAEC003.06
3	Define the joint Gaussian density function of two random variables.	Remember	CAEC003.06
4	Explain the concept of Linear Transformations of Gaussian Random Variables.	Understand	CAEC003.07
5	Show that var(X+Y)=var(x)+var(Y), if X &Y are independent random variables.	Understand	CAEC003.07

6	Define expectation for a function g(x,y) where X ,Y are two random variables.	Remember	CAEC003.06
7	Define Joint Central Moments.	Remember	CAEC003.07
8	Explain Joint Characteristic Functions.	Remember	CAEC003.07
9	Define Joint density function for a Gaussian Random Variables X and Y.	Remember	CAEC003.07
10	Discuss the concept of Transformations of Multiple Random Variables.	Understand	CAEC003.07
11	Write Characteristic Functions of uniform random variable.	Understand	CAEC003.07
12	Write Moment Generating Function of uniform random variable.	Understand	CAEC003.07
PART – B (LONG ANSWER QUESTIONS)			
1	Define and explain joint probability density function for two random variables.	Remember	CAEC003.06
2	State and explain joint probability distribution function for two random variables.	Remember	CAEC003.06
3	Show that the density function of the sum of two statistically independent random variables is the convolution of their individual density functions	Understand	CAEC003.06
4	Given the function $g(x, y) = be^{-x}\cos y$; $0 \le x \le 2, 0 \le y \le \pi/2$, = 0 ; otherwise Find the value of the constant so that is a valid probability density function.	Understand	CAEC003.06
5	Derive the expression for distribution and density function of a sum of two random variables.	Understand	CAEC003.06
6	A joint probability density function is $f_{XY}(x, y) = 1/ab$; $0 < x < a, 0 < y < b$, = 0 ; otherwise Find the value of the constant so that is a valid probability density function	Understand	CAEC003.06
CIE II			
7	Discuss the joint Gaussian density function of two random variables.	Understand	CAEC003.06
8	Show that the variance of a weighted sum of uncorrelated random variables equals the weighted sum of the variances of the random variables.	Understand	CAEC003.06
9	Explain joint moments about the origin and joint central moments.	Understand	CAEC003.07
10	State and prove the properties of joint characteristics function and joint covariance function.	Remember	CAEC003.09
11	Two Gaussian random variables X_1 and X_2 are defined by the mean $[\overline{X}] = \begin{bmatrix} 2 \\ 1 \end{bmatrix} [C_X] = \begin{bmatrix} 5 & \frac{-2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & 4 \end{bmatrix}$ and covariance matrices Two new random variables Y_1 and Y_2 are formed using the transformation $[T] = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ Find $[C_Y]$, correlation coefficient (ρ)	Remember	CAEC003.09
12	Statistically independent random variables X and Y have moments $m_{10}=2$, $m_{20}=14$ and $m_{11}=-6$. Find second central moment μ_{22}.	Remember	CAEC003.09
PART – C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)			
1	Calculate the marginal PDFs $f_X(x)$ and $f_Y(y)$ The joint Pdf of X and Y is $f_{XY}(x, y) = 5y/4$ $-1 \le x \le 1, -2 \le y \le 1$, =0 otherwise.	Understand	CAEC003.06

2	Find f_X (x) and f_Y (y), the marginal PDFs of X and Y .The joint probability density function of random variables X and Y is f_{XY} (x, y) = $6(x + y^2)/5$ $0 \leq x \leq 1, 0 \leq y \leq 1$, = 0, otherwise.	Remember	CAEC003.06
3	Find the PDF of W = X+ Y. X and Y have joint Pdf f_{XY} (x, y) = 1/15 $0 \leq x \leq 3, 0 \leq y \leq 5$, =0 otherwise.	Understand	CAEC003.06
4	Find the value of constant c .for The joint probability density function of X&Y is $f_{X,Y}(x,y)= c(2x+y)$; $0 \leq x \leq 2, 0 \leq y \leq 3$ 0; else	Remember	CAEC003.06
5	The joint density function of two random variables X and Y is $f_{xy}(x,y)=(x+y)^2$;-1<x<1 and -3<y<3 =0;otherwise Find $F_x(x)$;$F_Y(y)$	Understand	CAEC003.06
6	A joint probability density function is $f_{XY}(x, y)$ = 1/ab ; 0 <x <a, 0 < y <b, = 0 ; otherwise Find $F_{XY}(x,y)$ also if a<b find $P[X + Y \leq 3/4]$	Understand	CAEC003.06
CIE II			
7	Find f(y/x) and f(x/y) for The joint density function of random variables X and Y is Fxy(x,y)=8xy ;0<x<1,0<y<1 =0 ;otherwise	Understand	CAEC003.06
8	Prove that a) Are X and Y Uncorrelated. b) Are X and Y Independent. for the X and Y be the random variables defined as X=Cosθ and Y=Sinθ where θ is a uniform random variable over (0, 2π)	Understand	CAEC003.06
9	Calculate the following i) The variance of the sum of X and Y ii) The variance of the difference of X and Y for two random variables X and Y have zero mean and variance $\sigma_X^2 = 16$ and $\sigma_Y^2 = 36$ and correlation coefficient is 0.5.	Remember	CAEC003.06
10	Find variance & covariance of X&Y. If E[X]=2, E[Y]=3, E[XY]=10, $E[X^2]$=9, and $E[Y^2]$=16.	Understand	CAEC003.09
11	Two random variables X and Y have the joint characteristic function $Ø_{X,Y}(w_1,w_2)=\exp(-2w_1^2-8w_2^2)$. Show that X and Y are both zero mean and that they are uncorrelated.	Remember	CAEC003.09

UNIT IV
STOCHASTIC PROCESSES – TEMPORAL CHARACTERISTICS

PART – A (SHORT ANSWER QUESTIONS)

1	Define random process.	Remember	CAEC003.10
2	Define ergodicity.	Remember	CAEC003.08
3	Define mean ergodic process.	Remember	CAEC003.08
4	State correlation ergodic process.	Remember	CAEC003.08
5	Explain the first order stationary process.	Understand	CAEC003.10
6	Discuss second order stationary process.	Understand	CAEC003.10
7	State wide sense stationary random process.	Remember	CAEC003.10
8	Define strict sense stationary random process.	Remember	CAEC003.10
9	Define auto correlation function of a random process.	Remember	CAEC003.10
10	Define cross correlation function of a random process.	Understand	CAEC003.09
11	Find the time average of the random process X(t)=A ,where A is a random variable .	Remember	CAEC003.09

12	Discuss briefly about time average and Ergodicity.	Understand	CAEC003.08
PART – B (LONG ANSWER QUESTIONS)			
1	Give classification of random processes and Write conditions for a wide sense stationary random process.	Understand	CAEC003.10
2	Assume that an ergodic random process X(t) has an auto correlation function $R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2}[1 + 4\cos(12\tau)]$ i) Find $\lvert \bar{X} \rvert$ ii) Does the process have a periodic component.	Understand	CAEC003.10
3	Given E[X(t)]=6 and Rxx(t,t+ τ)= 36+25 exp(-τ) for a random process X(t). Indicate which of the following statements are true based on what is known with certainty: X(t) i. is first order stationary ii. has total average power of 61W iii. is ergodic iv. is wide sense stationary v. has a periodic component	Understand	CAEC003.10
4	State and prove any four properties of cross correlation function and cross covariance function.	Remember	CAEC003.09
5	Explain and prove any four properties of auto correlation function and auto covariance function.	Understand	CAEC003.09
6	A Gaussian random process has an auto correlation function $R_{XY}(\tau) = 6\exp\left[-\frac{\lvert\tau\rvert}{2}\right]$. Determine a covariance matrix for the random variables X(t), X(t+1) and X(t+2) and X(t+3).	Remember	CAEC003.09
7	If X(t) is a stationary random process having a mean value E[X(t)]=3 and auto correlation function $R_{XX}(\tau) = 9 + 2e^{-\lvert\tau\rvert}$ find i. The mean value and ii. The variance of the random variable $y = \int_0^2 x(t)\,dt$	Understand	CAEC003.09
8	Show that the process X(t)= A Cos (w_0t+θ) is wide sense stationery if it is assumed that A and w_0 are constants and θ is uniformly distributed random variable over the interval (0,2π).	Remember	CAEC003.09
9	(a) State and prove the properties of Cross correlation function. (b) Explain about Poisson Random process and also find its mean and variance.	Understand	CAEC003.09
10	A random process is defined by Y(t) = X(t) cos(ω_0t+θ) where X(t) is wide sense stationary random process that amplitude-modulates a carrier of constant angular frequency ω_0 with a random phase θ independent of X(t) and uniformly distributed on (−π, π). Determine E[Y(t)] and autocorrelation of Y (t).	Remember	CAEC003.09
11	Define a random process by X(t)=Acos(t), where A is a Gaussian Random Variable with zero mean variance σ_A^2 i. Find the density function of X(0) and X(1) ii. Is X(t) stationary in any sense	Remember	CAEC003.09
PART – C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)			
1	Find whether X(t) is wide sense stationary or not. If A random process is given as X(t) = At, where A is a uniformly distributed random variable on (0,2).	Remember	CAEC003.10

2	Let two random processes X(t) and Y(t) be defined by X(t) = A $Cos(w_0t)$ + B $Sin(w_0t)$ and Y(t) = B $Cos(w_0t)$ - A $Sin(w_0t)$. Where A and B are random variables and w_0 is constant. Show that X(t) and Y(t) are jointly wide sense stationery, assume A and B are uncorrelated zero- mean random variables with same variance.	Understand	CAEC003.10
3	Determine whether $X_1(t)$ and $X_2(t)$ are jointly wide sense stationary. X(t) is a wide sense stationary random process. For each process $X_i(t)$ defined below (a) $X_1(t) = X(t + a)$ (b) $X_2(t) = X(at)$	Remember	CAEC003.10
4	Find mean, variance and average power for a stationary ergodic random processes has the auto correlation function with the periodic components as $R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$	Understand	CAEC003.10
5	A random process is defined as X(t)=A $\cos(w_ct+ \theta)$ where θ is a uniform random variable over$(0,2\pi)$ .Verify the process is ergodic in the mean sense and auto correlation sense.	Remember	CAEC003.09
6	Given the random process X(t)=A $\sin(wt+ \theta)$ where A and w are constants and θ is a random variable uniformly distributed on the interval $(-\pi ,\pi)$ define a new random process Y(t)= $X^2(t)$. Find the auto correlation function of Y(t).	Understand	CAEC003.09
7	X(t) is a stationary random process with a mean of 3 and an auto correlation function of $9+2e^{-\lvert\tau\rvert}$. Find the variance of the random variable.	Remember	CAEC003.09
8	Find E[Z], E[Z^2] and var(z) if the function of time Z(t) = $X_1\cos\omega_0t$-$X_2\sin\omega_0t$ is a random process. If X_1 and $X_2$are independent Gaussian random variables, each with zero mean and variance σ^2.	Remember	CAEC003.10
9	Statistically independent zero mean random process X(t) and Y(t) have auto-correlation functions $Rxx(\tau)=e^{-\lvert\tau\rvert}$ and $R_{YY}(\tau)=\cos(2\pi\tau)$ Find the auto-correlation function of the sum w(t)=x(t)+y(t).	Remember	CAEC003.10
10	Statistically independent zero mean random process X(t) and Y(t) have auto-correlation functions $Rxx(\tau)=e^{-\lvert\tau\rvert}$ and $R_{YY}(\tau)=\cos(2\pi\tau)$ Find the auto-correlation function of the difference w(t)=x(t)-y(t).	Remember	CAEC003.10
11	Let N(t) be a Zero-mean wide-sense stationary noise process for which R_{NN} (t)=(N0/2)$\delta(\tau)$ where N0>0 is a finite constant. Determine N(t) is mean-ergodic.	Understand	CAEC003.10

UNIT V
STOCHASTIC PROCESSES – SPECTRAL CHARACTERISTICS

PART – A (SHORT ANSWER QUESTIONS

1	Define wiener khinchine relations	Remember	CAEC003.11
2	State any two properties of cross-power density spectrum.	Understand	CAEC003.11
3	Define cross –spectral density and its examples.	Remember	CAEC003.11
4	Explain any two uses of spectral density. .	Understand	CAEC003.11
5	Define power density spectrum.	Remember	CAEC003.11
6	State any two properties of power density spectrum	Remember	CAEC003.11
7	State any two properties of an auto correlation function.	Remember	CAEC003.11
8	Define cross correlation and its properties.	Understand	CAEC003.11
9	Prove that $R_{XY}(\tau) = R_{YX}(-\tau)$	Remember	CAEC003.11
10	Explain any two properties of cross correlation.	Understand	CAEC003.09
11	Write relation between autocorrelation and power spectral density	Understand	CAEC003.09
12	Write relation between output power spectral density and input power spectral density of a linear system.	Understand	CAEC003.09

PART – B (LONG ANSWER QUESTIONS)			
1	Discuss the concept of power density spectrum in detail and derive the expression for it.	Understand	CAEC003.12
2	Discuss the concept of cross power density spectrum in detail and derive the expression for it.	Understand	CAEC003.11
3	Explain the concept of cross power spectral density of input and output of a linear system.	Understand	CAEC003.11
4	Find the autocorrelation function corresponding to the power spectrum $S_{XX}(\omega) = \frac{8}{(9+\omega^2)^2}$	Understand	CAEC003.11
5	State and derive the properties of power density spectrum and prove wiener khinchien relations.	Remember	CAEC003.11
6	State and derive the properties of cross power density spectrum and define the power density spectrum of a system response.	Remember	CAEC003.13
7	Derive the relation between power spectrum and auto correlation function.	Understand	CAEC003.11
8	Derive the relation between cross power spectrum and cross correlation function.	Understand	CAEC003.11
9	Explain power spectrums for discrete-time random processes and sequences.	Remember	CAEC003.14
10	The auto correlation function of a random process X(t) is $R_{XX}(\tau) = 3 + e^{-4\tau^2}$ .find the power spectrum of X(t).	Understand	CAEC003.14
11	A random process has the power spectrum density $S_{XX}(\omega) = \frac{6\omega^2}{1+\omega^4}$. Find the average power of the process.	Understand	CAEC003.11
12	Find the power spectrum corresponding to the autocorrelation function $R_{XX}(\tau) = [\cos(\alpha\tau) + \sin(\alpha\|\tau\|)]e^{-\alpha\|\tau\|}$ $\alpha>0$ is a constant	Remember	CAEC003.14
PART – C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)			
1	Find an expression for its power spectral density Sxx(ω).Let the auto correlation function of a certain random process X (t) be given by R xx(τ) = (A ^{2}/2)cos(ωτ) .	Remember	CAEC003.13
2	Describe the power spectral density function For a wide sense stationary process X(t) has autocorrelation function R xx (τ) =Ae−b|τ | where b > 0. S_X(f) and calculate the average power E[X^2(t)].	Understand	CAEC003.11
3	Find the average power in a random process defined by X(t)=A cos(w_ot+Θ) where A and w_o are constants and Θ is a random variable uniformly distributed on the interval (0,π/2).	Understand	CAEC003.11
4	Find the autocorrelation function The power Spectral density of X(t) is given by Sxx(w)=1/(1+$w^{2)}$ for w>0.	Remember	CAEC003.12
5	The auto correlation function of an a periodic random process is R_{XX}(T)= $e^{-\|\tau\|}$. Find the PSD and average power of the signal.	Remember	CAEC003.12
6	The cross spectral density of two random process X (t) and Y (t) is S_{XY}(w)=1+(jw/k) for $–k<w<k$ and 0 elsewhere Where $k > 0$. Find the cross correlation function between the processes.	Remember	CAEC003.12
7	A random process has the power density spectrum S_{XX}(w)=w^2/(w^2+1). Find the average power in the random process.	Remember	CAEC003.12
8	Estimate the power spectral density of a stationary random process for which auto correlation function is R_{XX}(τ)=6.$e^{-\alpha\|\tau\|}$.	Understand	CAEC003.12

9	Find i) The average power of the process ii) The Auto correlation function Of a random process Y(t) has the power spectral density $S_{YY}(\omega)=9/(\omega^2+64)$	Remember	CAEC003.12
10	Find the cross correlation function of sin(wt) and cos(wt) and hence find its cross power spectral density..	Remember	CAEC003.12
11	Determine the given is a valid power density spectrum. $S_{XX}(\omega) = \frac{\omega^2}{\omega^6 + 3\omega^2 + 3}$	Remember	CAEC003.12
12	Find the cross correlation function corresponding to the Cross Power Spectrum $S_{XX}(\omega) = \frac{6}{(9+\omega^2)(3+j\omega)^2}$	Understand	CAEC003.12

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