INSTITUTE OF AERONAUTICAL ENGINEERING
$\square$ (Autonomous)
Dundigal, Hyderabad - 500043

## MODEL QUESTION PAPER

Four Year B.Tech III Semester End Examinations, November-2019
Regulations: $\mathbf{R 1 8}$
DISCRETE MATHEMATICAL STRUCTURES

## Answer ONE Question from each module <br> All Questions Carry Equal Marks <br> All parts of the question must be answered in one place only <br> MODULE - I

1. a) Show that $(\mathrm{p} \rightarrow(\mathrm{q} \rightarrow \mathrm{r})) \rightarrow((\mathrm{p} \rightarrow \mathrm{q}) \rightarrow(\mathrm{p} \rightarrow \mathrm{r}))$ is a Tautology using truth table.
b) Show that the statement "Every positive integer is the sum of squares of three integers" is false
a) Construct the truth table for the formula $(\mathrm{P} V \mathrm{Q}) \mathrm{V} \neg \mathrm{P}$
2. a) Construct the truth table for the formula ( $\mathrm{P}, ~ \mathrm{Q}$ ) $\mathrm{V} \neg \mathrm{P}$
b) Explain about the tautological implications and $\log$ eal equivalence using theorem.

## MODULE - II

3. a) Show that a relation $R$ defined on the set of real numbers as (a, b) $R(c, d)$ if $a^{2}+b^{2}=c^{2}+d^{2}$. Show that $R$ is an equivalence relation.
b) Let $\mathrm{X}=\{1,2,3,4\}$ and $\mathrm{R}=\{\langle\mathrm{x}, \mathrm{y}\rangle \mid \mathrm{x}>\mathrm{y}\}$. Draw the diagram of the graph R and also give its matrix.
4. a) Illustrate the following function definition with graph. Let X and Y be any two sets. A relation ffrom X to Y is called a function if for every $\mathrm{x} \in \mathrm{X}$ there is a unique $\mathrm{y} \in \mathrm{Y}$ such that $(\mathrm{x}, \mathrm{y}) \in \mathrm{f}$.
b) Let $X=\{1,2,3\}, Y=\{p, q\}$, and $Z=\{a, b\}$. Also let $f: X \rightarrow Y$ be $f=\{\langle 1, p\rangle,\langle 2, p\rangle,\langle 3, q\rangle\}$ and $\mathrm{g}: \mathrm{Y} \rightarrow \mathrm{Z}$ be given by $\mathrm{g}=\{\langle\mathrm{p}, \mathrm{b}\rangle,\langle\mathrm{q}, \mathrm{b}\rangle\}$. Find gof.

## MODULE - III

5. a) Show that the intersection of any two congruence relations on a set is also a congruence relation.
b) Let $\left(\mathrm{Z}_{4},+_{4}\right)$ and ( $\mathrm{B},+$ ) be the algebraic system. Show that ( $\mathrm{B},+$ ) is a homomorphic image of $\left(\mathrm{Z}_{4},+_{4}\right)$.
6. a) Prove using the theorem by showing that the composition of semi group homomorphism is also a semi group homomorphism.
b) Let $(\mathrm{N},+)$ be the algebraic system of natural numbers. Define an equivalence relation E on N such that $x_{1} E x_{2}$ iff either $x_{1}-x_{2}$ or $x_{2}-x_{1}$ is divisible by 4 . Show that $E$ is a congruence relation and that the homomorphism g defined is the natural homomorphism associated with E .
MODULE - IV
7. a) What is the solution of the recurrence relation $a_{n}=6 a_{n-1}-9 a_{n-2}$ for $n \geq 2$ given that $a_{0}=1, a_{1}=6$.
b) Find the recurrence relation for the Fibonacci sequence.
8. a) A computer system considers a string of decimal digits a valid codeword if it contains an even number of 0 digits. For instance, 1230407869 is valid, whereas 120987045608 is not valid. Let an be the number of valid n-digit codeword's. find the recurrence relation for an.
b) Find $r$ recurrence relation for Cn , the number of ways to parenthesize the product of $\mathrm{n}+1$ numbers, $x_{0}, x_{1} x_{2} \ldots x_{n}$, to specify the order of multiplication. For example, $C_{3}=5$ because there are five ways to parenthesize $\mathrm{x}_{0}, \mathrm{x}_{1} \mathrm{x}_{2} \mathrm{x}_{3}$ to determine the order of multiplication: $\left(\left(\mathrm{x}_{0} \cdot \mathrm{x}_{1}\right) \mathrm{x}_{2}\right) \cdot \mathrm{x}_{3}\left(\mathrm{x}_{0} \cdot\left(\mathrm{x}_{1}\right) \mathrm{x}_{2}\right) \cdot \mathrm{x}_{3}\left(\mathrm{x}_{0}\right.$. $\left.\left.x_{1}\right)\left(x_{2}\right) \cdot\left(x_{3}\right) x_{0} \cdot\left(\left(x_{1}\right) x_{2}\right) \cdot x_{3}\right) x_{0} .\left(x_{1}\left(x_{2} \cdot x_{3}\right)\right)$.

## MODULE - V

9 a) Prove that if $G$ is connected graph with $n$ vertices and ( $n-1$ ) edges then $G$ is a tree.
b) Show that the graphs G and H displayed in following Figure 1 are isomorphic.


G


H

Figure 1
10. a) Prove that the chromatic number of a tree is always $2 \&$ chromatic polynomial is $\lambda(\lambda-1)^{\mathrm{n}-1}$.
b) Show that neither graph displayed in following Figure 2 has a Hamilton circuit.


G


H

Figure 2

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## COURSE OBJECTIVES

The course should enable the students to:

| I | Describe the logical and mathematical foundations, and study abstract models of computation. |
| :---: | :--- |
| II | Illustrate the limitations of predicate logic. |
| III | Define modern algebra for constructing and writing mathematical proofs. |
| IV | Solve the practical examples of sets, functions, relations and recurrence relations. |
| V | Recognize the patterns that arise in graph problems and use this knowledge for constructing the <br> trees and spanning trees. |

## COURSE OUTCOMES (COs):

| CO 1 | To understand the concepts associated with Mathematical Logic and Predicate calculus |
| :--- | :--- |
| CO 2 | Ability to learn the basic concepts about relations, functions and to draw different diagrams like <br> Lattice, Hasse diagrams. |
| CO 3 | To understand the concepts of Algebraic Structures And Combinatorics . |
| CO 4 | To describe various types of recurrence relations and the methods to find out their solutions . |
| CO 5 | To understand the basic concepts associated with Graphs and Trees. |

## COURSE LEARNING OUTCOMES (CLOs):

| ACSB04.01 | Understand logical connectives and compound prepositions for building compound statements. |
| :--- | :--- |
| ACSB04.02 | Learn the formal symbols and use the preposition logic and predicate logic to solve problems on <br> logical equivalences and implications. |
| ACSB04.03 | Memorize different scientific notations to simplify the logical statements. |
| ACSB04.04 | Prepare valid arguments from the given propositional statements by using rules of inference. |
| ACSB04.05 | Identify ordered pairs to form a binary relation from the given sets. |
| ACSB04.06 | Construct directed graph and a matrix representation using a binary relation on finite order pairs. |
| ACSB04.07 | Identify the properties of relations to check for equivalence relation and partial order relation and <br> compute relations using operations on relations. |
| ACSB04.08 | Construct a hasse diagram to recognize the relevant partial ordered sets from the given binary <br> relation. |
| ACSB04.09 | Describe the types of functions (one to one, on-to, bijective, Identity and constant function). |
| ACSB04.10 | Implement the concept of the inverse and recursive functions to get an optimized solution for an <br> appropriate problem. |
| ACSB04.11 | Use the concept of lattices (Greatest Lower Bound (GLB) and Least Upper Bound (LUB) to <br> represent a defined finite set in multi- dimension applications. |
| ACSB04.12 | Explain about the properties and types of lattices (bounded and distributivelattice). |
| ACSB04.13 | Construct different algebraic structures by using concepts of groups, sub groups, monoids and <br> rings. |
| ACSB04.14 | Understand binomial and multinomial theorems to compute the coefficients for the given <br> expansions. |
| ACSB04.15 | Understand the concept of homomorphism and isomorphism of semi-groups. |
| ACSB04.16 | Analyze the given sets by using inclusion and exclusion principle. |


| ACSB04.17 | Identify the different counting techniques (permutations) related to mathematics and computer <br> science. |
| :--- | :--- |
| ACSB04.18 | Solve discrete probability and set problems by using permutations and combinatorics. |
| ACSB04.19 | Identify the series of expansion torepresent the sequence by using generatingfunctions. |
| ACSB04.20 | Identify the general solution for first-order and second-order linear homogeneous recurrence <br> relations. |
| ACSB04.21 | Identify the roots of second and higher order linear non-homogeneous recurrence relations. |
| ACSB04.22 | Understand the use of graphs and trees as representation tools in a variety of context. |
| ACSB04.23 | Identify Euler's and Hamilton rule for a simple connected graph in NP-complete problems. |
| ACSB04.24 | Construct a spanning tree by using search techniques (Depth First Search and Breadth First <br> Search). |
| ACSB04.25 | Construct a minimal spanning tree by using Kruskal's and Prim's algorithm in order to obtain a <br> solution for a real time problem. |
| ACSB04.26 | Possess the knowledge and skills for employability and to succeed in national and international <br> level competitive exams. |

## MAPPING OF SEMESTER END EXAM TO COURSE LEARNINIG OUTCOMES

| SEEQuestionNo |  | Course Learning Outcomes |  | Course Outcomes | $\qquad$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | a | ACSB04.02 | Learn the formal symbols and use the preposition logic and predicate logic to solve problems on logical equivalences and implications. | CO 1 | Understand |
|  | b | ACSB04.04 | Prepare valid arguments from the given propositional statements by using rules of inference. | CO 1 | Remember |
| 2 | a | ACSB04.01 | Understand logical connectives and compound prepositions for building compound statements. | CO 1 | Understand |
|  | b | ACSB04.02 | Learn the formal symbols and use the preposition logic and predicate logic to solve problems on logical equivalences and implications. | CO 1 | Understand |
| 3 | a | ACSB04.05 | Identify ordered pairs to form a binary relation from the given sets. | CO 2 | Remember |
|  | b | ACSB04.06 | Construct directed graph and a matrix representation using a binary relation on finite order pairs. | CO 2 | Remember |
| 4 | a | ACSB04.09 | Describe the types of functions (one to one, on-to, bijective, Identity and constant function). | CO 2 | Understand |
|  | b | ACSB04.09 | Describe the types of functions (one to one, on-to, bijective, Identity and constant function). | CO 2 | Understand |
| 5 | a | ACSB04.13 | Construct different algebraic structures by using concepts of groups, sub groups, monoids and rings. | CO 3 | Remember |
|  | b | ACSB04.13 | Construct different algebraic structures by using concepts of groups, sub groups, monoids and rings. | CO 3 | Remember |
| 6 | a | ACSB04.15 | Understand the concept of homomorphism and isomorphism of semi-groups. | CO 3 | Understand |
|  | b | ACSB04.15 | Understand the concept of homomorphism and isomorphism of semi-groups. | CO 3 | Understand |
| 7 | a | ACSB04.21 | Identify the roots of second and higher order linear non-homogeneous recurrence relations. | CO 4 | Remember |
|  | b | ACSB04.21 | Identify the roots of second and higher order linear non-homogeneous recurrence relations. | CO 4 | Remember |
| 8 | a | ACSB04.21 | Identify the roots of second and higher order linear non-homogeneous recurrence relations. | CO 4 | Remember |
|  | b | ACSB04.21 | Identify the roots of second and higher order linear non-homogeneous recurrence relations. | CO 4 | Remember |


| 9 | a | ACSB04.22 | Understand the use of graphs and trees as <br> representation tools in a variety of context. | CO 5 | Understand |
| :---: | :---: | :---: | :--- | :---: | :---: |
|  | b | ACSB04.22 | Understand the use of graphs and trees as <br> representation tools in a variety of context. | CO 5 | Understand |
| 10 | a | ACSB04.23 | Identify Euler's and Hamilton rule for a simple <br> connected graph in NP-complete problems. | CO 5 | Remember |
|  | b | ACSB04.23 | Identify Euler's and Hamilton rule for a simple <br> connected graph in NP-complete problems. | CO 5 | Remember |

